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Multilingual SEIR public opinion propagation model with social enhancement mechanism and cross transmission mechanisms

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The propagation of public opinion in multilingual environments presents unique challenges due to the diversity of languages, cultures, and values. This study develops an SEIR-based model tailored for multilingual contexts, incorporating mechanisms such as social enhancement, forgetting, and cross-transmission. The model's purpose is to improve transparency, inclusivity, and effectiveness in public opinion management, particularly in diverse linguistic settings. By emphasizing democratic engagement and avoiding social control, the model provides tools for managing public opinion that promote fairness and transparency. The model was validated using real Twitter data related to COVID-19 across multiple languages, including English, Spanish, and Catalan. Key results demonstrate that the model effectively captures the dynamics of opinion propagation, particularly in languages with fewer users, where opinion spread tends to be more predictable. By addressing cultural and linguistic differences, this study offers an inclusive approach to public opinion management. The inclusivity ensures that different cultural groups, regardless of language, are fairly represented in public discourse. This research contributes to the ethical management of public opinion, providing valuable insights for policymakers and analysts in multilingual societies.

Keywords Multilingual public opinion, SEIR model, Social enhancement effect, Cross-transmission mechanism, Optimal management strategy

Public opinion encompasses the collective views, attitudes, and sentiments of a society regarding events or issues. As a social psychological phenomenon, public opinion both reflects a society's needs, contradictions, and trends, and significantly influences its decisions, behaviors, and development. The propagation of public opinion involves the formation, diffusion, and evolution of societal viewpoints, engaging multiple stakeholders and media channels, as well as various influencing factors. Characterized by its breadth, speed, diversity, and unpredictability, the propagation of public opinion facilitates communication, coordination, and progress, while also potentially triggering social conflicts, turmoil, and crises.

Owing to their similarities to information transmission processes, epidemiological mathematical models are widely applied in the study of online public opinion propagation.[1] The study of information propagation dynamics originated from the analysis of rumor spreading. Allport,[2] a pioneer in rumor propagation research, analyzed the process primarily from a psychological perspective. In 1964, Goffman and Newill observed that the spread of rumors and infectious diseases shares similarities.[3] Following the DK model,[4] research into information propagation dynamics expanded to include Maki and Thomson's MT model,[5] and its subsequent modifications by Sudbury[6] and Belen.[7] Recent advances in complex network research have opened new avenues for studies in communication dynamics. Pastor-Satorras and Vespignani[8] conducted effective analyses of infectious disease propagation across regular, random, and small-world networks. Zanette,[9,10] a pioneer in the study of rumor propagation within complex networks, highlighted a propagation threshold in small-world networks and the significant influence of network structures on this process. The diverse topologies of complex networks have spurred significant interest and yielded substantial findings in rumor propagation dynamics. This includes models for information propagation across various complex networks such as random, small-world,

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homogeneous, heterogeneous, large-scale scale-free networks, those with adjustable clustering coefficients, time-varying, and multilayer networks.[11–24]

With the advent of globalization and informatization, multilingualism, defined as the coexistence and interaction of diverse language groups, has become prevalent in society. Public opinion propagation in multilingual environment becomes increasingly complex and uncertain, as varying cultures, values and cognitive styles among language groups contribute to the diversity, differentiation, and polarization of public opinions. Particularly since the onset of the big data era, the integration of artificial intelligence with big data technologies has given rise to the current AIGC technology. The impact of AIGC technology on public opinion propagation is multifaceted, involving: (1) enhanced content production efficiency and scale, reduced labor costs, met demands for vast information volumes, and boosted content quality and innovativeness, thereby strengthening communicative power and influence; (2) enabled multi-modal, multi-language, and multi-scene content generation, broadening communication reach and facilitating cross-cultural, cross-regional, and cross-platform exchanges. Therefore, in the AIGC era effectively describing, analyzing, guiding, and regulating public opinion in multilingual environments represents significant challenges in public opinion management.

Public opinion is a complex and dynamic phenomenon that plays a critical role in shaping societal behavior, policies, and governance. In multilingual environments, the propagation of public opinion becomes more intricate due to the interplay of different languages, cultures, and values. This paper develops a public opinion management strategy aimed at optimizing transparency, inclusivity, and effectiveness in guiding public discourse, especially in multilingual contexts.

As noted in references,[25–34] existing studies often overlook the social enhancement effect. Neglecting this effect in the process of information dissemination in a multilingual environment can lead to significant miscalculations—either underestimating or overestimating the effectiveness, impact, and persistence of information. This oversight can distort public opinion analysis, resulting in either an overemphasis or an underestimation of the positive or negative consequences of information spread, and ultimately, a loss of control over the management of public opinion.

To address these issues, this article proposes a dynamic model of Public Opinion Propagation that incorporates social enhancement effects. By utilizing both the forgetting mechanism and the cross-propagation mechanism, the model evaluates how public opinion interacts with different language groups, assessing its stability and sensitivity. The proposed strategy, based on optimal management theory, is designed to effectively guide and regulate public opinion by adjusting network structures and parameters.

Numerical simulations confirm the feasibility and applicability of this model, offering an innovative approach for managing public opinion in multilingual environments. The model has practical applications, particularly in public service contexts where managing public opinion is crucial. During public crises—such as natural disasters or public health emergencies—this model can ensure accurate information dissemination and prevent panic. By emphasizing guidance over management, the model supports public service departments in mitigating misinformation and maintaining stability in multilingual societies.

This study also considers the challenges posed by linguistic and cultural diversity in public opinion dynamics. By accounting for these differences, the model promotes an inclusive approach to public opinion management, ensuring that various cultural groups, regardless of language, are fairly represented in public discourse. This approach not only adheres to ethical standards but also enhances the model's relevance and applicability to real-world multilingual societies. The main contributions and innovations of this paper include:

(1) Social Enhancement Effect: This model introduces parameters describing social influence among speakers of different languages. This effect facilitates or inhibits the spread of public opinion across linguistic communities.
(2) Forgetting and cross-transmission Mechanisms: This model integrates a forgetting mechanism that reflects how information disappears over time and a cross-transfer mechanism that evaluates interactions between different language groups, significantly improving the applicability and predictive accuracy of the model.
(3) Optimal management Theory: management strategies based on optimal management theory aim to adjust network structure and parameters, effectively induce and manage public opinion, while minimizing costs and risks.

The paper is organized as follows: Chapter 2 presents the SEIR-based public opinion propagation model, incorporating social enhancement, forgetting mechanisms, and cross-transmission effects in multilingual settings. Chapter 3 explores the model's basic properties, focusing on the parameters influencing opinion transmission and the mathematical foundation of the model. Chapter 4 analyzes the model's stability and global dynamics in a bilingual environment, examining equilibrium points. Chapter 5 proposes optimal strategies for managing public opinion, using control theory to mitigate crises in multilingual societies. Chapter 6 describes simulation experiments that validate the model's accuracy and the effectiveness of the proposed management strategies. Chapter 7 applies the model to a real-world case study using multilingual Twitter data, validating the model's performance in capturing opinion dynamics. Chapter 8 summarizes key findings, emphasizing the model's applicability in multilingual settings and its practical implications for public opinion management.

Model building

This section introduces a new SEIR public opinion propagation model for multilingual environment, incorporating social enhancement effects, forgetting mechanisms, and cross-transmission mechanisms.

It is assumed that the total population is divided into groups based on language spoken. For instance, the first group comprises speakers of the first language, the second group speakers of the second language, and so forth. Additionally, public opinion is considered to be transmitted through n different languages, where n is a finite positive integer. Each group is categorized into four types: (1) susceptible, individuals who have not yet encountered public opinion information but are at risk; (2) latent, individuals exposed to public opinion information without significant attitudes or behaviors; (3) infectious, individuals who display strong attitudes or behaviors after exposure to public opinion, influencing others; (4) recovered, individuals previously exposed to public opinion but now disengaged for various reasons. At time t, the susceptible, latent, infectious, and recovered individuals in group i are represented as $S_i(t)$, $E_i(t)$, $I_i(t)$, and $R_i(t)$, respectively. In group i, the total population $N_i(t)$ is given by the sum $S_i(t) + E_i(t) + I_i(t) + R_i(t)$. The dynamics of the model are illustrated in Fig. 1, given that some individuals are multilingual, communication across different groups is possible. Based on the preceding analysis, the SEIR public opinion propagation model in a multilingual environment is described as follows:

$$\begin{cases} \dfrac{\mathrm{d}S_i(t)}{\mathrm{d}t} = b_i - \displaystyle\sum_{j=1}^{n} \beta_{ij} I_j(t) S_i(t) - d_i S_i(t), \\ \dfrac{\mathrm{d}E_i(t)}{\mathrm{d}t} = \displaystyle\sum_{j=1}^{n} \beta_{ij} I_j(t) S_i(t) - \alpha_i E_i(t) - d_i E_i(t), \\ \dfrac{\mathrm{d}I_i(t)}{\mathrm{d}t} = \alpha_i E_i(t) - \left(\gamma_i + \displaystyle\sum_{j=1}^{n} \mu_{ij} + \sum_{j=1}^{n} \eta_{ji} + d_i\right) I_i(t) + \displaystyle\sum_{j=1}^{n} \eta_{ij} I_j(t), \\ \dfrac{\mathrm{d}R_i(t)}{\mathrm{d}t} = \gamma_i I_i(t) - d_i R_i(t). \end{cases} \tag{1}$$

where, $i = 1, 2, |\cdots, n$ denotes the population of the i-th language and n is the number of language varieties. The initial conditions of system (1) can be set to $S_i(0) > 0$, $E_i(0) > 0$, $I_i(0) > 0$, $R_i(0) > 0$. Incorporating the birth rate b_i and removal rate d_i of populations speaking language i into the bilingual SEIR public opinion propagation model addresses the dynamic nature of user populations on social media platforms. On platforms like Twitter, user engagement is in constant flux, with new users joining discussions and others disengaging over time. This dynamic is particularly crucial in a bilingual environment, where the interaction between different linguistic groups can significantly impact the spread of public opinion. By modeling b_i and d_i as time-dependent variables, the model captures these fluctuations, offering a more accurate reflection of the real-world propagation process. This approach allows for a deeper understanding of how public opinion evolves, especially in the long term, where the entry and exit of individuals within each linguistic group influence the sustainability and reach of information dissemination. This enhanced realism provides a nuanced perspective on how public opinion spreads across multilingual social networks, which is critical for developing effective opinion management strategies. The parameters in the model are summarized as shown below:

b_i: Birth rate of the population speaking language i.

d_i: Removal rate of the population speaking language i.

β_{ii}: Transmission rate of language i, defined as the probability that a susceptible individual becomes a latent individuals carrier upon contact with an infectious individuals.

β_{ij}: Probability that the population speaking language i is influenced by the population speaking language j, resulting in the conversion of susceptible individuals into latent individuals, also known as the cross-transmission rate.

α_i: Probability that a carrier within the population speaking language i becomes an infectious individual.

γ_i: Recovery rate of infectious individuals in the population speaking language i.

μ_{ij}: Forgetting rate of infectious individuals in the population speaking language i, applicable when $i = j$,$u_{ij} = 0$.

η_{ij}: Social enhancement effect of the population speaking language j on the population speaking language i, defined as the effect on the change in numbers of latent individuals and infectious individuals between these populations, applicable when $i = j$, $\eta_{ij} = 0$.

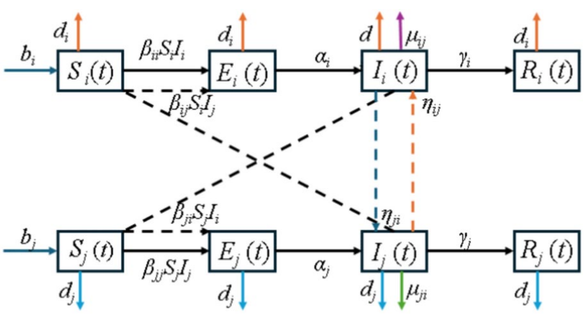

Fig. 1. Schematic of the propagation dynamics of the model.

For the birth rate b_i and removal rate d_i in the model, these two parameters are a function of time t in terms of the relatively long propagation period of the public opinion propagation problem, and we discuss the laws of these two parameters over time in the following:

$$b_i = \alpha_i \cdot H\left(t\right) + \chi_i \cdot U_i\left(t\right) + \sum_{j \neq i} \theta_{ij} \cdot C_{ij}\left(t\right),$$

where:

α_i: The weight of opinion trends influencing the growth of users speaking language i.
$H\left(t\right)$: The function representing changes in opinion intensity over time.
χ_i: The weight of user engagement affecting the growth of users speaking language i.
$U_i\left(t\right)$: The engagement level of users speaking language i as a function of time.
θ_{ij}: The cross-linguistic influence coefficient, representing the impact of language j on the birth rate of language.
$C_{ij}\left(t\right)$: The cross-linguistic interaction function, quantifying how language j influences users of language i. This function can be influenced by factors such as linguistic similarity, cultural proximity, and interaction frequency between users of different languages.

$$d_i = \gamma_i \cdot F\left(t\right) + \upsilon_i \cdot \left(1 - U_i\left(t\right)\right) + \sum_{j \neq i} \rho_{ij} \cdot D_{ij}\left(t\right),$$

where:

γ_i: The weight of opinion trend declines with the removal of users speaking language i.
$F\left(t\right)$: The decay function of opinion intensity over time.
υ_i: The weight of user interest decline on the removal of users speaking language i.
$\left(1 - U_i\left(t\right)\right)$: The level of declining interest among users speaking language i.
ρ_{ij}: The cross-linguistic disengagement coefficient, indicating the impact of language j on the removal rate of language i.
$D_{ij}\left(t\right)$: The cross-linguistic disengagement function, measuring how interactions with language j lead to the disengagement of users speaking language i. Influencing factors include linguistic differences, cultural conflicts, or conflicting viewpoints.

The following is a further detailed discussion of the functions of opinion fervor over time, language i user engagement over time, cross-language interaction functions, functions of opinion fervor decay over time, and cross-language disengagement functions involved in the birth rate time function and removal rate time function.

Public opinion heat as a function of time $H\left(t\right)$.

Opinion heat usually reflects how much attention a topic receives on social media, which can be measured by the number of tweets, search index, etc. We can use exponential growth or decay models combined with a saturation function to represent it:

$$H\left(t\right) = \frac{H_0 \cdot e^{r \cdot t}}{1 + \frac{H_0}{K}\left(e^{r \cdot t} - 1\right)},$$

Among them:
H_0: Initial public opinion heat.
r: Growth rate of public opinion heat.
K: The saturation value of public opinion heat.
t: Time.

The formula indicates that the heat of public opinion grows rapidly initially and then gradually becomes saturated.

language i user engagement over time $U_i\left(t\right)$.

User engagement can be measured by the activity levels of users, such as the frequency of posts, comments, and retweets. Considering that engagement may gradually increase over time and then decrease as opinion intensity declines, we can use the following formula:

$$U_i\left(t\right) = U_{i,0} \cdot \left(1 - e^{-\lambda_i \cdot H(t)}\right),$$

where:

$U_{i,0}$: Maximum engagement level of users speaking language i.
λ_i: The sensitivity of user engagement to changes in the heat of public opinion.
$H\left(t\right)$: Opinion heat as a function of time.

This formula indicates that user participation increases when the public opinion is hot and gradually decreases when the public opinion is hot.

Cross-Language Interactive Functions $C_{ij}\left(t\right)$.

Cross-language interaction can be expressed as the frequency and extent of communication between users of different languages, and can be constructed based on the speed at which public opinion spreads between two language groups:

$$C_{ij}\left(t\right)=\varsigma_{ij}\cdot H_j\left(t\right)\cdot\zeta_{ij},$$

where:

ς_{ij}:The influence coefficient of language j on the spread to language i.

$H_j\left(t\right)$:Opinion intensity of language j.

ζ_{ij}:Linguistic and cultural similarity between languages i and j(ranging from 0 to 1).

This function indicates that the heat of public opinion in the language j and the similarity between the two languages together determine the intensity of cross-linguistic interactions.

Decay of Public Opinion Heat as a Function of Time $F\left(t\right)$.

The decay of public opinion fervor can usually be represented by an exponential decay function to reflect the gradual waning of interest in a topic:

$$F\left(t\right)=F_0\cdot e^{\mu\cdot t},$$

Among them:

F_0: The initial public opinion heat.

μ: The public opinion heat decay rate, which reflects the speed of public opinion heat decline.

t:Time.

Cross-language detachment functions $D_{ij}\left(t\right)$

$$D_{ij}\left(t\right)=\delta_{ij}\cdot\left(1-\zeta_{ij}\right)\cdot H_j\left(t\right),$$

Among them:

δ_{ij}: Influence coefficient of language j on the removal of users speaking language i.

$\left(1-\zeta_{ij}\right)$: Linguistic and cultural variability (opposite of ζ_{ij}) .

$H_j\left(t\right)$: Opinion intensity of language j.

This formula indicates that when the linguistic and cultural differences between two languages are large, and the opinion intensity of language j is high, users speaking language i are more likely to disengage from the discussion.

In the model, if $\eta_{ij}>0$, the population speaking language j facilitates public opinion propagation in the population speaking language i. If $\eta_{ij}<0$, it suppresses public opinion propagation in the population speaking language i. If $\eta_{ij}=0$, it has no effect on the public opinion propagation in the population speaking language i. The parameter η_{ij}, which defines the social enhancement effect, is detailed as follows:

$$\eta_{ij}=k_{ij}.C_{ij}\left(\frac{I_j}{N_j}-\theta\right) \tag{2}$$

Among them:

k_{ij}: This parameter represents the base influence ability of language group j on the public opinion propagation of language group i, in the absence of other factors such as cultural differences and social networks. when $k_{ij}>0$, a higher value signifies stronger base influence, with key factors including language similarity, communication frequency, and interaction history.

