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An intelligent assessment of factors affecting new generations in the era of the internet and new media using intuitionistic hesitant fuzzy sugeno-weber aggregation operators

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The network of millions of linked devices that exchange resources is known as the Internet. The rapid expansion of digital connectivity necessitates an assessment and understanding of its multifaceted impact on younger generations. There are many benefits and drawbacks to using the Internet, including issues with human health, education and learning habits, communication and relationships, and privacy and security. Almost every area of life involves some degree of uncertainty and fuzziness. The best method for practically reducing ambiguity when determining the best option from fuzzy and uncertain data is multi-attribute decision-making (MADM). The research aims to develop a comprehensive decision framework for examining Internet effects on youth generations through fundamental behavioral and social elements. So, in this regard, the Intuitionistic Hesitant Fuzzy set (IHFS) is an effective and flexible tool for the investigation of uncertain information. The Sugeno-Weber operational (SWO) rules are a more generalized and flexible approach than other existing triangular norms (TN) and triangular conorm (TCN). Using SWOs and IHFS information, we construct the intuitionistic hesitant fuzzy Sugeno-Weber weighted averaging (IHFSWWA) and intuitionistic hesitant fuzzy Sugeno-Weber weighted geometric (IHFSWWG) operators. Also, we investigate some fundamental axioms of aggregation operators (AOs), like monotonicity, boundedness, and idempotency. We construct the MADM algorithm based on the developed theory, and some solve real-life numerical examples for assessment of factors affecting the new generations given the internet. To validate the effectiveness and practical applicability, a comparison analysis was conducted between the proposed IHFSWWA and IHFSWWG approaches with existing AOs. The conclusion is discussed in the last section.

Keywords Internet and new media, Multi-attribute decision-making, Intuitionistic hesitant fuzzy set, Sugeno weber TN and TCN, Aggregation operators

The crisp set theory was the sole foundation for decision-making sciences in the past. There are only two ways to describe the concept of crisp set theory: Yes or no. But in many complex fuzziness-related scenarios nowadays, crisp set theory falls short. In 1965, Zadeh[1] presented the original idea of a fuzzy set (FS), which is defined by the membership value (MD) in the range of the interval [0,1], as a solution to these kinds of problems. In the decision-making sciences, this was a significant discovery. However, the value of non-membership degrees (NMDs) cannot be explained by FS. To get around this, Atanassov[2], 1986, presented the concept of an intuitionistic fuzzy (IF) set (IFS) that incorporates both MD and NMD inside the interval [0,1]. This concept has

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caught the attention of various scholars and has been utilized in evaluating and selecting e-learning websites[3], and for site selection for software operating units[4].

Overview of hesitant fuzzy set

Torra[5] was the first to introduce the hesitant FS (HFS) in FS theory. Because it extends the fuzzy structure into the hesitant fuzzy structure, it is a dependable and adaptable tool for precisely representing uncertain and fuzzy data. Torra also derived specific fundamental operational rules and essential HFS axioms. Additionally, Wang et al.[6] modified the structure of IFS into the concept of an IHFS. Extending the MD and NMD range is the main idea behind HFS. In essential FS, an element can be a member of a set with a specific MD that falls between [0,1]. Instead of allowing only one MD possibility for an element, HFS permits several MD options. Many mathematicians worked a lot in the field of decision-making science using the concept of IHFS, like the concept of group decision-making based on the IHFS presented by Beg and Rashid[7] and the solution of MADM problems using the IHFS environment given by Ali et al.[8]. Moreover, Seikh et al.[9] solving matrix games with hesitant fuzzy pay-offs, and Karmakar and Seikh[10] defines a nonlinear programming approach for hesitant noncooperative fuzzy matrix games. The concept of D-IHFS for decision-making problems was proposed by Li and Chen[11], and Meng and Li[12] developed the methodology based on the time sequential problem using hesitant fuzzy structures.

