A helping hand

"Grossmann, you have to help me, or else I'll go crazy!"

One hundred years ago last month, a research paper appeared in the journal *Zeitschrift für Mathematik und Physik*, co-authored by a Swiss mathematician from ETH Zürich. Written with a physicist colleague, the paper presented the results of a two-year collaborative effort, which had flowered from a friendship formed when the two men were students some years before. Their paper of January 1914 stands as a historical monument to the power of timely collaboration.

The physicist lacked the mathematical knowledge to give his deepest insights concrete form. The mathematician helped to show him how, bringing recent mathematical theories to bear. The result was a paper entitled 'Outline of a general theory of relativity and a theory of gravitation'.

Most physicists will have guessed by now that the physicist in question was Albert Einstein. The mathematician was one Marcel Grossmann. History records that Einstein discovered general relativity in complete form in 1915, yet what many physicists might not know is just how significant a role Grossmann played in giving that theory its final form. Indeed, the Einstein–Grossmann paper of 1914 came within a hair's breadth of describing the correct equations of general relativity. "We had come at that time already very close to the solution," Einstein later wrote. "At the time we lacked only a few relations of a formal nature, without which the link between the formulas and already known laws cannot be attained."

Even today, however, no one has written a biography of Marcel Grossmann. He never claimed to be a co-inventor of the theory, merely a helper. He and Einstein remained friends for two decades until Grossmann's death in 1936. The historical evidence — as historian Tim Sauer points out in a new examination of Einstein's notebooks and other documents of the period (http://arXiv.org/abs/1312.4068) — clearly shows that Grossmann played a hugely influential role in pushing Einstein towards his final theory.

Open almost any modern textbook on general relativity and you'll encounter several chapters of mathematical preliminaries. For example, a popular undergraduate textbook by Bernard Schutz starts out covering topics such as covariant and contravariant tensors, measures of curvature on differentiable manifolds, the covariant derivative and more. This mathematical machinery and language, Sauer argues, was originally brought into the theory largely by Grossmann. He not only pointed Einstein to crucial mathematical papers — such as a 1901 article on the absolute differential calculus by Gregorio Ricci-Curbastro and Tullio Levi-Civita — but took real steps in forging the emerging theory into these terms.

Indeed, it seems it was Grossmann who kicked off the use in physics of the now ubiquitous term 'tensor', favouring the idea that equations written in terms of tensors would be manifestly of the same form in all coordinate systems. Sauer notes that Ricci-Curbastro and Levi-Civita had never used 'tensor' in this context, instead calling these objects "systèmes covariants ou contrevariants". Grossmann, Sauer's study of the notebooks suggests, "actually helped clarify the very mathematical status of the objects that were entering the centre stage of their theoretical efforts."

Einstein's notebooks make pointed reference to Grossmann's contributions to the development of the mathematical formalism. On one page, where an expression appears for the Riemann–Christoffel tensor — an object with four indices capturing the local curvature of a manifold — Einstein has written "Grossmann tensor of fourth manifold". It was Grossmann who showed how to reduce this tensor to an object with only two indices that would be suitable for a gravitational field equation, pointing the way to the correct mathematical objects needed to develop fully covariant field equations.

I'm sure none of this is news to physicists working with general relativity, who have long known of Grossmann's contributions. A prominent series of international conferences even bears his name. Yet it might also be said that Grossmann not only helped forge general relativity, but was instrumental in changing the mathematical language of physics. His tensor notation left a mark not only on the form of general relativity, but indeed on all fundamental theories developed since. Every physicist studying the rudiments of quantum field theory learns to write down Lagrangian functions in manifestly covariant form, thereby ensuring the relativistic invariance of the resulting theory.

It may be, of course, that someone in the role of Grossmann would have existed, even if Grossmann himself had not. Gravity is intimately linked with spacetime curvature and differential geometry. Even if Einstein had failed to see precisely how, someone else presumably would have eventually succeeded, learning the right mathematical language on their own, or meeting someone, like Grossmann, who could help. This kind of mathematics — a step up in abstraction from the mathematics of classical mechanics — was increasingly in the air.

At the time of Grossman and Einstein, Emmy Noether was making major advances in abstract algebra and group theory, and bringing these into the foundations of field theory. Following general relativity came the Hilbert-space formulation of quantum theory, Paul Dirac's relativistic quantum theory, field theories with broken symmetries and the rise of mathematical physics as the guiding light of physics. Grossmann's imprint is all over today's theories of cosmology and string theory.

After his work with Einstein, Marcel Grossmann did further research in synthetic and non-Euclidean geometry, but also became very active outside of research. In the years following the First World War, he helped found and run a twice-weekly magazine promoting open debate on Swiss national affairs and spent considerable energy in efforts to reform the Swiss education system. Beneficial education, in his view, would not only instil specific knowledge but should "create a frame of mind that enables the young person at the end of his studies to swim about without swimming coach and without swimming rings, even when the current would flow in unexpected directions."

He remained all the time a staunch believer in broad education, perhaps instructed from his own experience with Einstein of the vast potential for a little knowledge from other fields to make a big difference.

When Einstein first approached his friend, seeking help in 1912, he did so with the following words: "Grossmann, you have to help me, or else I'll go crazy!" It is fortunate for physics that Grossman helped. ❐

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