C_{ij}: This parameter quantifies the cultural proximity between two language groups, affecting opinion propagation. Values range from 0 to 1, where a value closer to 1 indicates greater cultural similarity and more efficient opinion propagation. Influential factors include shared values, beliefs, customs, and art forms.

$\frac{I_j}{N_j}$: This represents the proportion of infectious individuals in language group j.

θ:This threshold parameter, denoted as θ, regulates the social enhancement effect—whether it promotes, inhibits, or has no effect on public opinion propagation—based on the proportion of infectious individuals. If this proportion exceeds θ, the effect is facilitating; below θ, it is suppressing; at θ, it has no effect.θ is established using historical data, expert judgement, or predefined psychosocial thresholds.

This formula accounts for the complexity and dynamics of communication effects across different language groups. By adjusting parameters k_{ij},C_{ij} and θ, it allows for precise management over the direction and magnitude of the social enhancement effect on opinion communication between these groups.

The forgetting mechanism used in this model is inspired by research in cognitive psychology, which shows that memory decay is a natural part of information retention. Baddeley's[35]model of working memory and subsequent studies have demonstrated that individuals are more likely to forget information when cognitive load increases or when the information is no longer relevant. This mechanism is particularly important in multilingual environments where different linguistic groups may have varying levels of exposure to the same information, influencing how quickly it is forgotten. In multilingual environment, the forgetting mechanism is influenced not only by time but also by interactions among different language groups. Consequently, the forgetting mechanism parameter μ_{ij} includes the following key components:

μ_i: represents the base forgetting rate for the language group i, which is the natural rate at which individuals forget information absent external influences. Influencing factors include cognitive ability, information complexity, and exposure frequency. This ratio is usually between 0 and 1, where 0 indicates no forgetting and 1 indicates immediate forgetting. Previous studies have extensively investigated the impact of basic forgetting mechanisms on information spreading, but have not discussed this in more detail.[36–41]

ϖ_{ij}: Indicating the adjustment of forgetting rate through cross linguistic social enhancement effect, that is, the forgetting rate of language group i is influenced by the social enhancement effect of language group j. The factors that affect this parameter include frequency of communication, mutual understanding, and resonance of opinions. Due to significant cultural differences, positive factors may promote forgetting, while negative factors may delay forgetting due to profound empathy or understanding.

C_{ij}: We quantify the cultural proximity between two language groups by evaluating how cultural similarity affects the dissemination of public opinion. Values range from 0 to 1, where values nearing 1 indicate closer cultural alignment and more efficient opinion spread. Influential factors include shared values, beliefs, customs, and artistic expressions, among others.

Q: represents the information quality factor, denoting how the quality of opinion information—specifically its credibility and relevance—affects the forgetting rate.

Considering the above elements, the equation defining the forgetting mechanism parameter μ_{ij} is as follows:

$$\mu_{ij} = \mu_i + \lambda\left(\varpi_{ij} \cdot C_{ij} - Q\right) \tag{3}$$

where λ represents a scaling factor that adjusts the influence of cross-language social enhancement and cultural similarity on the base forgetting rate.ϖ_{ij} can assume positive values to facilitate forgetting or negative values to slow forgetting, depending on the nature of the interaction between the two languages.C_{ij} measures the effect of cultural similarity, typically ranging from 0 to 1.Q represents the effect of information quality, where higher quality information is likely to be remembered longer, thereby slowing the forgetting rate.

The cross-propagation mechanism in this model draws from communication theory, particularly the two-step flow of communication model by Katz and Lazarsfeld[42]. This theory suggests that opinion leaders play a key role in spreading information across different social groups. In multilingual environments, linguistic and cultural differences add another layer of complexity to this process. As noted by Triandis[43], cultural proximity between groups can either facilitate or inhibit the spread of public opinion. This model incorporates these factors to simulate how opinions propagate between different language groups, taking into account the linguistic and cultural barriers that may exist.

In the study of multilingual Public Opinion Propagation, traditional models have focused on factors such as rumor spreading and network dynamics. However, these models often overlook critical aspects of human behavior, including how individuals forget information over time (memory decay) and how opinions spread across different language groups (cross-propagation). Research in cognitive psychology, such as Baddeley's work on memory, demonstrates that forgetting is influenced by several factors including cognitive load and emotional salience. For example, Anderson and Schooler[44]showed how frequency and relevance of information affect how long individuals retain it. These findings are particularly important when modeling how public opinion evolves over time in multilingual environments. Additionally, communication theory, such as the two-step flow of communication model proposed by Katz and Lazarsfeld, highlights the importance of opinion leaders in spreading information across diverse social groups. In multilingual environments, this cross-propagation mechanism is influenced by linguistic proximity and cultural differences, as noted in Triandis' work on cross-cultural psychology. By integrating these social, psychological, and communication factors into a mathematical model, we aim to provide a more accurate representation of public opinion dynamics in multilingual contexts.

Remark 2.1 Many previous studies have overlooked the social enhancement effect on public opinion propagation in multilingual environment. Given the differences in values, beliefs, customs, and art forms across various languages and ethnic groups, it is crucial to consider how these differences, via social enhancement effects, influence public opinion propagation.

Remark 2.2 Few studies have examined the impact of the forgetting mechanism in multilingual contexts. Previous research focused on monolingual environments, neglecting factors like communication frequency and mutual understanding between language groups. This study redefines the forgetting mechanism by incorporating cultural differences, such as values and beliefs, offering a perspective that captures the complexities of opinion propagation in multilingual societies.

Remark 2.3 Cross-propagation plays a crucial role in multilingual public opinion dissemination, influenced by linguistic and cultural proximity. This mechanism facilitates or inhibits opinion spread depending on the degree of interaction between different language groups. By incorporating these factors, the model more accurately simulates how opinions propagate across multilingual societies.

To facilitate our research, we will use a bilingual environment as a model to investigate the mechanism of public opinion propagation in multilingual settings, setting $i = 1, 2$. The model of public opinion propagation in a bilingual environment is

$$\begin{cases} \frac{\mathrm{d}S_1(t)}{\mathrm{d}t} = b_1 - \beta_{11}S_1(t)I_1(t) - \beta_{12}S_1(t)I_2(t) - d_1S_1(t), \\ \frac{\mathrm{d}E_1(t)}{\mathrm{d}t} = \beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t) - \alpha_1E_1(t) - d_1E_1(t), \\ \frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = \alpha_1E_1(t) - (\gamma_1 + \mu_{12} + \eta_{21} + d_1)I_1(t) + \eta_{12}I_2(t), \\ \frac{\mathrm{d}R_1(t)}{\mathrm{d}t} = \gamma_1I_1(t) - d_1R_1(t), \\ \frac{\mathrm{d}S_2(t)}{\mathrm{d}t} = b_2 - \beta_{22}S_2(t)I_2(t) - \beta_{21}S_2(t)I_1(t) - d_2S_2(t), \\ \frac{\mathrm{d}E_2(t)}{\mathrm{d}t} = \beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t) - \alpha_2E_2(t) - d_2E_2(t), \\ \frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = \alpha_2E_2(t) - (\gamma_2 + \mu_{21} + \eta_{12} + d_2)I_2(t) + \eta_{21}I_1(t), \\ \frac{\mathrm{d}R_2(t)}{\mathrm{d}t} = \gamma_2I_2(t) - d_2R_2(t). \end{cases} \tag{4}$$

Model analysis
Properties of solutions

Lemma 1 The positive invariant set of the system (4) with positive invariant set is as follows.

$$\Omega = \Big\{(S_1(t), E_1(t), I_1(t), R_1(t), S_2(t), E_2(t), I_2(t), R_2(t)) \in \mathbf{R}_+^8 \,|S_i(t) \\ +E_i(t) + I_i(t) + R_i(t) < \frac{b_i}{\delta_i}, i = 1, 2\Big\}$$

Proof Based on the four equations of the system (4) and let $\varepsilon_i = \min\left[d_1, (\mu_{12} + d_1 + \eta_{21})\right]$, so we have that

$$\begin{aligned} &\frac{\mathrm{d}S_i(t)}{\mathrm{d}t} + \frac{\mathrm{d}E_i(t)}{\mathrm{d}t} + \frac{\mathrm{d}I_i(t)}{\mathrm{d}t} + \frac{\mathrm{d}R_i(t)}{\mathrm{d}t} \\ &= b_i + \eta_{ij}I_i(t) - d_i(S_i(t) + E_i(t) + R_i(t)) - (\mu_{ij} + d_i + \eta_{ji})I_i(t) \\ &\le b_i - d_i(S_i(t) + E_i(t) + R_i(t)) - (\mu_{ij} + d_i + \eta_{ji})I_i(t) \\ &\le b_i - \varepsilon_i(S_i(t) + E_i(t) + I_i(t) + R_i(t)), i = 1, 2. \end{aligned} \tag{5}$$

There by obtaining

$$\lim_{t\to+\infty} \sup S_i(t) + E_i(t) + I_i(t) + R_i(t) \le \frac{b_i}{\varepsilon_i}, i = 1, 2.$$

This implies that the feasible region Ω is orthogonally invariant for system (4).

Lemma 2 For all $t > 0$ and $i = 1, 2$, the solutions $S_i(t)$, $E_i(t)$, $I_i(t)$, $R_i(t)$ of system (4) with initial conditions $S_i(0) > 0$, $E_i(0) > 0$, $I_i(0) > 0$, $R_i(0) > 0$ are positive.

Proof From the initial conditions of system (1), we know that $S_i(0) > 0$, $E_i(0) > 0$, $I_i(0) > 0$, $R_i(0) > 0$, $i = 1, 2$.

indicate

$$h(t) = \min_t \{S_i(0), E_i(0), I_i(0), R_i(0), i = 1, 2\}.$$

All we need to do to prove that $S_i(t)$, $E_i(t)$, $I_i(t)$, and $R_i(t)$ are positive at any $t > 0$ moment is to prove that $h(t) > 0$ is positive at any $t > 0$ moment. Based on reductio ad absurdum, assume that there exists a positive number t_1 such that.

$h(t_1) = 0$, and when $t \in (0, t_1)$,$h(t) > 0$.

Below we discuss the function $h(t)$ in four cases.

Suppose there exists a positive integer $i_1 \in \{1, 2\}$ such that $h(t_1) = S_{i_1}(t_1) = 0$. From system (4) we know that:

$$\frac{\mathrm{d}S_{i1}(t)}{\mathrm{d}t} = b_{i1} - \beta_{i1}S_{i1}(t)I_{i1}(t) - \beta'_{i1}S_{i1}(t)I'_{i1}(t) - d_{i1}S_{i1}(t),$$

when $t = t_1$, and by substituting the assumption $S_{i_1}(t_1) = 0$ into the above formula, we obtain the following:

$$\frac{\mathrm{d}S_{i_1}(t)}{\mathrm{d}t}|_{t=t_1} = b_{_{i_1}} > 0$$

This is consistent with the assumption $S_{i_1}(t_1) = 0$ and the premise that $S_{i_1}(t)$ begins to decrease at $t = t_1$. Therefore,$S_{i_1}(t)$ is always positive when $t > 0$.

Suppose there exists a positive integer $i_1 \in \{1, 2\}$ such that $h(t_1) = E_{i_1}(t_1)$. From system (4) we know that:

$$\frac{\mathrm{d}E_{i1}(t)}{\mathrm{d}t} = \beta_{i1} S_{i1}(t) I_{i1}(t) + \beta'_{i1} S_{i1}(t) I'_{i1}(t) - \alpha_1 E_{i1}(t) - d_1 E_{i1}(t),$$

when $t = t_1$, and by substituting the assumption $E_{i_1}(t_1) = 0$ into the above formula, we obtain the following:

$$\frac{\mathrm{d}E_{i1}(t)}{\mathrm{d}t} |t = t_1 = \beta_{i1} S_{i1}(t) I_{i1}(t) + \beta'_{i1} S_{i1}(t) I'_{i1}(t),$$

therefore $S_{i_1}(t) \geq 0$, $I_{i_1}(t) \geq 0$ and $I_{i'_1}(t) \geq 0$, $\beta_{i_1} > 0$, $\beta'_{i_1} > 0$,so.

$\frac{\mathrm{d}E_{i_1}(t)}{\mathrm{d}t} |_{t=t_1} > 0$.

This is consistent with the assumption $E_{i_1}(t_1) = 0$ and the premise that $E_{i_1}(t)$ begins to decrease at $t = t_1$. Therefore,$E_{i_1}(t)$ is always positive when $t > 0$.

Suppose there exists a positive integer $i_1 \in \{1, 2\}$ such that $h(t_1) = I_{i_1}(t_1)$. From system (4) we know that:

$$\frac{\mathrm{d}I_i(t)}{\mathrm{d}t} = \alpha_{i1} E_{i1}(t) - \left(\gamma_{i1} + \mu_{i1i'_1} + \eta_{i'_1 i1} + d_{i1}\right) I_{i1}(t) + \eta_{i'_1 i1} I'_{i1}(t),$$

when $t = t_1$, and by substituting the assumption $I_{i_1}(t_1) = 0$ into the above formula, we obtain the following:

$$\frac{\mathrm{d}I_{i_1}(t)}{\mathrm{d}t} |_{t=t_1} = \alpha_{i_1} E_{i_1}(t),$$

therefore $E_{i_1}(t) \geq 0$ and $\alpha_{i_1} > 0$,so.

$\frac{\mathrm{d}I_{i_1}(t)}{\mathrm{d}t} |t = t_1 > 0$.

This is consistent with the assumption $I_{i_1}(t_1) = 0$ and the premise that $I_{i_1}(t)$ begins to decrease at $t = t_1$. Therefore,$I_{i_1}(t)$ is always positive when $t > 0$.

Suppose there exists a positive integer $i_1 \in \{1, 2\}$ such that $h(t_1) = R_{i1}(t_1)$. From system (4) we know that:

$$\frac{\mathrm{d}R_{i1}(t)}{\mathrm{d}t} = \gamma_{i1} I_{i1}(t) - d_{i1} R_{i1}(t),$$

when $t = t_1$, and by substituting the assumption $R_{i_1}(t_1) = 0$ into the above formula, we obtain the following:

$$\frac{\mathrm{d}R_{i1}(t)}{\mathrm{d}t} = \gamma_{i1} I_{i1}(t),$$

therefore, $I_{i1}(t) \geq 0$ and,$\gamma_{i1} > 0$, so.

$\frac{\mathrm{d}R_{i1}(t)}{\mathrm{d}t} |t = t_1 > 0$.

This is consistent with the assumption $R_{i_1}(t_1) = 0$ and the premise that $R_{i_1}(t)$ begins to decrease at $t = t_1$. Therefore,$R_{i_1}(t)$ is always positive when $t > 0$.

Thus we have that the solution $S_i(t)$, $E_i(t)$, $I_i(t)$, $R_i(t)$ of system (4) with initial conditions $S_i(0) > 0$, $E_i(0) > 0$, $I_i(0) > 0$, $R_i(0) > 0$ is positive for all $t > 0$ and $i = 1, 2$.

Basic regeneration number

It is easy to see that model (4) has a disease-free equilibrium point as shown below:

$$E^0 = \left(S_1^0, 0, 0, 0, S_2^0, 0, 0, 0\right), S_i^0 = \frac{b_i}{d_i}, i = 1, 2$$

Define the endemic disease equilibrium points within Ω as $E^* = (S_1^*, E_1^*, I_1^*, R_1^*, S_2^*, E_2^*, I_2^*, R_2^*)$.

Next, the familiar next-generation matrix method is utilized to find the fundamental regeneration number of system (4). We define the vector $\chi = (S_1, S_2, E_1, E_2, I_1, I_2, R_1, R_2)^T$. From this we obtain the 8×8 matrices F, V, V^{-1} of:

$$F = \begin{pmatrix} 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, V = \begin{pmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & V_3 & 0 \\ 0 & 0 & V_4 & 0 \\ 0 & 0 & V_5 & V_6 \end{pmatrix}, V^{-1} = \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & M_3 & 0 \\ 0 & 0 & M_4 & 0 \\ 0 & 0 & M_5 & M_6 \end{pmatrix}$$

Among them $F_1 = \begin{pmatrix} \beta_{11}S_1 & \beta_{12}S_1 \\ \beta_{21}S_2 & \beta_{22}S_2 \end{pmatrix}$, $V_1 = diag\left\{-d_1, -d_2\right\}$, $V_2 = diag\left\{\alpha_1 + d_1, \alpha_2 + d_2\right\}$, $V_3 = diag\left\{-\left(\gamma_1 + \mu_{12} + \eta_{21}\right), -\left(\gamma_2 + \mu_{21} + \eta_{12}\right)\right\}$,$V_4 = diag\left\{\gamma_1 + \mu_{12} + d_1, \gamma_2 + \mu_{21} + d_2\right\}$, $V_5 = diag\left\{\eta_{21}, \eta_{12}\right\}$, $V_6 = diag\left\{-d_1, -d_2\right\}$, $M_1 = diag\left\{-\frac{1}{d_1}, -\frac{1}{d_2}\right\}$,$M_2 = diag\left\{-\frac{1}{d_1+\alpha_1}, -\frac{1}{d_2+\alpha_2}\right\}$, $M_3 = diag\left\{\frac{\mu_{12}+\gamma_1+\eta_{21}}{(d_1+\alpha_1)(d_1+\mu_{12}+\gamma_1)}, \frac{\mu_{21}+\gamma_2+\eta_{12}}{(d_2+\alpha_2)(d_2+\mu_{21}+\gamma_2)}\right\}$, $M_4 = diag\left\{\frac{1}{d_1+\mu_{12}+\gamma_1}, \frac{1}{d_2+\mu_{21}+\gamma_2}\right\}$, $M_5 = diag\left\{\frac{\eta_{21}}{d_1(d_1+\mu_{12}+\gamma_1)}, \frac{\eta_{12}}{d_2(d_2+\mu_{21}+\gamma_2)}\right\}$, $M_6 = diag\left\{-\frac{1}{d_1}, -\frac{1}{d_2}\right\}$.