The MADM methodology

MADM is a crucial method for data aggregation in FS theory. FS theory is an essential technique that makes use of the MADM methodology in the fields of artificial intelligence, machine learning, and data sciences. The MADM technique was used by numerous researchers in various fields to define hesitant fuzzy structures for intuitionistic hesitant fuzzy values (IHFVs). For example, the solution of MADM using an extended IHFS framework proposed by Li and xu[13] and an entropy measure-based HFS environment for the MADM approach developed by Su et al.[14]. A complex IHFS is suggested by Ahmed et al.[15], single-valued neutrosophic IHFS environment for MADM was diagnosed by Imran et al.[16], and the MADM technique using the TOPSIS method based on the IHFS discussed by Aazagreyir et al.[17]. The risk assessment of COVID-19 infection using the IHFS based on the MADM approach presented by Tyagi and Tyagi[18], and the solution of MADM issues using IHFVs-based information using Einstein's operation derived by Faizi et al.[19].

Aggregation operators and t-norm and t-conorm

In the modern age, the assessment of uncertain and fuzzy information AOs is one of the best instruments for complicated information aggregation. A large number of scholars have worked on decision-making techniques, such as Liu et al.[20], who proposed the AOs based on the IHFS framework for data aggregation operators and some new weighted hybrid AOs for IHFS derived by Liao and xu[21]. Some induced AOs[22], Fermatean AOs[23], power AOs (PAOs) and similarity measures (SMs) for decision-making science for IHFS-based information are discussed by Mahmood et al.[24]. A few new AOs-based linguistic scales and VIKOR methods were discussed by Faizi et al.[25], and Linguistic scale-based AOs using IHFVs were discussed by Liu et al.[26].

Numerous traditional operational norms for various fuzzy structures failed to aggregate information as FS theory advanced. In this case, data scientists must develop novel approaches for complex data aggregation. Menger proposed the idea of a triangular norm as a solution to this kind of issue[27]. New multiple TN and TCN were later defined by numerous mathematicians, like the Archimedean TNM and TCN-based AOs discussed by Peng et al.[28], Dombi TNM-based AOs discussed by Amin et al.[29], Aczel-Alsina-based AOs utilized by Imran et al.[30] and Wang and Liu[31] proposed the Einstein TCN and TNs for FS theory. Frank TN and TCN operations are defined by Frank[32] and Weber[33] derived some new TN and TCN called Sugeno-Weber TCN and TN. This idea offers more flexibility by providing an adjustable parameter which offers a wide range for the decision maker to express their thoughts precisely and is more reliable in the MADM issues.

Background of application and literature review

The media has a significant impact on the younger generation by influencing their beliefs, attitudes, and actions. Young people are exposed to a wide range of opinions, international concerns, and cultural trends at a never-before-seen pace through social media, news, and entertainment. In addition to promoting awareness, creativity, and social connectedness, this exposure can result in inflated expectations, mental health issues, and pressure to live up to predetermined standards.

For the assessment of the effect of media on the new generation and overall society, many scholars have proposed many approaches. For example, the performance evaluation of media networks using FS theory is discussed by Trestian et al.[34], and the usage of social media assessment based on the fuzzy integral domain discussed by Hu et al.[35]. The solution of digital coverage using the MADM approach is presented by Hsu et al.[36], and Tavana et al.[37] proposed the MADM methodology for the investigation of the best social media platform.

Although the existing models like FS, IF, and HFS are effective in dealing with uncertainty in the decision-making problem and offer to deal with imprecision and vagueness. However, the extension of IHFS offers great flexibility particularly when both MD and NMDs hesitations are present. Traditional AOs also lack the adaptability to process such nuanced information structures. So, this gap motivates the use of IHFS by integrating it with SWO.

