Electromagnetic Radiation from Wave-Guides and Horns

RECENTLY, Levine and Schwinger[1] have obtained a rigorous solution of the problem of radiation of sound from an unflanged circular pipe. Their method hinges on the solution of a certain integral equation, which they solve by a decomposition of its Fourier transform into two parts, each of which can be shown, from its behaviour at infinity, to be a constant. This method of decomposition seems to be successful in a large number of problems in which integral equations of a certain type appear, and I have applied it to the radiation of electromagnetic waves from rectangular wave-guides. A number of useful and interesting results have been obtained, which seem worth recording.

A rectangular wave-guide of dimensions $a \times b$ $(a > b)$ is fed by a dominant mode with the electrical polarization parallel to the side b. The guide is open at one end, which is unflanged. The voltage reflexion coefficient, r, at the mouth, can be found rigorously in rather simple forms when either of the sides is infinite.

If a is infinite, it is found that

$$r = \exp\left(-\tfrac{1}{2}kb\right) \exp j \left\{\frac{kb}{\pi}\left(\log\frac{kb}{4\pi} - 1 + \gamma\right) + 2\sum_{1}^{\infty}\left(\sin^{-1}\frac{kb}{2m\pi} - \frac{kb}{2m\pi}\right)\right\},$$

where $k = 2\pi/\lambda$, λ being the free space wave-length.

The infinite series is very rapidly convergent, only a few terms being needed, even when $b = \lambda$. The phase is shown plotted against b/λ in Fig. 1.

The result is of immediate use in the case of an unflanged horn of dimensions $D \times b$, located between parallel walls (for example, of a 'cheese' mirror) of separation D (polarization parallel to the walls). It is only necessary to modify the above expression by replacing k by $\sqrt{k^2 - \pi^2/D^2}$.

When the side b is infinite, the form for r is

$$r = \left(\frac{2a}{\lambda} - \sqrt{\frac{4a^2}{\lambda^2} - 1}\right) \exp\left(-\frac{\pi}{2}\sqrt{\frac{4a^2}{\lambda^2} - 1}\right) \times$$

$$\exp j \left\{-\frac{k'a}{\pi}\left(\log\frac{2\lambda}{a} + 1 - \gamma\right) + \tan^{-1}\frac{k'a}{\pi} + 2\sum_{1}^{\infty}\left(\sin^{-1}\sqrt{\frac{\frac{a^2}{\lambda^2} - \frac{1}{4}}{n^2 + n}} - \sqrt{\frac{\frac{a^2}{\lambda^2} - \frac{1}{4}}{n}}\right)\right\}$$

where $k' = \sqrt{k^2 - \pi^2/a^2}$.

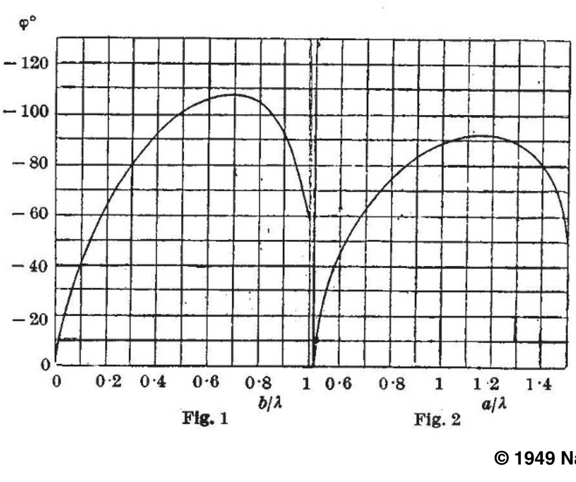

Fig. 1 Fig. 2

Again, the infinite series is very rapidly convergent. The phase is shown in Fig. 2 plotted against a/λ.

The result can be used in unmodified form if the wave-guide end takes the form of an unflanged horn of dimensions $D \times b$, located between two walls of separation D, the polarization being perpendicular to the walls.

It is hoped to publish full details of the calculations, together with further results for general size wave-guides, with openings with finite flanges, in the near future.

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[1] Levine, H., and Schwinger, J., *Phys. Rev.*, **73**, No. 4 (Feb. 15, 1948

Calibration of Electron-Sensitive Emulsions

CALIBRATION of Kodak *NT*4 photographic emulsions for β-particles has now been extended to an upper energy limit of 250 keV. The method used for calibration has been reported in an earlier letter in *Nature*[1]. New observations have been made at β-ray energies of 40, 60, 100, 147, 200 and 250 keV. in emulsion coatings of 200 microns thickness.

Mean values for the total range of the β-particle tracks and for the number of grains per track at a series of energy values are given in Table 1. The figures include all systematic observations to date.

The mean density of grains at different distances along the tracks from the last grain has been evaluated from all tracks in the highest two energy groups of Table 1 and from one track of 300 keV. energy. The mean number of branches per hundred microns of track has been determined from a larger group of 143 tracks having a total of 43 branches. Both sets of results are shown in Table 2. The count of grain number in any portion of a track includes the grains belonging to any branch produced in that portion.

All standard errors in Tables 1 and 2 are deduced from the corresponding standard deviations. The fact that the percentage standard deviations of grain counts are somewhat less than the percentage standard deviations of ranges justifies the use of grain counts for energy measurements, provided that the sensitivity and degree of development of the emulsion are tested by comparing grain counts with ranges for a limited number of tracks.

The calculated ranges shown in Table 1 have been obtained by graphical integration according to the equation

$$R = \int_0^E \left(1\Big/\frac{dE}{dR}\right) dE, \qquad (1)$$

using the Bethe relativity formula for the stopping power of hydrogen[2], generalized for heavy stopping atoms as suggested by Bethe[3], and adapted by the method of Cuer[4] for composite materials. The resulting formula, which is subject to the limitations of the Born approximation, is: