may depend on the outcome of the efforts of those whose names are listed at the end of this book, and all will surely join in wishing them every success in their vitally important work. F. C. BAWDEN

AERONOMY

The Upper Atmosphere

By Prof. H. S. W. Massey and Dr. R. L. F. Boyd. Pp. xii+333+28 plates. (London: Hutchinson and Co. (Publishers), Ltd., 1958.) 63*s.* net.

THE science of the upper region of the terrestrial atmosphere, where dissociation and ionization are important, has many facets. Because of the various interrelations that exist, none of these can be treated properly in isolation. Yet few people have the time and ability to become experts in the whole of this rapidly expanding science (which has been given the name 'aeronomy', following a suggestion by Prof. Sydney Chapman). For some years past there has been a need for an up-to-date book giving a survey of the entire field in simple terms so that a specialist in any one branch can readily obtain at least a general understanding of the work being done in other branches; for example, so that a radio man may learn something of the nightglow without bothering himself with the notation of molecular spectra, and so that a photochemist may learn something of the theory of the diurnal variations in the geomagnetic field without mastering the mathematics involved. There has also been a need for an elementary text which can be recommended to post-graduate students. Prof. Massey and Dr. Boyd do much to meet these needs by their admirable new book, "The Upper Atmosphere", though, regrettably, they do not include any references to the original literature.

The first two chapters are introductory in character. One of them recalls the basic principles of the relevant physics in elementary sections on electricity and magnetism, on wave motion of various types and on atoms and molecules; the other gives a short review of the main properties and phenomena of the upper atmosphere, anticipating the later, more detailed accounts.

These are followed by three chapters on the principal types of probe used—sound, radio and material. The chapter on material probes is particularly valuable in that it contains much information on balloons and rockets which has never been collected together before. Towards the end of the book there is a related chapter on artificial satellites, giving an interesting discussion of some of the important investigations which may be carried out with their aid.

The remaining chapters cover the following: the chemisphere and the ionosphere; the night-glow and the aurora; aerial tides and magnetic effects; solar, magnetic and ionospheric disturbances; meteors; cosmic rays. As would be expected, the exposition is lucid.

In addition to the six plates in colour and the twenty-two plates in black and white, there are a large number of instructive line drawings, many of them specially prepared. An excellent index adds considerably to the usefulness of the book, which should be in the library of every serious research worker in any branch of aeronomy.

D. R. BATES

DIOPHANTINE ANALYSIS

An Introduction to Diophantine Approximation

By Dr. J. W. S. Cassels. (Cambridge Tracts in Mathematics and Mathematical Physics.) Pp. x+166. (Cambridge: At the University Press, 1957.) 22*s.* 6*d.* net.

THE approximation to an irrational θ by rationals p/q depends on the distance between $q\theta$ and the nearest integer, a distance which Dr. J. W. S. Cassels writes as $\|q\theta\|$. This specifies the homogeneous problem; the inhomogeneous problem concerns $\|q\theta - \alpha\|$, and each problem can be generalized to that of simultaneous approximation to a finite number of θ's. In Chapters 1 and 3 of this Cambridge Tract, the classical material is neatly and efficiently displayed. Chapter 2, a little more difficult, deals with the curious Markoff 'chain', which disposes of one exceptional θ after another, and the related results about quadratic forms; this includes a new and hitherto unpublished 'isolation' theorem due to C. A. Rogers. In Chapter 4 on the distribution of the fractional part of $q\theta$ in the interval (0,1) or of the simultaneous fractional parts for a finite number of θ's, the method of trigonometrical sums, due to Weyl and developed by Vinogradoff, is employed. The next chapter discusses a type of duality between problems concerning related (transposed) sets of linear forms, first examined by Mahler. Chapter 6 is particularly interesting, since it deals with Roth's theorem, that if ξ is any algebraic number, that is, a root of an equation:

$$a_0u^n + a_1u^{n-1} + \ldots + a_n = 0$$

where the a's are integers, then there are only a finite number of pairs of integers p,q such that:

$$|\xi - p/q| < 1/q^k$$

if $k > 2$. Since there are infinitely many solutions if $k = 2$, Roth's result is an end to a story which began about fifty years ago with Thue's remarkable theorem, that the equation:

$$a_0x^n + a_1x^{n-1}y + \ldots + a_ny^n = c \quad (n \geqslant 3; \ c \text{ integral})$$

(where the form on the left is irreducible) cannot have an infinite number of integer solutions x,y. To obtain this, Thue used a weaker form of Roth's theorem, with $k = \frac{1}{2}n + 1$, a value improved by Siegel (1921) and Dyson (1947), until in 1955 Roth closed the gap completely with the best possible value of k; notably it is independent of n. The International Congress of Mathematicians, meeting at Edinburgh in August this year, recognized the brilliance of Roth's work by awarding him one of its two Fields Medals.

If $C > 0$, the inequality $\|q\theta\| < C/q$ has infinitely many solutions for almost all θ, that is, save for θ's forming a set of zero Lebesgue measure. This is the basis of Chapter 7, on metrical theory, developed in a series of papers by Dr. Cassels during 1950–51. The final, rather highly technical, chapter discusses the Pisot–Vijiyaraghavan numbers.

Dr. Cassels has selected his material carefully and expounds it with crisp clarity. The volume is a genuine introduction, for the reader requires no special knowledge of number theory; even in the final two chapters, only a very rudimentary acquaintance with Lebesgue measure and with algebraic numbers is supposed. Yet the author manages to bring us into effective contact with several important lines of present-day research, and his book must rank high among the famous Cambridge Tracts. T. A. A. BROADBENT