Motivation and primary research

In today's rapidly evolving digital landscape, the Internet and new media have significantly influenced decision-making processes across various domains. However, handling uncertainty and hesitation in such environments remains a critical challenge. In the past, numerous researchers have proposed various methods for resolving the

MADM, including hesitant FS and HFS. However, complex information cannot be handled by these structures, especially when dealing with intuitionistic hesitant fuzzy values (IHFVs). To address this issue, we propose the IHFS framework, a strong tool for evaluating ambiguous and fuzzy data, which is an advanced decision-making framework incorporating Intuitionistic Hesitant Fuzzy Sugeno-Weber AOs (IHFSWAO). By integrating these operators, our study enhances the accuracy and reliability of decision-making in complex and uncertain environments. Moreover, this structure allows decision-makers to aggregate a large variety of data more efficiently. The following is a discussion of the key findings as shown in Fig. 1.

So, the IHFS was chosen due to its ability to better handle uncertainty and hesitation in decision-making compared to traditional FS. In many real-world scenarios, decision-makers often face multiple possible membership values instead of a single crisp or fuzzy value. IHFS extends IFS by incorporating hesitation degrees, making it a more flexible and expressive tool for representing uncertain information.

Practical relevance of IHFS
IHFS has significant applications in MADM for handling conflicting and uncertain criteria in selection problems, risk assessment and management by addressing imprecise risk factors in finance, healthcare, and engineering, enhancing decision support systems where human hesitation exists, and managing uncertain and incomplete information in complex environments.

As compared to other fuzzy set models such as Type-1 Fuzzy, IFS, and Hesitant Fuzzy Sets, IHFS provides the following characteristics such as:
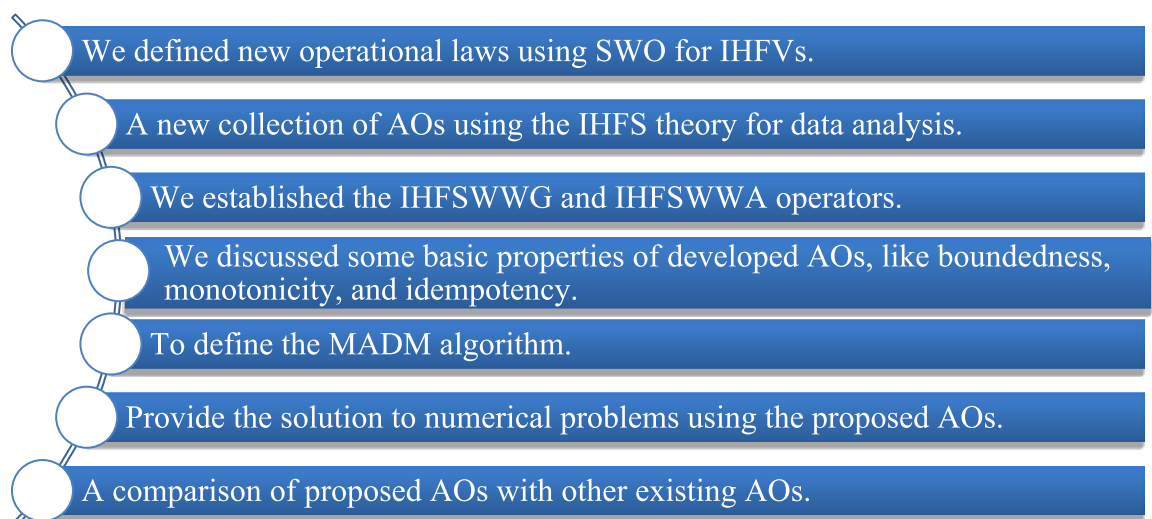
   i.   Greater flexibility: It allows multiple membership values rather than a single degree.
  ii.   Improved uncertainty handling: Unlike standard IFS, which considers only membership and non-membership, IHFS captures hesitation, making it more precise.
 iii.   Better decision support: The additional layer of hesitation improves decision accuracy in ambiguous scenarios.

By integrating Sugeno-Weber AOs, our research further enhances IHFS applications, making them more computationally robust and practically applicable in real-world decision-making.