For convenience, we define.

$$h_i = \frac{\mu_{ij} + \gamma_i + \eta_{ji}}{\left(d_i + \alpha_i\right)\left(d_i + \mu_{ij} + \gamma_i\right)},\ p_i = \frac{1}{d_i + \mu_{ij} + \gamma_i},\ q_i = \frac{\eta_{ji}}{d_i\left(d_i + \mu_{ij} + \gamma_i\right)},\ i = 1, 2,\ j = 1, 2$$

From this it follows that the basic regeneration number $\Re_0$ satisfies the following equation:

$$\Re_0 = \rho\left(F_1 M_4\right) = \rho\left(\beta_{ij} S_i^0 p_i\right)_{2\times 2} \tag{6}$$

where ρ denotes the spectral radius[45] .

Remark 3.1 According to reference [46], the disease-free equilibrium point E^0 of system (4) is locally asymptotically stable when $R_0 < 1$; the disease-free equilibrium point E^0 of system (4) is locally unstable when $R_0 > 1$.

Stability of equilibrium point

This section investigates the global stability of the SEIR public opinion propagation model in bilingual environment.

Disease-free equilibrium points

Lemma 3 when $R_0 < 1$, the disease-free equilibrium points E^0 of system (4) is globally asymptotically stable.

Proof From the previous formulation we know that $S = \left(S_1, S_2\right)^T$, we also define $M\left(S\right) = \left(\frac{\beta_{ij}S_i}{d_i+\mu_{ij}+\gamma_i}\right)_{2\times 2}$, $M\left(S^0\right) = \left(\frac{\beta_{ij}S_i^0}{d_i+\mu_{ij}+\gamma_i}\right)_{2\times 2}$, $i = 1, 2, j = 1, 2$.

Obviously, it is known that $S_i < S_i^0$, $i = 1, 2$. According to reference [47], it is known that $0 < M\left(S\right) < M\left(S^0\right) = M_0$. Hence $B = \left(\beta_{ij}\right)_{2\times 2}$ is irreducible and hence it can be concluded that M_0, $M\left(S\right)$, $M_0 + M\left(S\right)$, are irreducible. Thus if $S \neq S^0$, then there is $\rho\left(M\left(S\right)\right) < \rho\left(M\left(S^0\right)\right)$.

If $R_0 < 1$ as well as $S \neq S^0$, there is $\rho\left(M\left(S\right)\right) < 1$. and when $M\left(S\right)I = I$, there is a mundane solution $I = 0$ (refer to reference [48]). Thus $R_0 < 1$, system (4) has only one disease-free equilibrium E^0 in Ω.

The following defines the Lyapunov function.

$$V\left(t\right) = \sum_{i=1}^{2} p_i\omega_i E_i\left(t\right) + \sum_{i=1}^{2} p_i\omega_i I_i\left(t\right)$$

where $E_i\left(t\right) = \left(E_1\left(t\right), E_2\left(t\right)\right)^T$,$I_i\left(t\right) = \left(I_1\left(t\right), I_2\left(t\right)\right)^T$ and $\omega^T = \left(\omega_1, \omega_2\right)$ are the positive left eigenvectors of $M\left(S\right)$ corresponding to $\rho\left(M\left(S\right)\right)$. Therefore, we can get

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = \sum_{i=1}^{2} p_i\omega_i \frac{\mathrm{d}E_i(t)}{\mathrm{d}t} + \sum_{i=1}^{2} p_i\omega_i \frac{\mathrm{d}I_i(t)}{\mathrm{d}t}$$

$$\sum_{i=1}^{2} p_i\omega_i \Bigg\{ S_i(t) \sum_{j=1}^{2} \beta_{ij} I_j(t) - \alpha_i E_i(t) - d_i E_i(t)$$

$$+\alpha_i E_i(t) - \gamma_i I_i(t) - \sum_{j=1}^{n} u_{ij} I_i(t) - \sum_{j=1}^{n} \eta_{ji} I_i(t) - d_i I_i(t) + \sum_{j=1}^{n} \eta_{ij} I_j(t) \Bigg\}$$

$$= \sum_{i=1}^{2}\sum_{j=1}^{2} p_i\omega_i\beta_{ij} S_i(t) I_j(t) - \sum_{i=1}^{2} p_i\omega_i d_i E_i(t)$$

$$- \sum_{i=1}^{2}\sum_{j=1}^{n} p_i\omega_i (\mu_{ij} + \eta_{ji}) I_i(t) - \sum_{i=1}^{2} p_i\omega_i (d_i + \gamma_i) I_i(t)$$

$$+ \sum_{i=1}^{2}\sum_{j=1}^{2} p_i\omega_i\eta_{ij} I_j(t) \le \sum_{i=1}^{2}\sum_{j=1}^{2} p_i\omega_i\beta_{ij} S_i(t) I_j(t)$$

$$- \sum_{i=1}^{2}\sum_{j=1}^{2} p_i\omega_i (d_i + \gamma_i + \mu_{ij}) I_i(t) = \omega^T (\rho(M(S)) I - I)$$

$$= \omega^T I (\rho(M(S)) - 1) \le 0.$$

When $V(t) = 0$, can obtain $I_i(t) = 0$, $i = 1, 2$, $j = 1, 2$. Substituting $I_i(t) = 0$ into system (4) yields that when $t \to \infty$, $S_i(t) \to \frac{b_i}{d_i}$, $E_i(t) \to 0$, $R_i(t) \to 0$. According to the Lasalle invariance principle[49], the disease-free equilibrium points E^0 of system (4) are globally asymptotically stable when $R_0 < 1$.

Endemic disease equilibrium points
In this section, we assume $R_0 > 1$, to discuss the stability of Endemic disease equilibrium points. According to the reference [50], it is known that when $R_0 > 1$, the Endemic disease equilibrium points $E^* = (S_1^*, E_1^*, I_1^*, R_1^*, S_2^*, E_2^*, I_2^*, R_2^*)$ exists at Ω^0 inside Ω. That is, the following condition is satisfied:

$$\begin{cases} b_i = -S_i^* \sum_{j=1}^{n} \beta_{ij} I_j^* - d_i S_i^*, \\ S_i^* \sum_{j=1}^{n} \beta_{ij} I_j^* = \alpha_i E_i^* + d_i E_i^*, \\ 0 = \alpha_i E_i^* - \gamma_i I_i^* - \sum_{j=1}^{n} u_{ij} I_i^* - \sum_{j=1}^{n} \eta_{ji} I_i^* - d_i I_i^* + \sum_{j=1}^{n} \eta_{ij} I_j^*, \\ 0 = \gamma_i I_i^* - d_i R_i^*, \end{cases} \tag{7}$$

which is $i = 1, 2$.

Next, prove that the Endemic disease equilibrium points E^* of system (4) is globally stable at Ω^0.

Lemma 4 when $R_0 > 1$ the Endemic disease equilibrium points E^* of system (4) is globally asymptotically stable.

Proof Consideration.

$$\begin{cases} V(t) = \sum_{i=1}^{2} v_i \left\{ S_i^* g\left(\frac{S_i(t)}{S_i^*}\right) + E_i^* g\left(\frac{E_i(t)}{E_i^*}\right) + I_i^* g\left(\frac{I_i(t)}{I_i^*}\right) + R_i^* g\left(\frac{R_i(t)}{R_i^*}\right) \right\}, \\ g(\chi) = \chi - 1 - \ln\chi \ge g(1) = 0, \text{for any} \chi > 0, \end{cases} \tag{8}$$

where v_1 and v_2 are normal numbers.

Set $w_i = \frac{S_i(t)}{S_i^*}$,$x_i = \frac{E_i(t)}{E_i^*}$,$y_i = \frac{I_i(t)}{I_i^*}$,$z_i = \frac{R_i(t)}{R_i^*}$,$i = 1, 2$.
From this

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t}=\sum_{i=1}^{2}v_i\left\{\left(1-\frac{1}{w_i}\right)\dot{S}_i+\left(1-\frac{1}{x_i}\right)\dot{E}_i+\left(1-\frac{1}{y_i}\right)\dot{I}_i+\left(1-\frac{1}{z_i}\right)\dot{R}_i\right\}.$$

According to Eqs. (4) and (7), it can be obtained that

$$\begin{aligned}\dot{S}_i&=-\sum_{j=1}^{2}\beta_{ij}\left(S_i(t)I_j(t)-S_i^*I_i^*\right)-d_i\left(S_i(t)-S_i^*\right)\\&=-\sum_{j=1}^{2}\beta_{ij}S_i^*I_i^*\left(w_iy_j-1\right)-d_iS_i^*\left(w_i-1\right),\end{aligned}$$

$$\begin{aligned}\dot{E}_i&=\sum_{j=1}^{n}\beta_{ij}\left(S_i(t)I_j(t)-S_i^*I_i^*\right)-\alpha_i\left(E_i(t)-E_i^*\right)-d_i\left(E_i(t)-E_i^*\right)\\&=\sum_{j=1}^{2}\beta_{ij}S_i^*I_i^*\left(w_iy_j-1\right)-\alpha_iE_i^*\left(x_i-1\right)-d_iE_i^*\left(x_i-1\right),\end{aligned}$$

$$\begin{aligned}\dot{I}_i&=\alpha_i\left(E_i(t)-E_i^*\right)-\gamma_i\left(I_j(t)-I_j^*\right)-\sum_{j=1}^{n}\mu_{ij}\left(I_j(t)-I_j^*\right)\\&-\sum_{j=1}^{2}\eta_{ji}\left(I_i(t)-I_i^*\right)-d_iI_i(t)+\sum_{j=1}^{2}\eta_{ij}\left(I_j(t)-I_j^*\right)\\&=\alpha_iE_i^*\left(x_i(t)-1\right)-\gamma_iI_i^*\left(y_i-1\right)-\sum_{j=1}^{2}\mu_{ij}I_i^*\left(y_i-1\right)\\&-\sum_{j=1}^{2}\eta_{ji}I_i^*\left(y_i-1\right)-d_iI_i^*\left(y_i-1\right)+\sum_{j=1}^{n}\eta_{ij}I_j^*\left(y_j-1\right),\end{aligned}$$

$$\begin{aligned}\dot{R}_i&=\gamma_i\left(I_i(t)-I_i^*\right)-d_i\left(R_i(t)-R_i^*\right)\\&=\gamma_iI_i^*\left(y_i-1\right)-d_iR_i^*\left(z_i-1\right).\end{aligned}$$

Then, it can be further obtained that

$$\begin{aligned}
\frac{\mathrm{d}V(t)}{\mathrm{d}t} &= \sum_{i=1}^{2} v_i \left\{ \left[\sum_{j=1}^{n} \beta_{ij} S_i^* I_i^* (1 - w_i y_j) \left(1 - \frac{1}{w_i}\right) - d_i S_i^* (w_i - 1) \left(1 - \frac{1}{w_i}\right) \right] \right. \\
&+ \left[\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* (w_i y_j - 1) \left(1 - \frac{1}{x_i}\right) - \alpha_i E_i^* (x_i - 1) \left(1 - \frac{1}{x_i}\right) - d_i E_i^* (x_i - 1) \left(1 - \frac{1}{x_i}\right) \right] \\
&+ \left[\alpha_i E_i^* (x_i - 1) \left(1 - \frac{1}{y_i}\right) - \gamma_i I_i^* (y_i - 1) \left(1 - \frac{1}{y_i}\right) - \sum_{j=1}^{2} \mu_{ij} I_i^* (y_i - 1) \left(1 - \frac{1}{y_i}\right) \right. \\
&\left. - \sum_{j=1}^{2} \eta_{ji} I_i^* (y_i - 1) \left(1 - \frac{1}{y_i}\right) - d_i I_i^* (y_i - 1) \left(1 - \frac{1}{y_i}\right) + \sum_{j=1}^{2} \eta_{ij} I_j^* (y_j - 1) \left(1 - \frac{1}{y_i}\right) \right] \\
&\left. + \left[\gamma_i I_i^* (y_i - 1) \left(1 - \frac{1}{z_i}\right) - d_i R_i^* (z_i - 1) \left(1 - \frac{1}{z_i}\right) \right] \right\} \\
&= \sum_{i=1}^{2} v_i \left\{ \left[\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* \left[-g\left(\frac{1}{w_i}\right) - g(w_i y_j) + g(y_j) \right] - d_i S_i^* \left[g(w_i) + g\left(\frac{1}{w_i}\right) \right] \right] \right. \\
&\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* \left[g(w_i y_j) - g\left(\frac{w_i y_j}{x_i}\right) + g\left(\frac{1}{x_i}\right) \right] - \alpha_i E_i^* \left[g(x_i) + g\left(\frac{1}{x_i}\right) \right] \\
&- d_i E_i^* \left[g(x_i) + g\left(\frac{1}{x_i}\right) \right] + \alpha_i E_i^* \left[g(x_i) - g\left(\frac{x_i}{y_i}\right) + g\left(\frac{1}{y_i}\right) \right] - \gamma_i I_i^* \left[g(y_i) + g\left(\frac{1}{y_i}\right) \right] \\
&- \sum_{j=1}^{2} \mu_{ij} I_i^* \left[g(y_i) + g\left(\frac{1}{y_i}\right) \right] - \sum_{j=1}^{2} \eta_{ji} I_i^* \left[g(y_i) + g\left(\frac{1}{y_i}\right) \right] - d_i I_i^* \left[g(y_i) + g\left(\frac{1}{y_i}\right) \right] \\
&+ \sum_{j=1}^{2} \eta_{ij} I_j^* \left[g(y_j) - g\left(\frac{y_j}{y_i}\right) + g\left(\frac{1}{y_i}\right) \right] + \gamma_i I_i^* \left[g(y_i) - g\left(\frac{y_i}{z_i}\right) + g\left(\frac{1}{z_i}\right) \right] \\
&\left. - d_i R_i^* \left[g(z_i) + g\left(\frac{1}{z_i}\right) \right] \right\} \\
&= -\sum_{i=1}^{2} \sum_{j=1}^{2} v_i (\beta_{ij} S_i^* I_i^* + d_i S_i^*) g\left(\frac{1}{w_i}\right) - \sum_{i=1}^{2} \sum_{j=1}^{2} v_i \beta_{ij} S_i^* I_i^* g\left(\frac{w_i y_j}{x_i}\right) - \sum_{i=1}^{2} v_i d_i E_i^* g(x_i) \\
&- \sum_{i=1}^{2} v_i \alpha_i E_i^* g\left(\frac{x_i}{y_i}\right) - \sum_{i=1}^{2} v_i \eta_{ij} I_j^* g\left(\frac{y_j}{y_i}\right) - \sum_{i=1}^{2} v_i \gamma_i I_i^* g\left(\frac{y_i}{z_i}\right) - d_i R_i^* g(z_i) + \gamma_i I_i^* g(y_i) \\
&+ \left[\sum_{j=1}^{2} v_j \left(\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* + \sum_{j=1}^{2} \eta_{ij} I_j^* \right) g(y_j) - \sum_{i=1}^{2} v_i \left(\gamma_i I_i^* + \sum_{j=1}^{2} \mu_{ij} I_i^* + d_i I_i^* + \sum_{j=1}^{2} \eta_{ji} I_i^* \right) g(y_i) \right]
\end{aligned}$$

According to reference [27] it is known as $\sum_{j=1}^{2} v_j \left(\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* + \sum_{j=1}^{2} \eta_{ij} I_j^* \right) g(y_j) = \sum_{i=1}^{2} v_i \left(\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* + \sum_{j=1}^{2} \eta_{ij} I_i^* \right) g(y_i)$

, and according to the second and third equations of the system of Eqs. (7) it is known as $\sum_{j=1}^{2} v_j \left(\sum_{j=1}^{2} \beta_{ij} S_i^* I_i^* + \sum_{j=1}^{2} \eta_{ij} I_j^* \right) g(y_j) = \sum_{i=1}^{2} v_i \left(\gamma_i I_i^* + \sum_{j=1}^{2} \mu_{ij} I_i^* + d_i I_i^* + \sum_{j=1}^{2} \eta_{ji} I_i^* \right) g(y_i)$.

To summarize, $\frac{\mathrm{d}V(t)}{\mathrm{d}t} < 0$. Note that if $S_i = S_i^*$, $E_i = E_i^*$, $I_i = I_i^*$, $R_i = R_i^*$, $i = 1, 2$, then $\frac{\mathrm{d}V(t)}{\mathrm{d}t} = 0$. Thus, with the above analysis as well as the Lasalle invariance principle, it is clear that the Endemic disease equilibrium points point E^* of system (4) is globally asymptotically stable when $R_0 > 1$.

Optimal management strategy

Managing internet-based public opinion effectively is vital and challenging in our interconnected world. The ubiquitous nature of the Internet and social media platforms accelerates the dissemination of news, impacting millions rapidly. Strategic management of online public opinion is crucial to maintain social stability, protect personal and corporate reputations, combat misinformation, and promote a constructive digital discourse. Optimal resource allocation is essential, considering the limited resources available. Strategic investments guided by optimal management theories ensure that resources are utilized efficiently, enhancing the impact of public opinion management efforts while minimizing waste.

