Galactic voids may be statistical

Excitement in the past few years about the apparent clumpiness of galactic clusters, leaving empty spaces in between, may be misplaced.

WHEN is a void real and not just a statistical artefact? The question has become topical and important because of recent reports that the Universe, statistically homogeneous though it may be, is also clumpy. There is no preferred direction in space, at least within the limits of experimental errors as they are, but nevertheless the visible matter is not merely collected into galaxies but the galaxies themselves appear to occur as clusters. More than that, as well as clusters there appear also to be voids, patches on the sky where there are no recognizable clusters of galaxies. Is this a phenomenon requiring a physical explanation? Or may the appearance of the voids be a statistical illusion which does not contradict the notion that galaxies and clusters thereof are distributed at random?

The circumstances are, or should be, familiar to cosmologists and astrophysicists. Observational bias is a recurring trouble. Whenever new techniques are used to search the sky, there is obviously a risk that the first novel objects to be found will have declared their presence only because they are exceptional. The only remedy is patience, but that is not as simple as it sounds. In spite of all the energy expended on the observation and interpretation of quasars in the past quarter of a century, for example, it is still not certain whether the apparent scarcity of quasars at a distance greater than that corresponding to a redshift parameter $z=3.5$ or thereabout implies either that quasars do not often form in more distant or younger galaxies or that there is an observational bias against the detection of more distant objects. The quasars are also the cause of a running battle between the US observer Arp and most other quasar watchers about his assertion that these objects turn out to be more often geometrically related to each other, in straight lines for example, than chance would allow.

The treatment of the problem of voids by David H. Politzer and John P. Preskill of the California Institute of Technology (*Phys. Rev. Lett.* **56**, 99; 1986) should be read by all those embroiled in arguments like these. Briefly, the conclusion is that the voids so far described on the surface of the sky are almost as likely to be consistent with randomness as with some physical process for concentrating galaxy clusters elsewhere, away from the voids. The curious feature of this article is that the underlying problem is so important and general, and the solution so simple, that one is bound to wonder why it has not been dealt with in a quite different context. At the very least, it will have cropped up as a problem for students to solve in a final examination at one of the better Victorian universities.

At first sight, indeed, there seems to be no problem. For the sake of definiteness, the case of the two-dimensional observable surface of the sky is a good place to start. The objects observed may be galaxies, or galaxies brighter than some arbitrary limit, or clusters, or clusters with redshifts between arbitrary limits. "Voids" will be only relatively empty; a patch of sky of area S will not be entirely devoid of objects, but, rather have fewer, perhaps many fewer, than most other patches of the same size. So the problem seems tractable by the elementary application of Poisson statistics. The chance that a patch of area S will contain k objects will be given by $(1/k)\ (nS)^k e^{-nS}$, where n is the average density of objects, just as it is the statistical distribution of the numbers of soldiers in the old Prussian army killed each year by being kicked by a horse. A void corresponds to the cases in which k is less than the average number in a patch of area S, given by nS.

The snag, unfortunately, is that a calculation that a specified patch in the sky will be a void, however defined, does not directly correspond to what people with telescopes actually observe. Physically, what matters is not that a specified patch on the sky should contain a void, but that the sky as a whole should contain some such patch. So the problem that matters is to calculate the probability that a void with predetermined shape and content will appear somewhere in the sky from the assumption that the objects being observed are distributed randomly. The trick, perhaps not surprisingly, has something in common with the process of swinging a telescope across the sky, searching for a void with characteristics specified in advance.

The principle is simple even if the algebra may be tedious. If a void is defined in advance as a circular patch that must contain no more than, say, k objects, the sensible course to follow is to cover the sky with a regular array of overlapping patches of that size, calculating the chance that one or more of them may correspond to a void, and then to allow the density of the overlapping patches to increase indefinitely. Technically, this is done by centring circles on the intersections of some kind of lattice structure whose spacings are then made indefinitely to shrink. The same kind of calculation can be carried through with other shapes of voids than circles.

Not surprisingly, the chance of finding a specified void somewhere in the sky turns out to be greater than that a particular patch will contain such a void. More significantly, numerical factors apart, the chances of finding predetermined voids are related to the Poisson distribution by factors of the form (nS) raised to the power two in the case of circular patches on a two-dimensional sky, but which may be raised to a higher power if the voids are elliptical in shape and allowed to assume any orientation. What this implies is that the presence of voids in the distribution of objects in the sky is certain to be much more common than has been assumed.

The argument applies directly to one of the first claims of the presence of a void in the distribution of rich galaxy clusters (Bachall, N. and Soneira, R. *Astrophys. J.* **262**, 419; 1982), which has been the stimulus for other such claims in the more recent literature. As Politzer and Preskill argue, the chance that the observed distribution has arisen from a random distribution of galaxies is something like 30 per cent, which goes to show that the occurrence, dramatic though it may appear, is nowhere out of the ordinary.

There is, of course, no reason why anybody should be upset by this turn of events. What stands out a mile is that simple expectations of the consequences of the random distribution of objects on the sky are unreliable. The surprise is that the accurate calculation has not been made before this. And even if it should be that voids are less remarkable than they have seemed in the past few years, that does not imply that they are entirely without interest.

For galaxies are not distributed at random. Large clusters are gravitationally bound and presumably collapsing. Much of the immediate interest in this field is that of measuring the dynamics of the clusters, which may throw light on their origin. This why Politzer and Preskill want to turn their problem around, and use the measured distribution of voids as a way of inferring something of non-random distribution of galaxies and of clusters of galaxies.

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