Objective and contribution
The primary objective of this research is to develop a robust and efficient aggregation framework for decision-making under uncertainty. This study aims to define AOs, including IHFSWWA and IHFSWWG, and discuss the fundamental properties of these operators, such as boundedness, monotonicity, and idempotency. Moreover, the formation of a decision-making algorithm using the proposed operators and to validate the effectiveness of the proposed framework, numerical examples and comparative analysis with existing methods are provided. So, the major contribution of this research includes;

   i.   Introduction of a new AOs using IHFS for handling uncertain information.
  ii.   Development of new operational laws using Sugeno-Weber operations for IHFVs.
 iii.   Definition and formalization of IHFSWWA and IHFSWWG operators, providing a more generalized and flexible approach than existing triangular norms.
  iv.   Discuss the key mathematical properties (boundedness, monotonicity, and idempotency) to ensure the reliability of the proposed operators.
   v.   Formation of a MADM algorithm and validation through real-world numerical applications.

We defined new operational laws using SWO for IHFVs.

A new collection of AOs using the IHFS theory for data analysis.

We established the IHFSWWG and IHFSWWA operators.

We discussed some basic properties of developed AOs, like boundedness, monotonicity, and idempotency.

To define the MADM algorithm.

Provide the solution to numerical problems using the proposed AOs.

A comparison of proposed AOs with other existing AOs.

Fig. 1. Key findings.

Organizations of proposed work

The remaining portion of this paper is structured as follows: Section"Preliminaries"examines some fundamental definitions of the suggested technique. New operating laws for SWO for the IHFS framework are covered in Section"Operational laws".The proposed IHFSWWA and IHFSWWG operators are discussed in Section"Proposed methodology". Section"MADM algorithm based on the developed approach"presents the MADM methodology, which is explained in detail by utilizing IHFS theory and a case study for the assessment of the effect of media on new generations discussed in Section"Case study". In Section"Numerical example", diagnosed IHFSWWA and IHFSWWG operators are used to solve a real-world numerical problem. In Section"Comparative Analysis with present AOs", the accuracy and applicability of the estimated results were examined by a detailed comparison with current methods. Section"Conclusion"discusses the conclusion.

Preliminaries

This section explores basic definitions related to the established approach. This segment defines IHFS and PyHFS, score function (SF), accuracy function (AF), SWTNM, and TCNM.

Definition 1. The IHFS I is defined on X[6]:

$$I = \{(\ \breve{\alpha}(x), \Psi(x)) : x \in X\} \tag{1}$$

where the MD is presented as $\breve{\alpha}_j(x) \in [0,1]$, $j = 1, 2, \ldots, n$, the NMD is presented as $\Psi(x) = \Psi_j(x) \in [0,1]$ with $0 \leq \breve{\alpha}(x) + \Psi(x) \leq 1$. In Addition, we provided neutral information, such as $\mathbb{r}(x) = \mathbb{r}_j(x) \in [0,1]$ and $\mathbb{r}(x) = (1 - (\breve{\alpha}_j(x) + \breve{\alpha}_j(x)))$, and the straightforward form of IHFV is indicated by $I_j = (\ \breve{\alpha}, \Psi) = \cup_{\left(\begin{smallmatrix}\breve{\alpha}_j(x) \in \breve{\alpha}(x), \\ \Psi_j(x) \in \Psi(x)\end{smallmatrix}\right)} \{\breve{\alpha}_j(x), \Psi_j(x)\}$.

Definition 2. Let IHFS $I_j = \{\breve{\alpha}_j(x), \Psi_j(x)\}$, $j = 1, 2, \ldots, n$, then the score value (SV) is given by[38]:

$$SV(I_j) = \left(\frac{1 + \breve{\alpha}_j(x) - \Psi_j(x)}{2}\right) \tag{2}$$

Accuracy value (AV) can be defined as follows:

$$AV(I_j) = \left(\frac{1 + \breve{\alpha}_j(x) - \Psi_j(x)}{2}\right) \tag{3}$$

Definition 3. Let IHFS $I_j = \{\breve{\alpha}_j(x), \Psi_j(x)\}$, $j = 1, 2, \ldots, n$ and $SF(I_1) > SF(I_2)$ then I_1 higher to I_2 [38].

   i. If $SF(I_1) < SF(I_2)$ then I_1 lower to I_2.
   ii. If $SF(I_1) > SF(I_2)$ then I_1 higher to I_2
   iii. If $SF(I_1) = SF(I_2)$ then I_1 equal to I_2

   a) If $AF(I_1) > AF(I_2)$ then I_1 higher to I_2.
   b) If $AF(I_1) < AF(I_2)$ then I_1 inferior to I_2.
   c) If $AF(I_1) > AF(I_2)$ then I_1 higher to I_2.