This study introduces two proactive strategies to manage public opinion dynamically: the Instant Public Alert System(u_{1i}) and Emergency Blocking Measures (u_{2i}). The Instant Public Alert System engages the public directly through timely alerts via SMS, app notifications, or social media, effectively reducing the spread of emergent public opinion trends. Emergency Blocking Measures curtail the dissemination of potentially harmful information by selectively disabling social media functionalities or restricting data flows, thus limiting public exposure and the escalation of public opinion crises. Both strategies are designed to swiftly mitigate the impact of volatile public opinion, ensuring rapid response and containment. The corresponding allowable management set is defined as $U = \{(u_1(t), u_2(t)) | 0 \le u_1(t) \le u_1^{\max}, 0 \le u_2(t) \le u_2^{\max}, t \in [0, T]\}$, where $u_1^{\max} \in [0,1]$ as well as $u_2^{\max} \in [0,1]$ are the upper bounds of $u_1(t)$ as well as $u_2(t)$. From this, we can obtain the SEIR model with management measures in a bilingual environment:

$$\begin{cases} \dfrac{\mathrm{d}S_1(t)}{\mathrm{d}t} = b_1 - \beta_{11}(1-u_{11}(t))S_1(t)I_1(t) - \beta_{12}(1-u_{11}(t))S_1(t)I_2(t) - d_1S_1(t), \\ \dfrac{\mathrm{d}E_1(t)}{\mathrm{d}t} = \beta_{11}(1-u_{11}(t))S_1(t)I_1(t) + \beta_{12}(1-u_{11}(t))S_1(t)I_2(t) - \alpha_1E_1(t) - d_1E_1(t), \\ \dfrac{\mathrm{d}I_1(t)}{\mathrm{d}t} = \alpha_1E_1(t) - [(\gamma_1 + u_{21}(t)) + \mu_{12} + \eta_{21} + d_1]I_1(t) + \eta_{12}I_2(t), \\ \dfrac{\mathrm{d}R_1(t)}{\mathrm{d}t} = (\gamma_1 + u_{21}(t))I_1(t) - d_1R_1(t), \\ \dfrac{\mathrm{d}S_2(t)}{\mathrm{d}t} = b_2 - \beta_{21}(1-u_{12}(t))S_2(t)I_1(t) - \beta_{22}(1-u_{12}(t))S_2(t)I_2(t) - d_2S_2(t), \\ \dfrac{\mathrm{d}E_2(t)}{\mathrm{d}t} = \beta_{22}(1-u_{12}(t))S_2(t)I_2(t) + \beta_{21}(1-u_{12}(t))S_2(t)I_1(t) - \alpha_2E_2(t) - d_2E_2(t), \\ \dfrac{\mathrm{d}I_2(t)}{\mathrm{d}t} = \alpha_2E_2(t) - [(\gamma_2 + u_{22}(t)) + \mu_{21} + \eta_{12} + d_2]I_2(t) + \eta_{21}I_1(t), \\ \dfrac{\mathrm{d}R_2(t)}{\mathrm{d}t} = (\gamma_2 + u_{22}(t))I_2(t) - d_2R_2(t), \end{cases} \tag{9}$$

where $u_{11}(t)$ and $u_{12}(t)$ represent the degree of management over the rate of exposure of susceptible populations to public opinion in bilingual groups.$u_{21}(t)$ and $u_{22}(t)$ represent the management levels for the infectious populations within the bilingual groups.

To more effectively management the spread of public opinion, we have optimized two management measures. Our goal is to minimize the number of affected individuals and reduce economic losses incurred from management measures during the period of public opinion spread online. Assume the economic losses due to public opinion influence are proportional to their number, with scaling factors B_1 and B_2 for the bilingual groups. Additionally, economic losses from management measures correlate with the square of their intensity, with scaling factors B_3, B_4, B_5, and B_6. Consequently, we derive the following objective function:

$$J(u_{11}, u_{12}, u_{21}, u_{22}) = \int_{t_0}^{t_f} \left[B_1I_1(t) + B_2I_2(t) + B_3u_{11}^2 + B_4u_{12}^2 + B_5u_{21}^2 + B_6u_{22}^2\right]\mathrm{d}t, \tag{10}$$

where t_0 and t_f are the initial moment and the ending moment, respectively. The main work in this section is to find the optimal management pair $(u_{11}^*, u_{12}^*, u_{21}^*, u_{22}^*)$, such that:

$$J(u_{11}^*, u_{12}^*, u_{21}^*, u_{22}^*) = \min_U J(u_{11}, u_{12}, u_{21}, u_{22}). \tag{11}$$

The management condition is $U = \left\{(u_{11}, u_{12}, u_{21}, u_{22}) \in L^1(t_0, t_f) | 0 \le u_{11}, u_{12}, u_{21}, u_{22} \le 1\right\}$. To find the optimal solution, the Lagrange function is made:

$$L(I_1, I_2, u_{11}, u_{12}, u_{21}, u_{22}) = B_1I_1(t) + B_2I_2(t) + B_3u_{11}^2 + B_4u_{12}^2 + B_5u_{21}^2 + B_6u_{22}^2. \tag{12}$$

The Hamiltonian function defining this management problem is:

$$\begin{aligned} &H(S_1, I_1, E_1, R_1, S_2, I_2, E_2, R_2, u_{11}, u_{12}, u_{21}, u_{22}, \lambda_i) \\ &= B_1I_1(t) + B_2I_2(t) + B_3u_{11}^2 + B_4u_{12}^2 + B_5u_{21}^2 \\ &+ B_6u_{22}^2 + \sum_{i=1}^{8} \lambda_i f_i \end{aligned} \tag{13}$$

where $\lambda_i\ (i = 1, 2, \cdots, 8)$ are the concomitant variables of the system and f_i is the right end function of the system (9), the necessary conditions for optimal management can be derived by applying Pontryagin's principle of extreme values.

The system's concomitant variables satisfy the following relationship with the Hamiltonian function.

$$\begin{cases} \frac{\mathrm{d}\lambda_1(t)}{\mathrm{d}t} = [\beta_{11}(1-u_{11}(t))I_1(t) + \beta_{12}(1-u_{11}(t))I_2(t)](\lambda_1(t) - \lambda_2(t)) + d_1\lambda_1(t), \\ \frac{\mathrm{d}\lambda_2(t)}{\mathrm{d}t} = (\alpha_1 + d_1)\lambda_2(t) - \alpha_1\lambda_3(t), \\ \frac{\mathrm{d}\lambda_3(t)}{\mathrm{d}t} = -B_1 + [\beta_{11}(1-u_{11}(t))S_1(t)](\lambda_1(t) - \lambda_2(t)) + [\gamma_1 + u_{21}(t) + \mu_{12} + \eta_{21} + d_1]\lambda_3(t) \\ -(\gamma_1 + u_{21}(t))\lambda_4(t) - \eta_{21}\lambda_7(t), \\ \frac{\mathrm{d}\lambda_4(t)}{\mathrm{d}t} = d_1\lambda_4(t), \\ \frac{\mathrm{d}\lambda_5(t)}{\mathrm{d}t} = [\beta_{21}(1-u_{12}(t))I_1(t) + \beta_{22}(1-u_{12}(t))I_2(t)](\lambda_5(t) - \lambda_6(t)) + d_2\lambda_5(t), \\ \frac{\mathrm{d}\lambda_6(t)}{\mathrm{d}t} = (\alpha_2 + d_2)\lambda_6(t) - \alpha_2\lambda_7(t), \\ \frac{\mathrm{d}\lambda_7(t)}{\mathrm{d}t} = -B_2 + [\beta_{22}(1-u_{12}(t))S_2(t)](\lambda_5(t) - \lambda_6(t)) + [\gamma_2 + u_{22}(t) + \mu_{21} + \eta_{12} + d_2]\lambda_7(t) \\ -(\gamma_2 + u_{22}(t))\lambda_8(t) - \eta_{12}\lambda_3(t), \\ \frac{\mathrm{d}\lambda_8(t)}{\mathrm{d}t} = d_2\lambda_8(t). \end{cases}$$

It follows that at the ending moment t_f, the transversality condition and the boundary condition satisfy $\lambda_i(t_f) = 0$.

Using the optimal management conditions can be obtained that:

$$\left.\frac{\partial H}{\partial u_{11}}\right|_{u_{11}=u_{11}^*} = 2B_3u_{11}^* - [\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t)) = 0 \Rightarrow$$
$$u_{11}^*(t) = \frac{[\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t))}{2B_3}$$
$$\left.\frac{\partial H}{\partial u_{12}}\right|_{u_{12}=u_{12}^*} = 2B_4u_{12}^* - [\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t)) = 0 \Rightarrow$$
$$u_{12}^*(t) = \frac{[\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t))}{2B_4}$$
$$\left.\frac{\partial H}{\partial u_{21}}\right|_{u_{21}=u_{21}^*} = 2B_5u_{21}^* + (\lambda_4(t) - \lambda_3(t))I_1(t) = 0 \Rightarrow$$
$$u_{21}^*(t) = \frac{(\lambda_3(t) - \lambda_4(t))I_1(t)}{2B_5}$$
$$\left.\frac{\partial H}{\partial u_{22}}\right|_{u_{22}=u_{22}^*} = 2B_6u_{22}^* + (\lambda_8(t) - \lambda_7(t))I_2(t) = 0 \Rightarrow$$
$$u_{22}^*(t) = \frac{(\lambda_7(t) - \lambda_8(t))I_2(t)}{2B_6}$$

Because $0 \le u_{11}, u_{12}, u_{21}, u_{22} \le 1$, utilizing the management space U results in several scenarios:

$$\begin{cases} u_{11}^*(t) = 0; \text{when} \frac{[\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t))}{2B_3} \le 0; \\ u_{11}^*(t) = \frac{[\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t))}{2B_3}; \\ \text{when } 0 < \frac{[\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t))}{2B_3} < u_{11}^{\max}(t); \\ u_{11}^*(t) = u_{11}^{\max}(t); \text{when } \frac{[\beta_{11}S_1(t)I_1(t) + \beta_{12}S_1(t)I_2(t)](\lambda_2(t) - \lambda_1(t))}{2B_3} \ge u_{11}^{\max}(t). \end{cases}$$

$$\begin{cases} u_{12}^*(t) = 0; \text{when} \frac{[\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t))}{2B_4} \le 0; \\ u_{12}^*(t) = \frac{[\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t))}{2B_4}; \\ \text{when } 0 < \frac{[\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t))}{2B_4} < u_{12}^{\max}(t); \\ u_{12}^*(t) = u_{12}^{\max}(t); \text{when } \frac{[\beta_{22}S_2(t)I_2(t) + \beta_{21}S_2(t)I_1(t)](\lambda_6(t) - \lambda_5(t))}{2B_4} \ge u_{12}^{\max}(t). \end{cases}$$

$$\begin{cases} u_{21}^{*}(t)=0;\text{when}\dfrac{(\lambda_3(t)-\lambda_4(t))I_1(t)}{2B_5}\le 0;\\ u_{21}^{*}(t)=\dfrac{(\lambda_3(t)-\lambda_4(t))I_1(t)}{2B_5};\text{when } 0<\dfrac{(\lambda_3(t)-\lambda_4(t))I_1(t)}{2B_5}<u_{21}^{\max}(t)\\ u_{21}^{*}(t)=u_{21}^{\max}(t);\text{when } \dfrac{(\lambda_3(t)-\lambda_4(t))I_1(t)}{2B_5}\ge u_{21}^{\max}(t). \end{cases}$$

$$\begin{cases} u_{22}^{*}(t)=0;\text{when}\dfrac{(\lambda_7(t)-\lambda_8(t))I_2(t)}{2B_6}\le 0;\\ u_{22}^{*}(t)=\dfrac{(\lambda_7(t)-\lambda_8(t))I_2(t)}{2B_6};\text{when } 0<\dfrac{(\lambda_7(t)-\lambda_8(t))I_2(t)}{2B_6}<u_{22}^{\max}(t)\\ u_{22}^{*}(t)=u_{22}^{\max}(t);\text{when } \dfrac{(\lambda_7(t)-\lambda_8(t))I_2(t)}{2B_6}\ge u_{22}^{\max}(t). \end{cases}$$

Therefore, the optimal management is:

$$u_{11}^{*}(t)=\max\left(\min\left(\frac{[\beta_{11}S_1(t)I_1(t)+\beta_{12}S_1(t)I_2(t)](\lambda_2(t)-\lambda_1(t))}{2B_3},u_{11}^{\max}(t)\right),0\right), \quad (14)$$

$$u_{12}^{*}(t)=\max\left(\min\left(\frac{[\beta_{22}S_2(t)I_2(t)+\beta_{21}S_2(t)I_1(t)](\lambda_6(t)-\lambda_5(t))}{2B_4},u_{12}^{\max}(t)\right),0\right), \quad (15)$$

$$u_{21}^{*}(t)=\max\left(\min\left(\frac{(\lambda_3(t)-\lambda_4(t))I_1(t)}{2B_5},u_{21}^{\max}(t)\right),0\right), \quad (16)$$

$$u_{22}^{*}(t)=\max\left(\min\left(\frac{(\lambda_7(t)-\lambda_8(t))I_2(t)}{2B_6},u_{22}^{\max}(t)\right),0\right). \quad (17)$$

Numerical simulation

In this section, numerical simulations will be conducted to illustrate the accuracy of the SEIR model, which takes into account the social enhancement effect, the forgetting mechanism, and the cross-transmission mechanism, in analyzing the trend of online public opinion propagation in a multilingual environment, as well as the effectiveness of the instantaneous public warning system and the emergency blocking measures in suppressing the dissemination of online public opinion.We determined the parameters in the model by minimizing the difference between the actual observed data and the predicted values, using the least squares method to set the parameter values.

In this model, we have selected specific inter language and intra language propagation parameters to describe the dynamic process ofPublic Opinion Propagation. The selection of these parameters is based on theoretical analysis and numerical simulation requirements, ensuring that the model can effectively reflect the complexity of Public Opinion Propagation in multilingual environments.

Language internal propagation parameters (β_{11} and β_{22}): These parameters describe the rate of information propagation within each language group. We have preliminary set these propagation parameters by analyzing the research results on information propagation rate in existing literature and referring to the relevant parameter ranges in the classical SEIR model. In numerical simulation, a reasonable combination of parameters was selected to achieve a stable state of the system. To ensure the rationality of the model, we further optimized the parameters using the least squares method. By minimizing the sum of squared errors between the simulation results and theoretical predictions, we accurately determined the final parameter values.

Inter language propagation parameters (β_{12} and β_{21}): These parameters reflect the rate of information propagation between different language groups. We have reasonably set initial parameters based on literature research on language similarity, cultural commonality, and frequency of interaction between groups. In numerical simulations, these parameters are adjusted for different cross linguistic communication scenarios to ensure that the model can reflect the complex interactions of public opinion communication in multilingual environments. Further optimize these parameters through the least squares method to ensure that the cross linguistic propagation process in the model can accurately capture the transmission and evolution of information between groups.

Other parameters (such as social reinforcement effect parameters and forgetting mechanism parameters): The social reinforcement effect parameter (η_{ij}) is set as the degree of mutual influence between different language groups, reflecting the interactive effects between groups. In the simulation, we choose parameter values that can demonstrate reasonable interactions between different language groups. The forgetting mechanism parameter (μ_{ij}) is set by referring to relevant research results in existing information forgetting models, ensuring that the attenuation characteristics of public opinion in different language groups can be accurately captured.

The reason for choosing the least squares method: The least squares method is a classic parameter optimization method that is particularly suitable for reducing the sum of squared errors between numerical simulation results and theoretical expected values. By minimizing the sum of squared errors, we can obtain an optimal set of parameters that enable the model to exhibit a stable and reasonable process of public opinion propagation in numerical simulations. This method helps us balance the influence of various parameters in complex multilingual propagation models, ensuring the stability and accuracy of simulation results.

It should be noted that in numerical simulations, birth rates and removal rates are treated as constant values rather than time functions. The main reason for simplifying these parameters to constants during the simulation process is to reduce the complexity and computational burden of the model. In practice, introducing time-dependent functions for all parameters will make the system more difficult to simulate and may not significantly affect the overall model behavior during certain analysis periods.

Stability of disease-free equilibrium points

In the studied model of SEIR public opinion propagation in a bilingual environment, by setting a specific set of parameters:$\beta_{11} = 0.0225$, $\beta_{12} = 0.01125$, $\beta_{21} = 0.015$,$\beta_{22} = 0.0075$,$\alpha_1 = 0.2$,$\alpha_1 = 0.25$,$\gamma_1 = 0.15$,$\gamma_2 = 0.17$,$d_1 = 0.007$,$d_2 = 0.009$,$b_1 = 0.008$,$b_2 = 0.01$,$\mu_{12} = 0.01$,$\eta_{21} = 0.02$,$\mu_{21} = 0.02\eta_{12} = 0.01$, $S_1 = 0.99$, $E_1 = 0$, $I_1 = 0.01$, $R_1 = 0$, $S_2 = 0.99$,$E_2 = 0$,$I_2 = 0.01$,$R_2 = 0$. We simulate the dynamic process of public opinion propagation, focusing on the local stability of the disease-free equilibrium points when $R_0 < 1$. The basic reproduction number R_0 represents the average number of secondary infections caused by an infectious individual, assuming no preventive measures are in place. When $R_0 < 1$, it is expected that the public opinion will not trigger large-scale propagation among the population.

As shown in Fig. 2, with the initial condition of a few infectious individuals, the number of infectious individuals in both populations significantly decreases over time and eventually converges to zero. This suggests that public opinion propagation is not self-sustaining for a given parameter configuration and that the system gradually reverts to a state with no public opinion propagation. This process is depicted in the figure by the two curves, representing the proportion of infectious individuals, gradually approaching the horizontal axis without any rebounds or fluctuations, further supporting the local stability of the disease-free equilibrium points.

