tionship described by equation (1), because M is always positive, it is immediately clear that, rather than resulting in a smaller F_i, the addition of magnetic field increases the needed magnitude of F_i. This implies the presence of even more unseen mass than in the unmagnetized case. Thus the addition of a magnetic field does not alleviate the dark matter problem; in this simplest case it exacerbates the problem.

The preceding analysis, couched in terms of an isolated (except for gravity) system, with $S = 0$, is that considered by Battaner *et al.* Now we turn briefly to the full virial equation, including the surface stresses contained in S of equation (1). The simple consequences, derived in the previous paragraph, of the virial relationship applied to the peripheral gas could be evaded by magnetic or pressure stresses, S, transmitted across the bounding surface.

This could be accomplished by a magnetic field anchored to a massive central object, but this may be shown to lead to a very large required field, contrary to observations (J. R. J. and E. H. L., manuscript in preparation). Alternatively, the expansive magnetic stress may be confined by the pressure of an external medium. A confining magnetic field would have to be a cosmological entity as strong as the confined magnetic field itself, filling the entire Universe, otherwise dynamical problems of confinement, similar to those described above, would simply arise on a larger scale. Or one could imagine high-pressure intergalactic gas (or cosmic rays) providing the pressure to confine the magnetic field. In either case the external confining pressure would have to be as high as the confined pressure to provide the needed virial equilibrium.

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SIR — Battaner *et al.*[1] argue that the flat rotation curves of galaxies such as M31 may be caused not by dark matter, but by magnetic tension. In essence they suggest that the centripetal force required to hold together a rapidly spinning disk of gas is provided by the tension of an azimuthal magnetic field B_Φ rather than by the inward gravitational attraction of dark matter. Here we show that a field strong enough to alter significantly a galaxy's rotation curve will cause the gaseous disk to flare unacceptably.

Because gas pressure makes a negligible contribution to the equation of horizontal equilibrium, the circular speed v_c, the streaming velocity of gas v_Φ and the magnetic field B_Φ are coupled by

$$v_\Phi^2 - v_c^2 = \frac{1}{2\mu_0 R^2 \rho} \frac{\partial}{\partial R}(R^2 B_\Phi^2) \quad (1)$$

where R is the galactocentric radius and ρ is the gas density. The magnetic field imposes two sorts of forces: azimuthal tension and pressure in both the radial and the vertical directions. The tension always imposes a net inward force, and thus tends to increase v_Φ. The radial pressure force $\propto \partial B_\Phi^2/\partial R$ acts inwards or outwards depending on whether B_Φ increases outwards or inwards. When $B_\Phi \propto 1/R$, the radial forces arising from tension and pressure balance exactly and $v_\Phi = v_c$.

As Battaner *et al.* remark, the field strength plotted in their Fig. 1 asymptotes to this form at large R. Consequently, this field makes a negligible contribution to v_Φ, contrary to the claims of Battaner *et al.* (see our Fig. 1).

We find that to make a field of the order

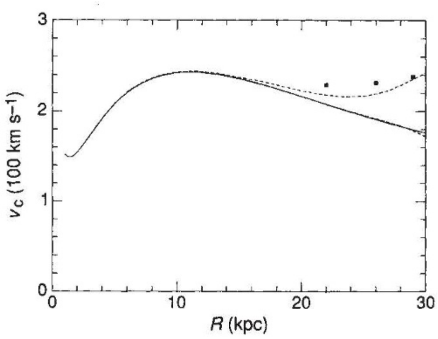

FIG. 1 Circular speed of Kent's maximum disk model[2] of M31 (full curve) and v_Φ calculated from $B_\Phi(R)$ as given in Fig. 1 of ref. 1 (lower dashed curve). The upper dashed curve shows $v_\Phi(R)$ for a model in which the B_Φ is twice as large as in ref. 1 for $R < 15$ kpc, and constant for $R > 15$ kpc. Data points from ref. 3.

of $B_\Phi \simeq 10\,\mu G$ advocated by Battaner *et al.* dynamically significant, the asymptotic magnetic pressure must be directed outwards with the result that both it and the tensile force tend to increase v_Φ (Fig. 1).

Magnetic pressure thickens the disk vertically by pushing gas upwards with a force $\propto \partial B_\Phi^2/\partial z$, which vanishes only in the totally implausible case in which B_Φ^2 does not fall off with increasing z. In reality one expects B_Φ^2 to fall off roughly with the gas density ρ. In this case the vertical pressure

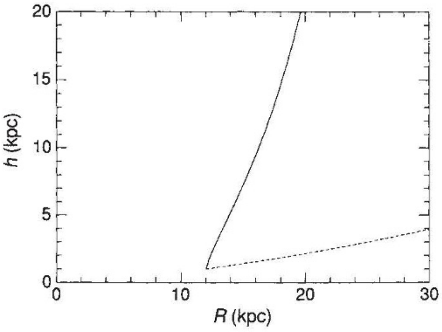

FIG. 2 Scale height $h(R)$ in the M31 model of ref. 1 if the rotation curve is held constant in the keplerian regime by a magnetic field (full curve) or is allowed to drop as it would in the absence of a magnetic field (dashed curve). The mass $M = 10^{11}\,M_\odot$ has been inferred from the measured luminosity $L_B = 2.5 \times 10^{10}\,L_\odot$ (ref. 2) and mass-to-light ratio $\gamma_B = 4$ (refs 4, 5). Disk scale length is $R_d = 4.3$ kpc as in ref. 1.

force is at least an order of magnitude greater than the radial tensile force $\sim B_\Phi^2/R$, as $R >> h$, the disk scale-height.

Figure 2 confirms this qualitative analysis by plotting $h(R)$ for $R \geqslant 12$ kpc as derived for M31 from the field strength shown in Fig. 1 of Battaner *et al.* from equation (1) above. Specifically, we have taken the gas to be isothermal with $B_\Phi^2 \propto \rho$, and solved equation (1) for $B_\Phi^2(R,0)$ under the Battaner *et al.* assumptions of a constant circular speed $v_c = 240$ km s^{-1} and a keplerian mass distribution. The gas temperature is implicitly determined by requiring $h(12\text{ kpc}) = 1$ kpc. Once $B_\Phi^2(R,0)$ is known, $h(R)$ follows immediately. For comparison, the dotted curve shows the variation of the scale height expected in the absence of a magnetic field for one choice of $h(12\text{ kpc})$.

The upward surge of the full curve in Fig. 2 demonstrates that a magnetic field large enough to change a keplerian rotation curve into a flat one will cause the gas disk to flare unacceptably. The origin of this thickening is simply that a magnetic field provides pressure in two directions and tension in only one. If the field's azimuthal tension is dynamically important, its pressure will blow the disk apart vertically. We have demonstrated the violence of this effect in only one illustrative case, but it is intuitively clear that the effect is a perfectly general one.

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