Operational laws

Definition 4. Let $I_j(\breve{\alpha}_j, \Psi_j)$, $j = 1,2$ are two IHFVs. Then[39];

   i. $I_1 \oplus I_2 = \cup_{\left(\begin{smallmatrix}\breve{\alpha}_1(x), \breve{\alpha}_2(x) \in \breve{\alpha}(x), \\ \Psi_1(x), \Psi_2(x) \in \Psi(x)\end{smallmatrix}\right)} (\breve{\alpha}_1(x) + \breve{\alpha}_2(x) - \breve{\alpha}_1(x)\breve{\alpha}_2(x), \Psi_1(x)\Psi_2(x))$

   ii. $I_1 \otimes I_2 = \cup_{\left(\begin{smallmatrix}\breve{\alpha}_1(x), \breve{\alpha}_2(x) \in \breve{\alpha}(x), \\ \Psi_1(x), \Psi_2(x) \in \Psi(x)\end{smallmatrix}\right)} (\breve{\alpha}_1(x)\breve{\alpha}_2(x), \Psi_1(x) + \Psi_2(x) - \Psi_1(x)\Psi_2(x))$

   iii. $\gamma I_1 = \cup_{\left(\begin{smallmatrix}\breve{\alpha}_1(x) \in \breve{\alpha}(x), \\ \Psi_1(x) \in \Psi(x)\end{smallmatrix}\right)} (1 - (1 - \breve{\alpha}_1(x))^\gamma, \Psi_1(x))$

   iv. $I_1^\gamma = \cup_{\left(\begin{smallmatrix}\breve{\alpha}_1(x) \in \breve{\alpha}(x), \\ \Psi_1(x) \in \Psi(x)\end{smallmatrix}\right)} (\breve{\alpha}_1(x)^\gamma, 1 - (1 - \Psi_1(x))^\gamma)$

Definition 5. The Sugeno-Weber sum (SWS) and Sugeno-Weber sum (SWP) are defined as follows[39]:

$$\chi^C = \begin{cases} \chi_d(\top, N^o), if\ C = -1 \\ \max\left(0, \frac{\top + N^o - 1 + CN^o}{1 + C}\right), if -1 < C < +\infty \\ \chi_p(\top, N^o), C = +\infty \end{cases} \tag{4}$$

and

$$\wp^C = \begin{cases} \wp_d\left(\top, N^O\right), if \ C = -1 \\ \min\left(1, \top + N^O - \left(\frac{C}{1+C}\right)\top N^O\right), if -1 < C < +\infty \\ \wp_p\left(\top, N^O\right), \ C = +\infty \end{cases} \quad (5)$$

where $\wp_d(\top, N^O)$ and $\chi_d\left(\top, N^O\right)$ known as drastic TCNM and TNM, respectively, and $\wp_p\left(\top, N^O\right)$ and $\chi_p\left(\top, N^O\right)$ known as the probabilistic sum of TCNM and TNM.