Additionally, the curve of $I(t)$ monotonically decreasing nature indicates that the system does not deviate from the disease-free equilibrium but continues to converge towards it over time. This trend aligns with

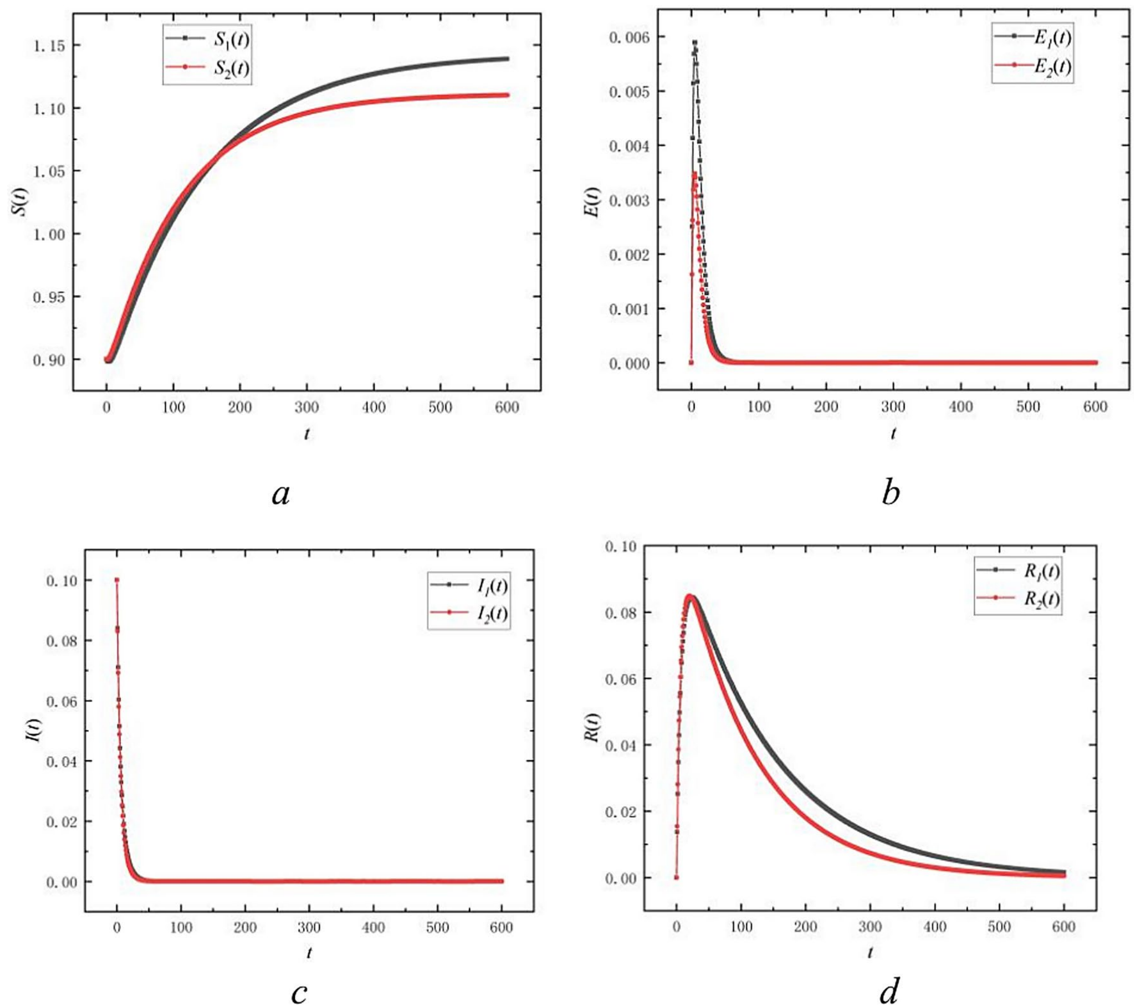

Fig. 2. Trends of four types of nodes over time in bilingual environment at $R_0 < 1$.

theoretical analyses showing that any small deviations from the disease-free equilibrium will dissipate over time, and in cases where $R_0 < 1$, the system will eventually return to a disease-free state.

In the studied model of SEIR public opinion propagation in a bilingual environment, by setting a specific set of parameters:$\beta_{11} = 0.0028$, $\beta_{12} = 0.0022$, $\beta_{21} = 0.0025$,$\beta_{22} = 0.002$,$\alpha_1 = 0.12$,$\alpha_2 = 0.11$,$\gamma_1 = 0.4$,$\gamma_2 = 0.45$,$d_1 = 0.001$,$d_2 = 0.009$,$b_1 = 0.003$, $b_2 = 0.003$, $\mu_{12} = 0.011$, $\eta_{21} = 0.0015$, $\mu_{21} = 0.009$, $\eta_{12} = 0.0017$, and by setting six initial conditions. We simulate the dynamics of public opinion propagation and analyze the global stability of the disease-free equilibrium points when $R_0 < 1$. Figure 3 depicts the dynamics of public opinion propagation under various scenarios, each characterized by distinct initial conditions. Specifically, it tracks the trajectory of the proportion of susceptible individuals $S(t)$ against the combined proportion of infectious individuals $I_1(t) + I_2(t)$ over time.

Figure 3 displays a series of curves originating from various starting points that converge over time towards common regions. This common region corresponds to the disease-free equilibrium points, where the proportion of infectious individuals approaches zero and the proportion of susceptible individuals remains constant. This trend indicates the global stability of the disease-free equilibrium points when the basic reproduction number (average new infections generated by an infectious individual during their infectious period) is less than 1. A basic reproduction number below 1 signifies that each infectious individual transmits the disease to fewer than one person on average, resulting in the epidemic's decline.

In the SEIR public opinion propagation model in bilingual environment, this global stability implies that irrespective of initial conditions, the system converges to a disease-free equilibrium where no new public opinions propagated. This phenomenon is illustrated by graphical trajectories showing a decreasing proportion of infectious individuals converging to zero and a stable proportion of susceptible individuals over time.

Therefore, numerical simulations confirm that the system's disease-free equilibrium is globally asymptotic stable when the basic reproduction number is below 1 in the SEIR public opinion propagation model in bilingual environment.

Stability of the endemic disease equilibrium points

In the studied model of SEIR public opinion propagation in bilingual environment, by setting a specific set of parameters:$\beta_{11} = 0.14$, $\beta_{12} = 0.07$, $\beta_{21} = 0.12$,$\beta_{22} = 0.06$,$\alpha_1 = 0.1$,$\alpha_2 = 0.1$,$\gamma_1 = 0.005$,$\gamma_2 = 0.005$,$d_1 = 0.007$,$d_2 = 0.007$,$b_1 = 0.008$,$b_2 = 0.008$,$\mu_{12} = 0.01$,$\eta_{21} = 0.01$,$\mu_{21} = 0.01$,$\eta_{12} = 0.01$,$S_1 = 0.97$, $E_1 = 0.01$, $I_1 = 0.01$, $R_1 = 0.01$, $S_2 = 0.97$, $E_2 = 0.01$, $I_2 = 0.01$, $R_2 = 0.01$. We observed key features of public opinion dynamics. The basic reproduction number R_0, a critical metric for public opinion propagation, exceeds 1 in this model, indicating that each infectious individual propagated the public opinion to more than one person. Figure 4 show that the proportion of susceptible individuals tends to decrease over time, while the proportion of infectious individuals decreases and then stabilize after an initial rise, and the proportion of recovered individuals increases and stabilize. This dynamic suggests that the public opinion persists in the population and reaches a state of endemic equilibrium under the parameter settings considered. The global asymptotic stability of this equilibrium is evidenced by stable, non-zero proportions of infectious individuals, suggesting that the public opinion will continue to propagated long-term without self-extinguishing. Our analysis indicates that the endemic disease equilibrium points achieves global asymptotic stability when $R_0 > 1$, ensuring that the public opinion propagation level remains stable without further interventions.

Optimal management

The SEIR public opinion propagation model with management measures in a bilingual environment is Eq. (9); the parameter values can be found in Table 1 below, and take $B_1 = 1$, $B_2 = 1$, $B_3 = 2$, $B_4 = 2$, $B_5 = 3$, $B_6 = 3$. Figures 5, 6, 7 gives the density change of each populations over time in the three scenario. It should

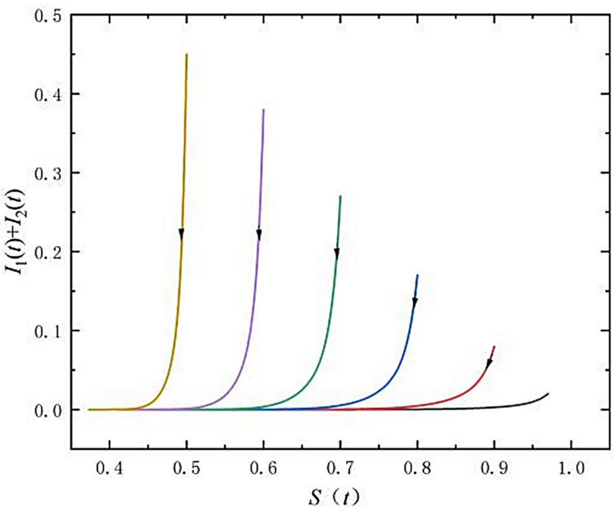

Fig. 3. Global asymptotic stability of the disease-free equilibrium points at $R_0 < 1$.

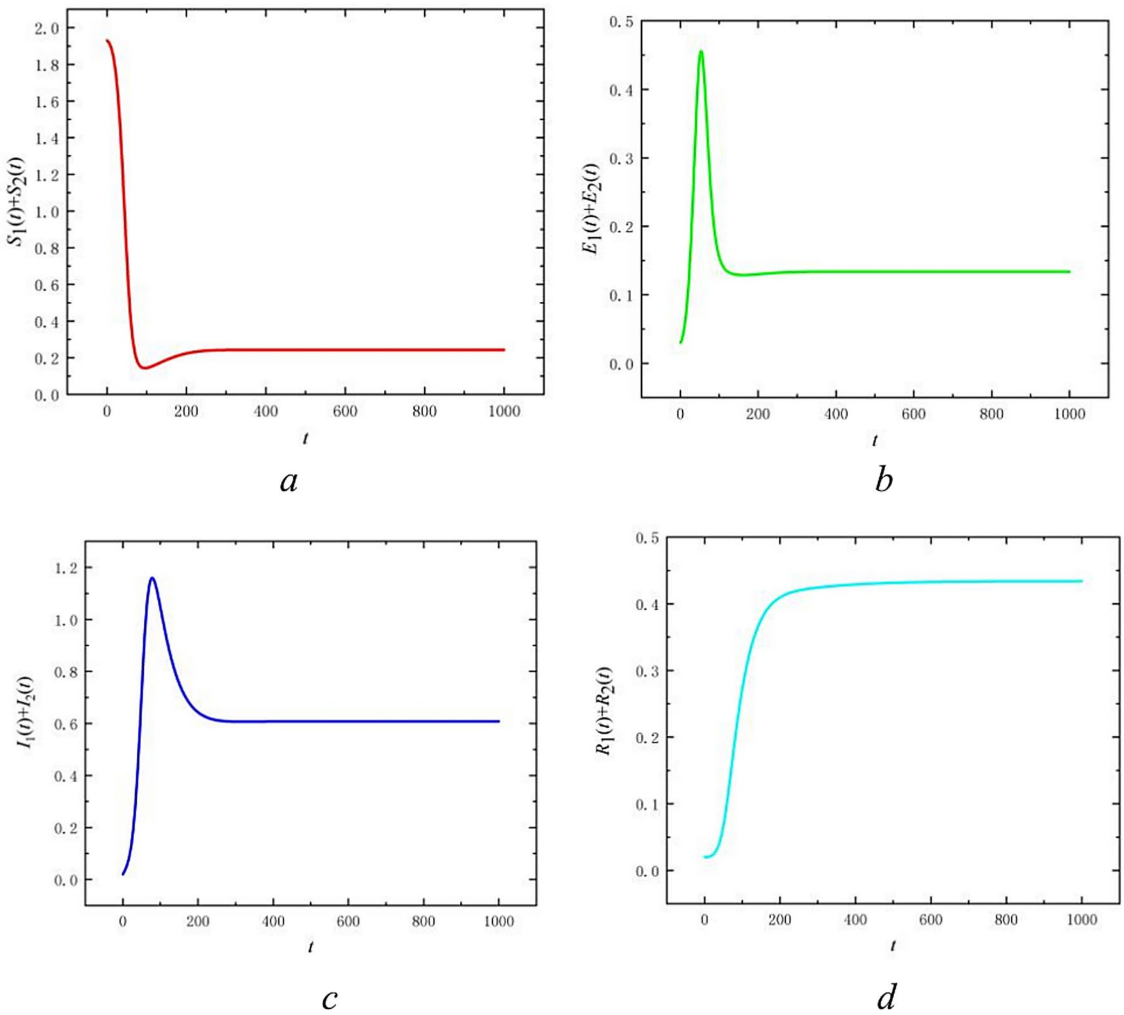

Fig. 4. Trends of four types of nodes over time in bilingual environment at $R_0 > 1$.

Parameter	Value	Parameter	Value
β_{11}	0.41	γ_2	0.09
β_{12}	0.36	d_1	0.04
β_{21}	0.34	d_2	0.05
β_{22}	0.39	b_1	0.07
α_1	0.20	b_2	0.08
α_2	0.21	μ_{12}	0.04
γ_1	0.08	μ_{21}	0.05
η_{12}	0.03	η_{21}	0.04

Table 1. Parameter settings.

be noted that under optimal management , the density of susceptible $S_i\,(t)$ and recovered individuals $R_i\,(t)$ increases, whereas the density of infectious $I_i\,(t)$ and latent individuals $E_i\,(t)$ rapidly decreases and stabilizes more quickly. Furthermore, the graph illustrates that management intensity gradually escalates from zero at the onset, reaches a peak, and subsequently declines to zero or to a minimal level.From the figure, it can be seen that the two optimal management strategies are effective in managing public opinion propagation in bilingual environments.

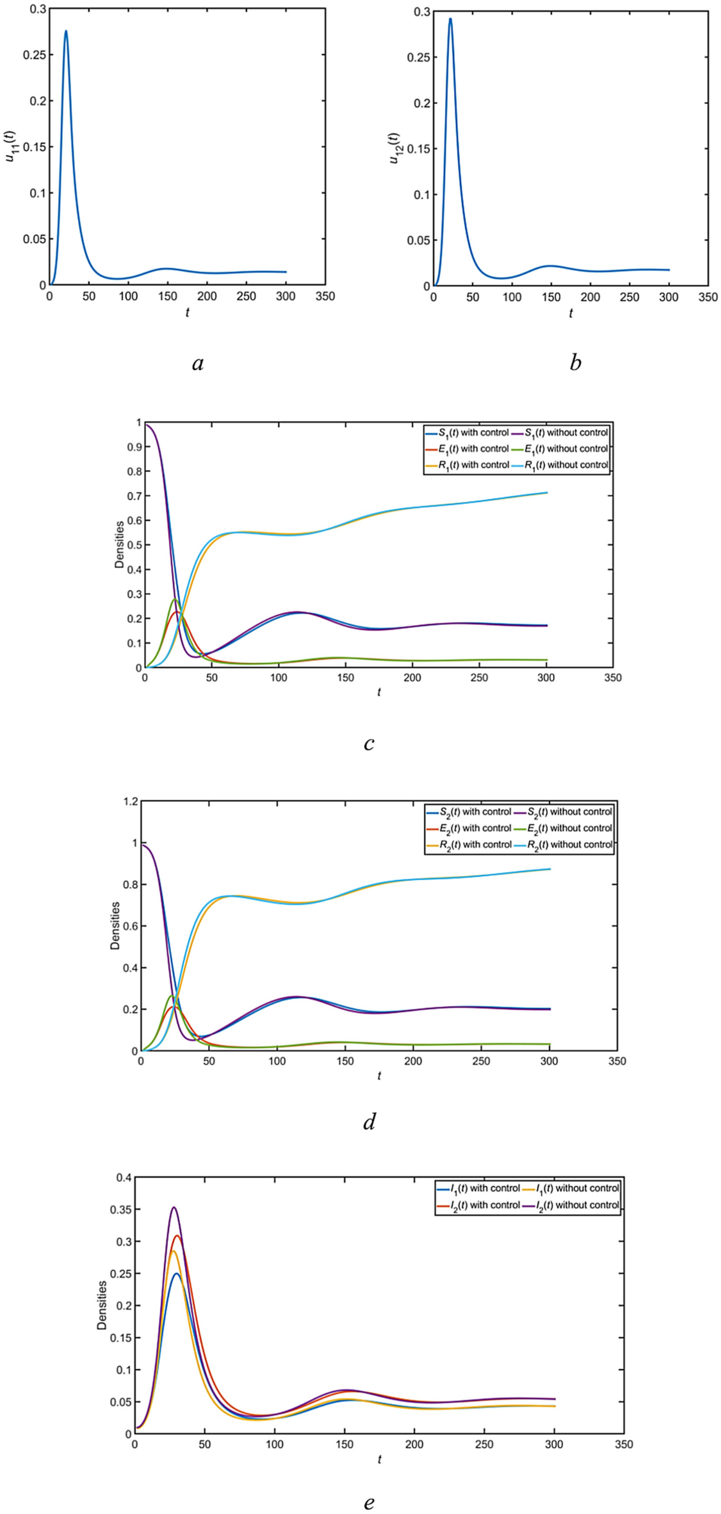

Fig. 5. The dynamic of $S_i\ (t)$, $R_i\ (t)$, $I_i\ (t)$, $E_i\ (t)$ and management trajectories for the first scenario($u_{11} > 0$, $u_{12} > 0$, $u_{21} = 0$, $u_{22} = 0$).

Model parameter analysis

The effect of propagation rate on the propagation of public opinion

To examine the effect of propagation rate β_{11} on public opinion, we use the data from Table 2 for numerical simulations. We hold all other parameter values constant and vary the propagation rate β_{11} at 0.1, 0.3, 0.5, 0.7, and 0.9. Figure 8a shows how the total infectious individuals $I_1 + I_2$ varies between bilingual groups at different β_{11} values. The figure illustrates that as β_{11} increases, the peak of public opinion contagion rises and occurs

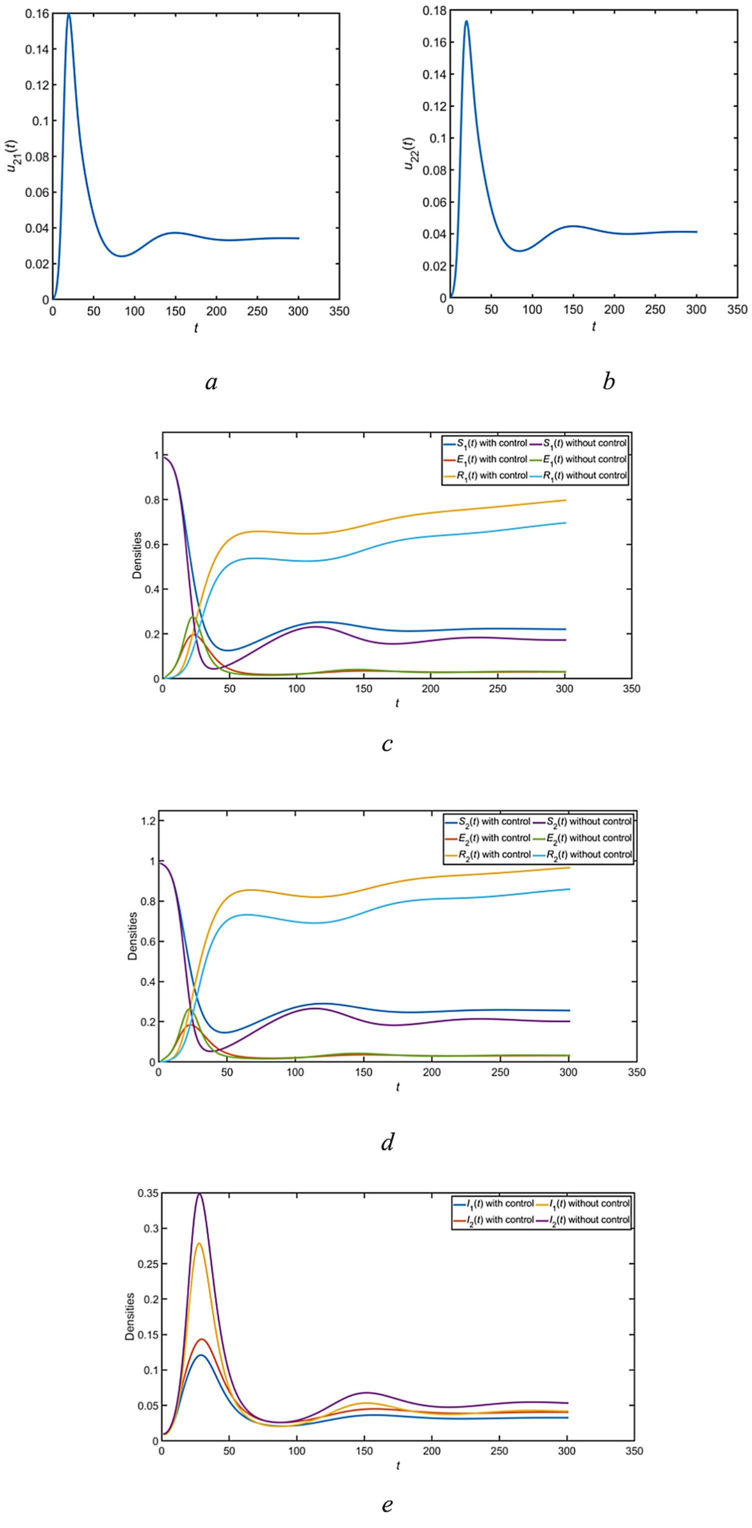

Fig. 6. The dynamic of $S_i\,(t)$, $R_i\,(t)$, $I_i\,(t)$, $E_i\,(t)$ and management trajectories for the second scenario($u_{11}=0, u_{12}=0, u_{21}>0, u_{22}>0$).

more rapidly. Consequently, a higher contagion rate results in a quicker and broader dissemination of public opinion within the first linguistic group.

To assess the effect of the propagation rate β_{22} on public opinion, we utilized data from Table 2 for numerical simulations, keeping all other parameter values constant and setting β_{22} at 0.05, 0.15, 0.25, 0.35, and 0.45. Figure 8b illustrates the effects of varying β_{22} values on public opinion propagation within the second linguistic group. As with β_{22}, increasing its value not only raises the peak but also accelerates the rate of opinion

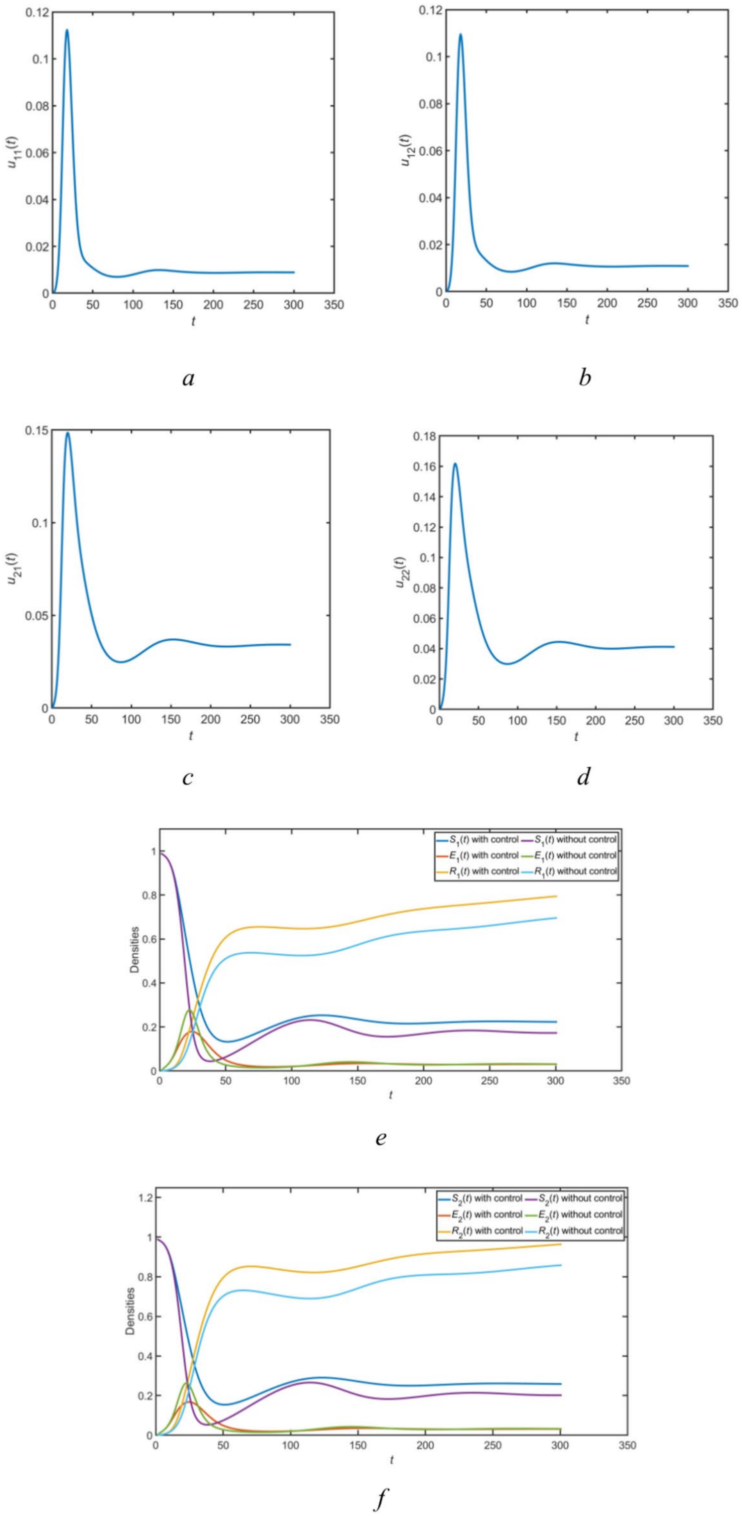

Fig. 7. The dynamic of $S_i\,(t)$, $R_i\,(t)$, $I_i\,(t)$, $E_i\,(t)$ and management trajectories for the third scenario($u_{11} > 0, u_{12} > 0, u_{21} > 0, u_{22} > 0$).

propagation, suggesting that both the speed and extent of propagation in the second linguistic group intensify with higher propagation rates.

The effect of cross-transmission mechanisms on the propagation of public opinion

To analyze the effect of the cross-transmission rate β_{12} on public opinion, we conducted numerical simulations using data from Table 2. All other parameters remained constant while β_{12} varied at 0.1, 0.3, 0.5, 0.7, and 0.9.

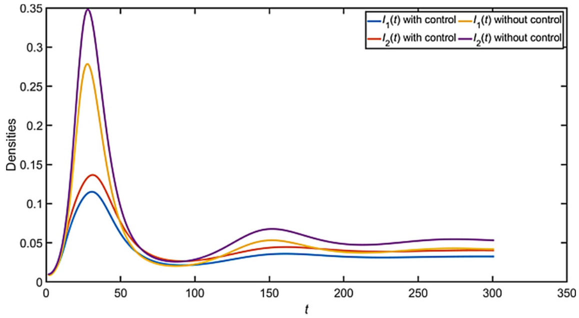

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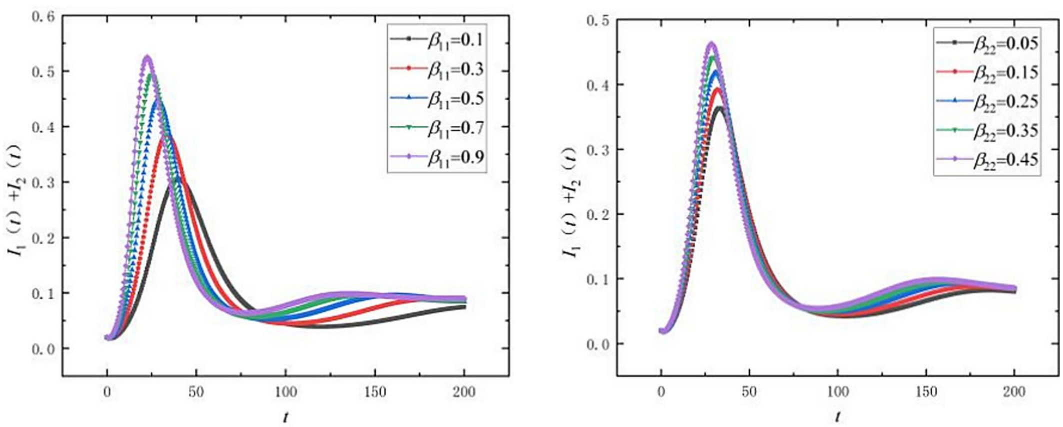
Figure 7. (continued)

Parameter	Value	Parameter	Value
β_{11}	0.40	γ_2	0.09
β_{12}	0.30	d_1	0.015
β_{21}	0.25	d_2	0.015
β_{22}	0.35	b_1	0.01
α_1	0.25	b_2	0.01
α_2	0.15	μ_{12}	0.04
γ_1	0.08	μ_{21}	0.05
η_{12}	0.03	η_{21}	0.06

Table 2. Parameter settings.



a b

Fig. 8. Effect of propagation rate on the propagation of public opinion.

Figure 9a illustrates how the total infectious individuals $I_1 + I_2$ respond in bilingual groups at various β_{12} values. The data indicate that higher β_{12} values accelerate the rate of public opinion propagation, particularly in the first language group, resulting in more rapid propagation and higher peak values. Increased propagation speeds result in infectious individuals propagate public opinions more rapidly, thereby shortening the duration of propagation. This acceleration is due to the enhanced β_{12} value facilitating faster transmission of opinions between the second and first language groups, thereby expediting public opinion propagation in the first language group.

To investigate the effect of cross-transmission rate β_{21} on public opinion, we used data from Table 2 for numerical simulations with all other parameters held constant. β_{21} was set at values of 0.05, 0.15, 0.25, 0.35, and 0.45. Figure 9b illustrates the effect of various β_{21} values on public opinion propagation in bilingual groups. This Figure illustrates that increasing β_{21} values accelerate the propagation of public opinion in the second language group. Additionally, a faster propagation speed results in higher peak levels of opinion spread. The acceleration

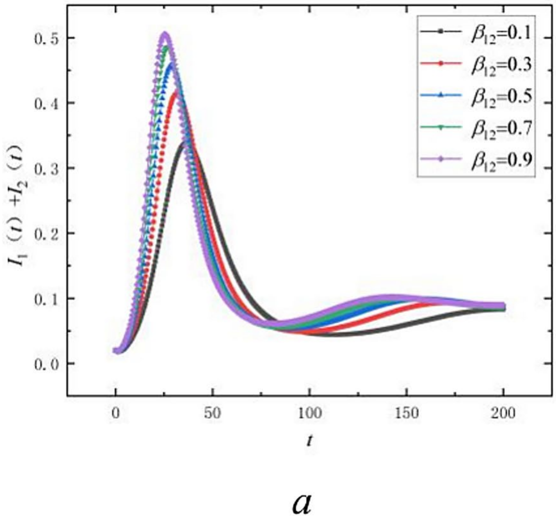


Fig. 9. Effect of cross-transmission mechanisms on the propagation of public opinion.

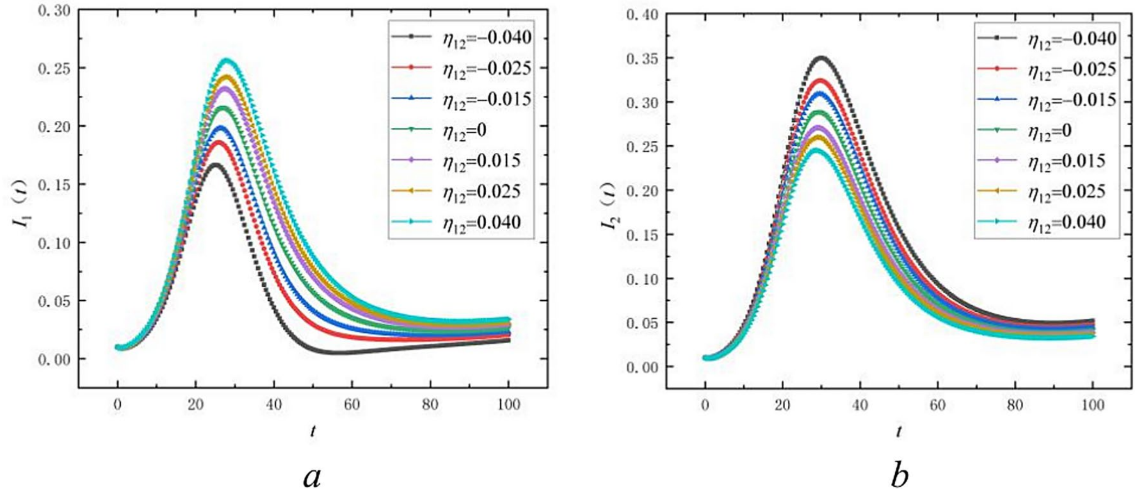


Fig. 10. Social Enhancement Effect η_{12} on the propagation of public opinion.

of opinion propagation in this group is directly attributed to the increased β_{21} values, which enhance the speed of spread.

The effect of social enhancement effects on the propagation of public opinion

When examining the impact of the social enhancement effect η_{12} on public opinion propagation, we conduct numerical simulations using the data from Table 2, keeping other parameter values unchanged. The values of η_{12} are selected as -0.04, -0.025, -0.015, 0, 0.15, 0.025, and 0.04. Equation (2) shows that η_{12} can be negative, zero, or positive, representing the following: $\eta_{ij} > 0$ indicates that the j-th language population facilitates opinion propagation in the i-th language population;$\eta_{ij} < 0$ indicates inhibition of public opinion propagation from the j-th language to the i-th language population; $\eta_{ij} = 0$ indicates no effect on public opinion propagation between the different language groups. From Fig. 10a, we observe changes in the infectious individuals I_1 within the first language group across different η_{12} values. Specifically, as η_{12} increases (from negative to positive), there is an advancement and increase in the peak number of infectious individuals I_1 within the first language group. This indicates that a positive η_{12} value suggests a facilitating effect of the second language group on public opinion contagion agents within the first language group. Conversely, a decrease in η_{12}(from positive to negative values) delays and reduces the peak of I_1, demonstrating an inhibitory effect. In Fig. 10b, we observe the variation of the infectious individuals I_2 within the second language group across different values of η_{12}. When $\eta_{12} = 0$, it indicates negligible opinion contagion interaction between groups. Similarly to I_1, the dynamics of infectious individuals in the second language group are affected by the change in η_{12}, albeit opposite to the effect on I_1; that is, with η_{12} (from negative to positive values), there is a delay and decrease in the peak number of infectious individuals I_2 in the second language group.

When analyzing the impact of the social enhancement effect η_{21} on public opinion propagation, we conducted numerical simulations using the data from Table 2. Other parameters in Table 2 remained constant, while η_{21} was varied at values of -0.04, -0.025, -0.015, 0, 0.15, 0.025, and 0.04. From Fig. 11a, we observe the variation of I_1 in the first language group under different values of η_{21}. Specifically, as η_{21} increases from negative to positive values, there is an advancement and increase in the peak number of infectious individuals I_2 in the second language group. This indicates that when $\eta_{21} > 0$ the first language group facilitates opinion contagion in the second language group. Conversely, as η_{21} decreases from positive to negative values, the peak of I_2 is delayed and decreases, indicating an inhibitory effect. Figure 11b illustrates the variation of infectious individuals I_1 in the first language group across different η_{21} values. Similarly to I_2, we observe a trend where changes in η_{21} affect the dynamics of infectious individuals in the first language group. However, this effect is opposite to that observed for I_2; specifically, there is a delay and decrease in the peak number of infectious individuals I_1 within the first language group as η_{21} transitions from negative to positive values.

The impact of social enhancement effects on Public Opinion Propagation in multilingual environment involves multiple interacting factors, such as foundational influence capacity, cultural proximity, contagion proportion, and threshold parameters. Foundational influence capacity reflects the basic level of interaction among diverse linguistic groups without further social or cultural influences. Higher values correlate with linguistic similarities, frequent communication, and positive historical interactions, facilitating faster information dissemination. Cultural proximity evaluates whether cultural similarities facilitate or impede information flow. Closer cultural ties lead to more efficient information dissemination, enhancing the spread of public opinion.

The contagion proportion indicates the percentage of group members actively involved in public opinion propagation. A higher contagion proportion signifies increased activity in disseminating information, which can amplify the impact of public opinion. The threshold parameter determines whether the social enhancement effect fosters, inhibits, or has no impact on Public Opinion Propagation, highlighting crucial conditions for inter-group interaction. When the contagion proportion exceeds the threshold, it promotes faster dissemination of public opinion; conversely, falling below the threshold can impede dissemination due to disparities in information acceptance or cultural isolation.

Therefore, the direction and magnitude of social enhancement effects directly influence the speed and scope of Public Opinion Propagation among diverse linguistic groups. These factors collectively reveal the complex mechanisms of Public Opinion Propagation in multilingual societies and provide new insights and strategies for opinion management in multicultural settings.

The effect of forgetting mechanism on the propagation of public opinion

When examining the impact of the forgetting mechanism μ_{12} on public opinion propagation, we utilized the data from Table 2 for numerical simulation, maintaining the constant values of other parameters. The values of μ_{12} were chosen as follows: 0.05, 0.15, 0.25, 0.35, and 0.45. From Fig., we observe the variation of the total infectious individuals $I_1 + I_2$ across different values of μ_{12} in the bilingual groups. As depicted in Fig. 12a, $I_1 + I_2$ represents the temporal evolution of the total number of infectious individuals across bilingual groups. With an increase in μ_{12}, we note a decrease in the peak value of $I_1 + I_2$ along with its rightward shift, signifying the attenuation of the speed and breadth of public opinion propagation due to the forgetting effect. The elevation of μ_{12} decelerates the pace and extent of public opinion propagation within the first language group.

When examining the impact of the forgetting mechanism μ_{21} on public opinion propagation, we utilize the data from Table 2 for numerical simulation, maintaining constant values for other parameters. The values of μ_{21} are as follows: 0.05, 0.15, 0.25, 0.35, and 0.45. Figure 12b illustrates the variations in the total infectious individuals $I_1 + I_2$ across the bilingual groups at different μ_{21} values. As depicted in Fig. 12b, an increase in μ_{21} results in a decrease in the peak value of $I_1 + I_2$, accompanied by a rightward shift, indicating once more that the speed of opinion propagation and its influence are curbed by the forgetting effect. The elevation of μ_{21} decelerates the propagation speed and breadth of public opinion within the second language group.

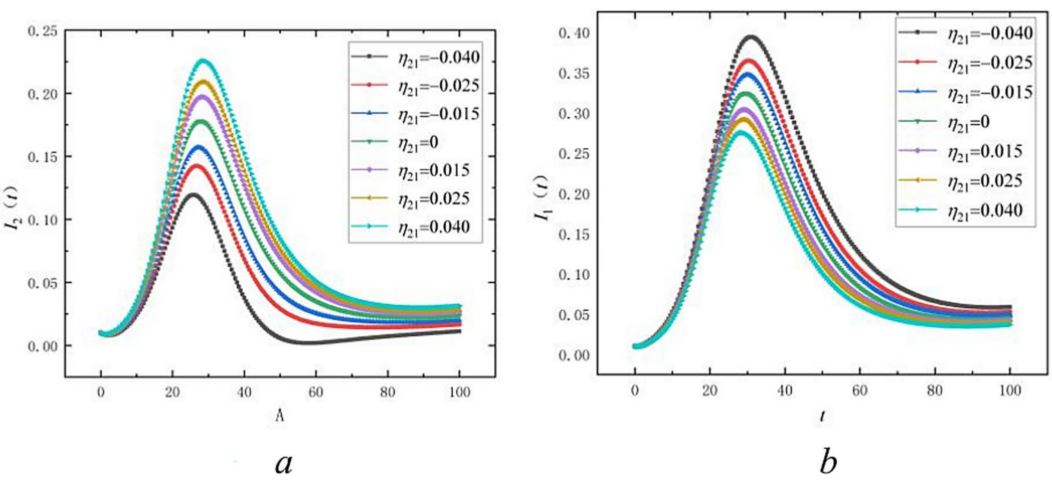


Fig. 11. Social Enhancement Effect η_{21} on the propagation of public opinion.

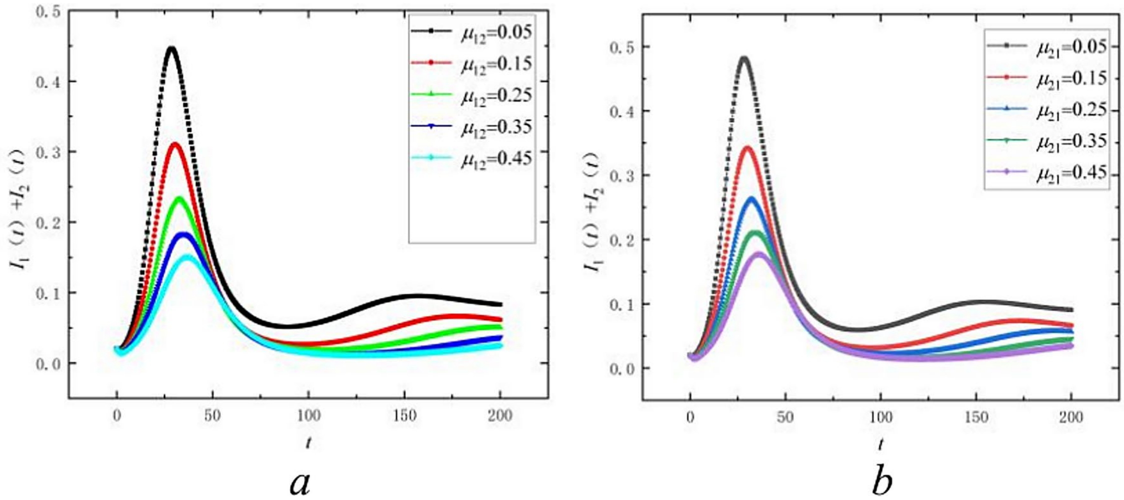


Fig. 12. The effect of forgetting mechanism on the propagation of public opinion.

Parameter	Value	Parameter	Value
β_{11}	0.50	γ_2	0.09
β_{12}	0.02	d_1	0.004
β_{21}	0.03	d_2	0.005
β_{22}	0.25	b_1	0.007
α_1	0.20	b_2	0.008
α_2	0.23	μ_{12}	0.02
γ_1	0.08	μ_{21}	0.02
η_{12}	0.02	η_{21}	0.02

Table 3. Parameter settings.

These simulation results clearly show the impact of factors like base forgetting rate, adjustments in cross-linguistic social enhancement effects, cultural proximity, and information quality on the forgetting mechanism in a multilingual setting. The base forgetting rate indicates the natural rate at which different linguistic groups forget information, unaffected by external factors. This ratio is influenced by cognitive ability, information complexity, and frequency of exposure. Meanwhile, the cross linguistic social enhancement effect indicates that communication frequency, understanding level, and information resonance between language groups can accelerate or slow down the rate of forgetting. Cultural similarity improves the efficiency of information dissemination, reduces forgetting rates, and enables culturally similar groups to more effectively retain information. The quality of information, especially its reliability and relevance, can effectively affect the retention of information over time, slowing down the rate of information forgetting.

Over time, with the continuous influx of new information and limited attention, old information is gradually forgotten. However, influenced by the factors mentioned earlier, this forgetting process varies greatly among different language groups. It not only affects the absorption capacity of information, but also affects the formation and dissemination process of public opinion. These findings highlight the necessity of considering these factors in global cross-cultural interaction.

The impact of asymmetric initial conditions on multilingual Public Opinion Propagation

In this simulation, we constructed a bilingual Public Opinion Propagation model with initial conditions set as asymmetric dissemination scenarios. Specifically, the susceptibility of the second language group is 100% ($S_2 = 1$), with 0 initial infected individuals ($I_2 = 0$), while the first language group has a certain proportion of initial infected individuals ($I_1 = 0.05$). The values of other parameters are shown in Table 3. This setting aims to simulate certain language groups being unaffected by public opinion in the early stages due to cultural isolation, information barriers, or limitations in communication channels, while another language group has already begun to spread public opinion. The design of such asymmetric initial conditions helps us understand how cross linguistic communication and information lag affect the dynamic evolution of two language groups in the process of Public Opinion Propagation.

From Fig. 13a, it can be seen that there are significant differences in the dissemination patterns of public opinion between the two language groups: the first language group (S_1, E_1, R_1): Initially, due to the presence of infected individuals ($I_1 = 0.05$), public opinion spreads rapidly in the first language group. As the number of infected individuals increases, the number of susceptible individuals (S_1) rapidly decreases, and the spread of public opinion enters the incubation period (E_1), gradually reaching its peak. As time goes by, the number of infected individuals decreases and the number of recovered individuals (R_1) gradually increases, indicating a gradual decline in public opinion. The restorer curve (R_1) shows that over time, public opinion in this language group has been effectively controlled and eventually stabilized. Second language groups (S_2, E_2, R_2): In contrast, due to the initial lack of infected individuals, the dissemination of public opinion in the second language group is significantly lagging behind. Public opinion spreads across languages from the first language group to the second language group (through parameters such as β_{12} and β_{21}). After a certain period of time, the susceptible individuals (S_2) in the second language group are gradually affected, and the transmission curve begins to emerge, with E_2 and R_2 starting to change. Eventually, the spread of public opinion among the second language group gradually reached its peak and then entered a period of recovery.

This asymmetric dissemination pattern indicates that there is a significant cross linguistic communication lag phenomenon in the dissemination of public opinion. Due to the initial lack of infected individuals, the second language group was largely unaffected in the early stages of Public Opinion Propagation. However, with the passage of time and the strengthening of cross linguistic transmission effects, this group eventually entered the cycle of Public Opinion Propagation.

Figure 13b shows the changes in the number of infected individuals (I_1, I_2) in two language groups over time, further revealing the asymmetry of Public Opinion Propagation:I_1 curve (infected individuals in the first language group): At the beginning, the number of infected individuals rapidly increases and reaches a peak in a short period of time, then gradually decreases and tends to stabilize. This trend reflects the rapid spread of public opinion within the first language community and the subsequent decrease in the number of infected individuals due to recovery or management measures. I_2 curve (infected individuals in the second language group): In contrast, the infected individuals curve in the second language group lags significantly behind. The number of infected individuals only began to significantly increase after a certain period of time, indicating that public opinion gradually expanded to the second language group through cross linguistic transmission. In the end, the trend of I_2 changes is consistent with I_1, but the entire propagation process is significantly lagging behind.

This phenomenon reveals the delayed effect of information transmission across languages. The increase in the number of infected individuals in the second language group is limited by interaction with the first language group (through cross linguistic transmission parameters) due to being in a completely susceptible state in the initial stage. As the public opinion of the first language group gradually enters a stable period, the number of infected individuals in the second language group begins to increase significantly.

The simulation results reveal the complexity of cross linguistic communication from the perspective of dynamic propagation. In reality, certain language groups may exhibit significant lag in the early stages of Public Opinion Propagation due to cultural isolation or differences in information dissemination channels. For example, certain language groups may not have received sufficient public opinion information in the first instance during a specific event, resulting in them starting to be influenced by public opinion after a longer period of time. On the other hand, the cross linguistic dissemination rate is reflected in the model through the β_{12} and β_{21} parameters, which simulate the social interaction intensity and information transmission efficiency between different language groups.

This model is of great significance to policy makers, social media platforms and public crisis managers. By understanding the lag effect in multilingual public opinion communication, relevant departments can formulate targeted emergency measures in advance, and carry out phased public opinion management and guidance based on the communication characteristics of different language groups.

The impact of network structure on the process of public opinion

To verify the impact of complex network topology on the process of public opinion propagation, three typical network structures were selected for comparison. Using the NetLogo software, simulation experiments were conducted on the SEIR-based public opinion propagation model in a bilingual environment across three

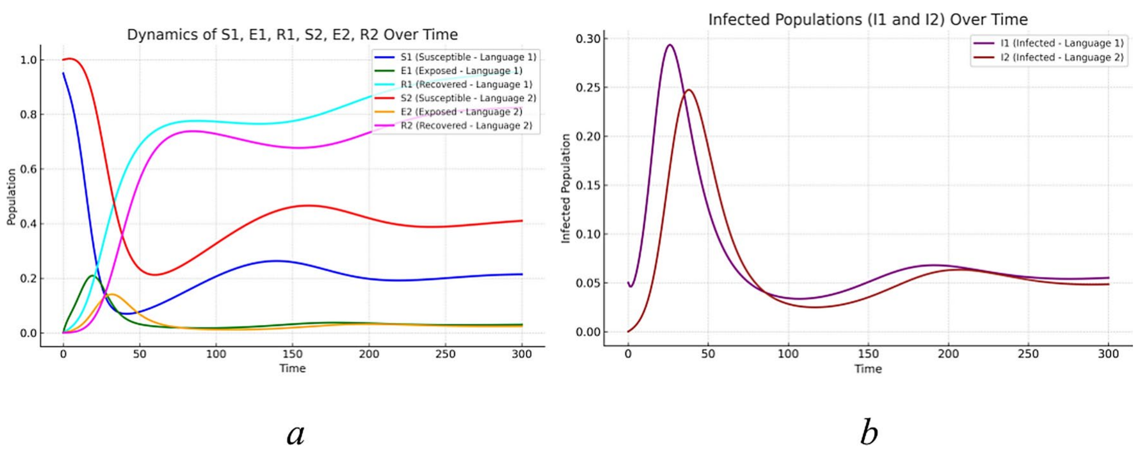


Fig.13. The impact of asymmetric initial conditions on multilingual Public Opinion Propagation.

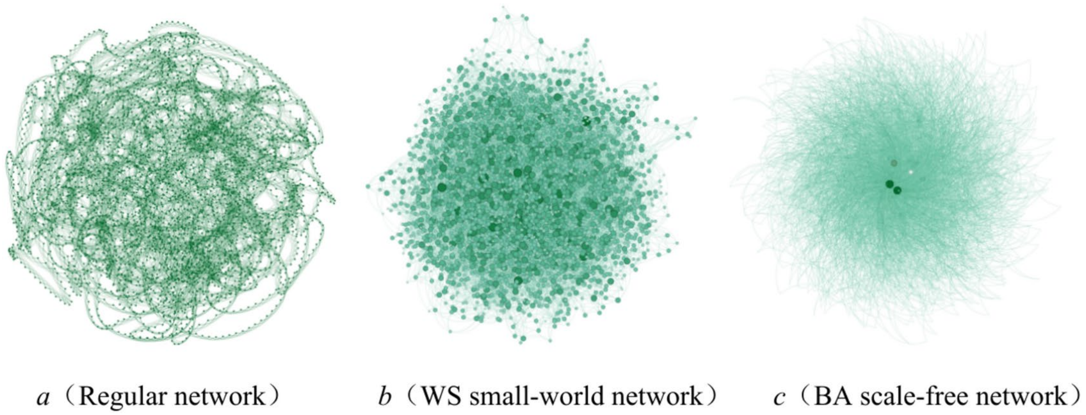


Fig. 14. Visualization of three types complex networks.

Parameter	Value	Parameter	Value
β_{11}	0.90	γ_2	0.07
β_{12}	0.80	d_1	0.05
β_{21}	0.85	d_2	0.05
β_{22}	0.80	b_1	0.01
α_1	0.40	b_2	0.01
α_2	0.30	μ_{12}	0.04
γ_1	0.09	μ_{21}	0.05
η_{12}	0.03	η_{21}	0.04

Table 4. Parameter settings.

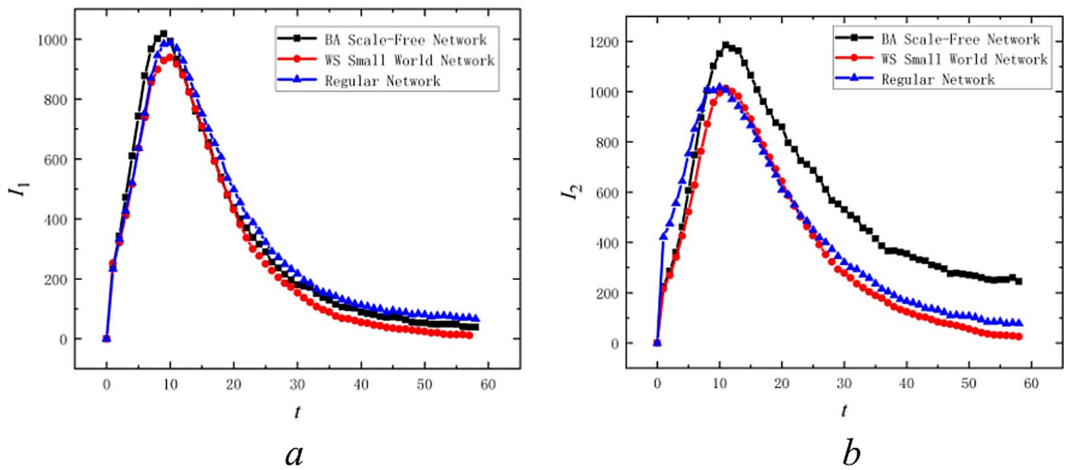


Fig. 15. The Impact of network structure on the process of public opinion.

networks: a regular network (4000 nodes, 16,000 edges), a WS small-world network (4000 nodes, 16,000 edges), and a BA scale-free network (4000 nodes, 8000 edges). As shown in Fig. 14, the visualization of these three classical complex networks is presented. The parameter settings are shown in Table 4.

Figure 15 simulates the temporal evolution of the densities of two different language propagators,I_1 and I_2, within three different network topologies. Figure 15a illustrates the propagation process of language 1 propagators across the three networks. The black curve represents the scale-free network, the red curve represents the small-world network, and the blue curve represents the regular network. The overall trend shows that all three curves initially rise to a peak, then decline, and eventually stabilize. As depicted, the black curve reaches the highest peak, followed by the blue curve, and the red curve has the lowest peak, but the red curve stabilizes the earliest. Figure 15b depicts the propagation process of language 2 propagators across the three networks. Similarly, the black curve represents the scale-free network, the red curve represents the small-world network, and the blue curve represents the regular network. In Fig. 14b, the differences in peak values among the three curves are even more pronounced, with the black curve having the highest peak, the blue curve the second highest, and the red curve the lowest. Again, the red curve stabilizes the earliest. The underlying reason is that the BA scale-free network is characterized by a power-law distribution of node connections, where the majority of nodes have few connections, while a few hub nodes have a large number of connections. These hub nodes play a critical role in the spread of public opinion, leading to the highest rise in propagator density. The small-world network, which lies between a completely regular network and a completely random network, introduces a small probability of long-distance connections by rewiring the original nearest-neighbor nodes. This network structure is characterized by most nodes being locally connected, with occasional long-distance connections, which facilitates the rapid spread of public opinion information.

Validation of the model on the Twitter dataset

In this section, we utilize a real dataset from Twitter[51] to evaluate the effectiveness of the proposed model. As of September 2021, this dataset contains over 2.2 billion tweets collected since January 22, 2020, covering discussions about COVID-19 and published in multiple languages. Among these multilingual tweets, English tweets have the highest proportion, followed by tweets in Spanish and Catalan. Consequently, we selected data from tweets related to COVID-19, published in English, Spanish, and Catalan during the first 100 days from January 22, 2020, as the dataset to validate the SEIR opinion propagation model proposed in this paper under a bilingual environment. Given the large volume of data, we define a user as an opinion spreader only if they post tweets related to COVID-19 in English, Spanish, or Catalan, without considering the number of likes received. As shown in Table 5, each column of the dataset contains specific information related to individual tweets and retweets. This information includes user ID, timestamp corresponding to the date and time of each tweet or retweet, the language used for the tweet, the status of the tweet, and other relevant details.

We employ the SEIR opinion propagation model under a bilingual environment, fitting I_1 to represent the temporal dynamics of English opinion spreaders on Twitter and I_2 to represent the temporal dynamics of Spanish and Catalan opinion spreaders. According to Twitter user data,[52] the daily active user count on Twitter in 2020 was 192 million, an increase from 152 million in 2019. Therefore, we assume that each user posts only one tweet per day and use this assumption to calculate the proportion of English tweets relative to the total daily active users, thereby estimating the proportion of English opinion spreaders. A similar method is applied to estimate the proportion of Spanish and Catalan opinion spreaders. The parameter settings for the fitting process are detailed in Table 6, and the fitting results are illustrated in Fig. 16.

Figure 16 illustrates the model's ability to capture the dynamics of public opinion propagation in a multilingual context, focusing on English and Spanish tweet activity related to COVID-19. The blue and green curves represent the actual tweet volumes in English and Spanish, respectively, while the red and orange dashed curves reflect the SEIR model's fitted results. The model incorporates three critical mechanisms: social enhancement, forgetting, and cross-transmission.

As shown in the figure, the model effectively fits the Spanish tweet data throughout both the early and later stages of the propagation process, demonstrating the influence of social enhancement—where active opinion spreaders amplify their impact within smaller, more homogenous language groups. In contrast, the forgetting mechanism, which accounts for the decay of engagement over time, is more evident in the English data, where the higher volume of tweets leads to more complex and unpredictable behavior, particularly in the later stages.

In conclusion, the SEIR opinion propagation model, applied within a bilingual environment, has demonstrated its effectiveness in capturing the dynamics of public opinion across multiple languages. The model shows particularly strong performance in fitting the propagation trends of Spanish and Catalan tweets, where opinion dynamics are more consistent and predictable. This superior fitting underscores the model's ability to adapt to smaller, more homogeneous linguistic groups, which exhibit less variability in opinion spread over time.

Compared to single-language models, the bilingual SEIR model offers a more comprehensive reflection of opinion propagation in multilingual and multicultural contexts. It captures the nuances of cross-transmission between languages, social enhancement within linguistic communities, and the decay of engagement through the forgetting mechanism. These features not only improve the model's precision and reliability but also highlight its capability to account for the complexities inherent in multilingual environments.

Tweet_ID	Language	Date created	···	Likes	Retweets
1,477,068,982,054,268,936	En	['2020', 'Jan', '22', '00:07:42']	···	0	0
1,477,070,543,174,225,927	En	['2020', 'Jan', '22', '00:13:54']	···	1	0
1,477,070,851,006,779,393	En	['2020', 'Jan', '22', '00:15:08']	···	0	0
1,477,067,130,210,295,809	Fr	['2020', 'Jan', '22', '00:00:21']	···	0	3
1,477,067,209,017,110,533	Fr	['2020', 'Jan', '22', '00:00:39']	···	0	3
···	···		···	···	···
1,477,068,038,793,043,973	Pt	['2020', 'Jan', '22', '00:03:57']	···	0	37
1,477,068,069,625,376,768	Pt	['2020', 'Jan', '22', '00:04:05']	···	0	37
1,477,068,091,574,165,508	Fr	['2020', 'Jan', '22', '00:04:10']		0	368

Table 5. Twitter real dataset.

Parameter	Value	Parameter	Value
β_{11}	0.85	γ_2	0.0004
β_{12}	0.65	d_1	0.00028
β_{21}	0.57	d_2	0.00045
β_{22}	0.37	b_1	0.007
α_1	0.0250	b_2	0.008
α_2	0.0028	μ_{12}	0.002
γ_1	0.0005	μ_{21}	0.003
η_{12}	0.002	η_{21}	0.003
$S_1(0)$	0.9995	$S_2(0)$	0.9994
$I_1(0)$	0.0005	$I_2(0)$	0.0006

Table 6. Parameter values of public opinion communication system in bilingual environments.

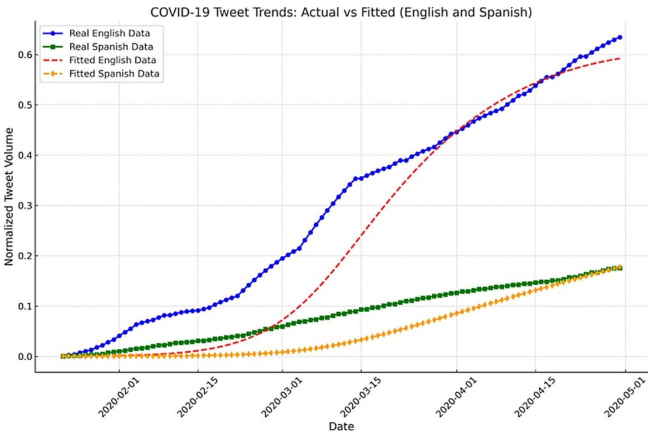


Fig.16. Fitting effect of Twitter real dataset.

This multilingual modeling approach broadens the model's applicability, enhancing its practicality for global opinion analysis. By incorporating diverse linguistic and cultural dynamics, the model becomes a valuable tool for more accurate and inclusive public opinion management in increasingly interconnected and multicultural societies.

Conclusion

This study focuses on understanding the dynamics of public opinion in multilingual environments where cultural and linguistic diversity introduces significant complexity to the spread of information. Traditional models often fail to capture these intricacies, leading to less accurate predictions of public opinion trends.

To address these limitations, this paper develops a comprehensive SEIR-based model tailored specifically for multilingual contexts. The model incorporates several advanced mechanisms, including social enhancement (reflecting the influence of opinions within and across language groups), forgetting mechanisms (showing how opinions diminish over time), and cross-transmission mechanisms (demonstrating how opinions spread between different linguistic communities).

A key aspect of this research is the acknowledgment and incorporation of the challenges posed by language and cultural diversity. By considering these differences, the model promotes an inclusive approach to public opinion management, ensuring that all cultural groups, regardless of their language, are fairly represented in public discourse. This inclusivity not only adheres to ethical standards but also significantly enhances the model's applicability to real-world multilingual societies.

The model was validated using real Twitter data related to COVID-19, which included multilingual tweets in English, Spanish, and Catalan. The results demonstrate that the model effectively captures the dynamics of opinion propagation across different languages, particularly in languages with fewer users, where opinion spread tends to be more predictable.

Key findings from the study include:

The model accurately reflects the temporal dynamics of opinion spread across different linguistic groups.

Social enhancement and cross-transmission mechanisms play crucial roles in shaping the spread and persistence of opinions in multilingual environments.

The model's predictive power is especially strong in languages with lower tweet volumes, such as Spanish and Catalan, where opinion trends are less complex.

In summary, this research contributes a robust and innovative framework for managing public opinion in multilingual societies. The integration of mechanisms to account for cultural and linguistic influences significantly improves the model's predictive power and applicability. As a result, the model offers valuable insights for policymakers and social media analysts looking to manage public opinion in increasingly globalized and multilingual contexts.

Data availability

The datasets generated and/or analyzed during the current study are available in the [GitHub] repository, [https://github.com/suyalatu1984/SuyalatuDong.git]. The datasets are also available from the corresponding a uthor upon reasonable request.

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References

1. Mingxin, Z. Survey on modeling and simulation of internet public opinion in China. *J. Syst. Simulation* **31**(10), 1983–1994 (2019).
2. Allport G W, Postman L .The Psychology of Rumor[M], USA Oxford, 1947.
3. Goffman, W. & Newill, V. A. Generalization of epidemic theory: an application to the transmission of ideas. *Nature* **204**(4955), 225–228 (1964).
4. Daley, D. J. & Kendall, D. G. Epidemics and rumors. *Nature* **204**(4963), 1118 (1964).
5. D. P. T. Maki, M. D. P. Maki, and M. Thompson, Mathematical models and applications: with emphasis on the social, life and management sciences. Englewood Cliffs, New Jersey. Prentice Hall, 1973.
6. Sudbury, A. The proportion of the population never hearing a rumor. *J. Appl. Probability* **22**(2), 443–446 (1985).
7. Belen, S., Kropat, E. & Weber, G. W. On the classical Maki-Thompson rumor model in continuous time. *Central Eur. J. Operations Res.* **19**(1), 1–17 (2011).
8. Pastor-Satorras, R. & Vespignani, A. Epidemic dynamics and endemic states in complex networks. *Phys. Rev. E* **63**(2), 066117 (2001).
9. Zanette, D. H. Critical behavior of propagation on small-world networks. *Phys. Rev. E* **64**(5), 059901 (2001).
10. Zanette, D. H. Dynamics of rumor propagation on small-world networks. *Phys. Rev. E* **65**(4), 041908 (2002).
11. Newman, M. E. J. & Watts, D. J. Scaling and percolation in the small-world network model. *Phys. Rev. E* **60**(6), 7332–7342. https://doi.org/10.1103/PhysRevE.60.7332 (1999).
12. Trpevski, D., Tang, W. K. S. & Kocarev, L. Model for rumor spreading over networks. *Phys. Rev. E* **81**(5), 056102 (2010).
13. Huang, J. & Jin, X. Preventing rumor spreading on small-world networks. *J. Syst. Sci. Complex.* **24**, 449–456 (2011).
14. Zhu, L., Zhou, M. & Zhang, Z. Dynamical analysis and control strategies of rumor spreading models in both homogeneous and heterogeneous networks. *J. Nonlinear Sci.* **30**, 2545–2576 (2020).
15. Liu, Z., Lai, Y. C. & Ye, N. Propagation and immunization of infection on general networks with both homogeneous and heterogeneous components. *Phys. Rev. E* https://doi.org/10.1103/PhysRevE.67.031911 (2003).
16. Yuan, T. et al. Stability analysis and optimal control of epidemic-like transmission model with nonlinear inhibition mechanism and time delay in both homogeneous and heterogeneous networks. *J. Math. Anal. Appl.* **526**(1), 127273 (2023).
17. Zhang, M., Qin, S. & Zhu, X. Information diffusion under public crisis in BA scale-free network based on SEIR model-Taking COVID-19 as an example. *Phys. A: Stat. Mech. Appl.* **571**, 125848 (2021).
18. Wang, Y. M. et al. Direct immune-SCIR public-opinion propagation model based on real-time online users. *Chinese Phys. B* **29**(10), 100204 (2020).
19. Ai, S. et al. CSRT rumor spreading model based on complex network. *Int. J. Intelligent Syst.* **36**(5), 1903–1913 (2021).
20. Qianqian L, Yijun L, Ruya T, et al. Opinion dynamics on triad scale free network[C]//Active Media Technology: 8th International Conference, AMT 2012, Macau, China, December 4–7, 2012. Proceedings 8. Springer Berlin Heidelberg, 2012: 445–450.
21. Jiang, J. et al. Rumor source identification in social networks with time-varying topology. *IEEE Trans. Depend. Secure Comput.* **15**(1), 166–179 (2016).
22. Zhuang, Y. & Yağan, O. Information propagation in clustered multilayer networks. *IEEE Trans. Network Sci. Eng.* **3**(4), 211–224 (2016).
23. Li, Y. & Wang, J. Cross-network propagation model of public opinion information and its control in coupled double-layer online social networks. *Aslib J. Inf. Manage.* **74**(2), 354–376 (2022).
24. Geng, L. et al. Modeling public opinion propagation in a multilayer network with SEIR model based on real social networks. *Eng. Appl. Artif. Intell.* **125**, 106719 (2023).
25. Li, J. et al. Dynamical analysis of rumor spreading model in multi-lingual environment and heterogeneous complex networks. *Inf. Sci.* **536**, 391–408 (2020).
26. Xia, Y., Jiang, H. & Yu, Z. Global dynamics of ILSR rumor spreading model with general nonlinear spreading rate in multi-lingual environment. *Chaos, Solitons Fractals* **154**, 111698 (2022).
27. Wang, J. et al. Global dynamics of the multi-lingual SIR rumor spreading model with cross-transmitted mechanism. *Chaos, Solitons Fractals* **126**, 148–157 (2019).
28. Ye, M. et al. Global analysis of a fractional-order reaction-diffusion rumor-spreading model in multi-lingual environment. *Eur. Phys. J. Plus* **138**(1), 102 (2023).
29. Yu, S. et al. The spread and control of rumors in a multilingual environment. *Nonlinear Dynamics* **100**, 2933–2951 (2020).
30. Xia, Y. et al. Dynamic analysis and optimal control of a reaction-diffusion rumor propagation model in multi-lingual environment. *J. Math. Anal. Appl.* **521**(2), 126967 (2023).
31. Wang, J. et al. Stability and Hopf bifurcation analysis of multi-lingual rumor spreading model with nonlinear inhibition mechanism. *Chaos, Solitons Fractals* **153**, 111464 (2021).
32. Yang, S. et al. Dynamics of the rumor-spreading model with hesitation mechanism in heterogenous networks and bilingual environment. *Adv. Diff. Equat.* **2020**, 1–21 (2020).
33. Bharti M, Jindal H. Automatic rumor detection model on social media[C]//2020 Sixth International Conference on Parallel, Distributed and Grid Computing (PDGC). IEEE, 2020: 367–371.
34. Li, J. et al. Dynamical analysis of rumor spreading model in homogeneous complex networks. *Appl. Math. Comput.* **359**, 374–385 (2019).
35. Baddeley A. Working memory [M]//Memory. Routledge, 2020: 71–111.
36. Zhao, L. et al. Rumor spreading model considering forgetting and remembering mechanisms in inhomogeneous networks. *Phys. A: Stat. Mech. Appl.* **392**(4), 987–994 (2013).
37. Zhao, L. et al. Rumor spreading model with consideration of forgetting mechanism: a case of online blogging Live journal. *Phys. A: Stat. Mech. Appl.* **390**(13), 2619–2625 (2011).

38. Zhao, L. et al. A rumor spreading model with variable forgetting rate. *Phys. A: Stat. Mech. Appl.* **392**(23), 6146–6154. https://doi.org/10.1016/j.physa.2013.07.080 (2013).
39. Zhao, L. et al. SIHR rumor spreading model in social networks. *Phys. A: Stat. Mech. Appl.* **391**(7), 2444–2453 (2012).
40. Dong, X., Liu, Y., Chao, W., Lian, Y. & Tang, D. A double-identity rumor spreading model. *Phys. A: Stat. Mech. Appl.* **528**, 121479. https://doi.org/10.1016/j.physa.2019.121479 (2019).
41. Afassinou, K. Analysis of the impact of education rate on the rumor spreading mechanism. *Phys. A: Stat. Mech. Appl.* **414**, 43–52. https://doi.org/10.1016/j.physa.2014.07.041 (2014).
42. Katz E, Lazarsfeld P F. Personal influence: The part played by people in the flow of mass communications. Free Press, 1959.
43. Triandis H C. Individualism and collectivism. Routledge, 2018.
44. Anderson, J. R. & Schooler, L. J. Reflections of the environment in memory. *Psych. Sci.* **2**(6), 396–408 (1991).
45. Diekmann, O., Heesterbeek, J. A. P. & Metz, J. A. J. On the definition and the computation of the basic reproduction ratio R 0 in models for infectious diseases in heterogeneous populations. *J. Math. Biol.* https://doi.org/10.1007/BF00178324 (1990).
46. Van den Driessche, P. & Watmough, J. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Math. Biosci.* **180**(1–2), 29–48 (2002).
47. Berman A,Plemmons RJ.Nonnegative matrices in the mathematical sciences.New York: Academic Press; 1979.
48. Muroya, Y., Enatsu, Y. & Kuniya, T. Global stability for a multi-group SIRS epidemic model with varying population sizes. *Nonlinear Anal.: Real World Appl.* **14**(3), 1693–1704 (2013).
49. La Salle J P. The stability of dynamical systems [M]. Society for Industrial and Applied Mathematics, 1976.
50. Smith H L, Waltman P. The theory of the chemostat: dynamics of microbial competition [M]. Cambridge university press, 1995.
51. Lopez, C. E. & Gallemore, C. An augmented multilingual Twitter dataset for studying the COVID-19 infodemic. *Social Network Anal. Mining* **11**(1), 102 (2021).
52. https://www.statista.com/topics/737/twitter/#topicOverview.

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Declarations

Competing interests
The authors declare no competing interests.

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