Definition 6. Let $I_j\left(\ddot{\alpha}_j, \Psi_j\right), j = 1$ be any IHFVs, then[40];

$$s\left(I_1\right) = \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \left\{ \begin{array}{c} \frac{1}{ordered(\ddot{\alpha}_1(x))}\sum_{j=1}^{n} \ddot{\alpha}_1\left(x\right) - \\ \frac{1}{ordered(\Psi_1(x))}\sum_{j=1}^{n} \Psi_1\left(x\right) - \frac{1}{ordered(\ddot{\alpha}_1(x))}\sum_{j=1}^{n} \ddot{\alpha}_1\left(x\right) \end{array} \right\}; S\left(I_1\right) \in [-1, 1] \quad (6)$$

$$A\left(I_1\right) = \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \left\{ \begin{array}{c} \frac{1}{ordered(\ddot{\alpha}_1(x))}\sum_{j=1}^{n} \ddot{\alpha}_1\left(x\right) - \\ \frac{1}{ordered(\Psi_1(x))}\sum_{j=1}^{n} \Psi_1\left(x\right) - \frac{1}{ordered(\ddot{\alpha}_1(x))}\sum_{j=1}^{n} \ddot{\alpha}_1\left(x\right) \end{array} \right\} \quad (7)$$

Proposed methodology

This section will use the operational regulations covered in Definition 4 and the TNM and TCNM concepts introduced in Definition 5 to diagnose a new collection of AOs known as IHFSWWA and IHFSWWG operators.

Definition 7. Let a collection of IHFVs $\tilde{\tilde{\upsilon}}_{\mathfrak{r}} = \left(\ddot{\alpha}_{\mathfrak{r}}, \Psi_{\mathfrak{r}}\right)$, where $(\mathfrak{r} = 1, 2, \ldots, n)$ and $\overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}}$ be the weight vector (WV) $\sum_{\mathfrak{r}=1}^{n} \overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}} = 1$, the parameter $C \geq 1$.

$$\text{IHFSWWA}\left(\tilde{\upsilon}_1, \tilde{\upsilon}_2, \ldots, \tilde{\upsilon}_n\right) = \overset{n}{\underset{\mathfrak{r}=1}{\oplus}} \ \tilde{\upsilon}_{\mathfrak{r}} \ \overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}}$$

Theorem 1. Let a collection of IHFVs $\tilde{\upsilon}_{\mathfrak{r}} = (\ddot{\alpha}, \Psi)$, where $(\mathfrak{r} = 1, 2, \ldots, n)$, $\overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}}$ be the WV, and the sum of WVs is 1, where $C \geq 1$. Then, the aggregation result is also an IHFV. The obtained outcomes are given as follows:

$$\text{IHFSWWA}\left(\tilde{\upsilon}_1, \tilde{\upsilon}_2, \ldots, \tilde{\upsilon}_n\right) = \cup \begin{pmatrix} \ddot{\alpha}_{\mathfrak{r}}\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_{\mathfrak{r}}\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \begin{pmatrix} \frac{1+C}{C}\left(1 - \prod_{\mathfrak{r}=1}^{n}\left(1 - \ddot{\alpha}_{\mathfrak{r}}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}}}\right), \\ \frac{1}{C}\left((1+C)\prod_{\mathfrak{r}=1}^{n}\left(\frac{C\Psi_{\mathfrak{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{\upsilon}}}_{\mathfrak{r}}} - 1\right) \end{pmatrix} \quad (8)$$

Proof: We prove this theorem by using the technique of mathematical induction.

Let two IHFVs such as:

$$\overline{\overline{\tilde{\upsilon}}}_1 = \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \begin{pmatrix} \frac{1+C}{C}\left(1 - \left(1 - \ddot{\alpha}_1\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{\upsilon}}}_1}\right), \frac{1}{C}\left((1+C)\left(\frac{C\Psi_1+1}{1+C}\right)^{\overline{\overline{\tilde{\upsilon}}}_1} - 1\right) \end{pmatrix}$$

$$\overline{\overline{\tilde{\upsilon}}}_2 = \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \begin{pmatrix} \frac{1+C}{C}\left(1 - \left(1 - \ddot{\alpha}_2\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{\upsilon}}}_2}\right), \frac{1}{C}\left((1+C)\left(\frac{C\Psi_2+1}{1+C}\right)^{\overline{\overline{\tilde{\upsilon}}}_2} - 1\right) \end{pmatrix}$$

$$\overline{\overline{\tilde{\upsilon}}}_1 \oplus \overline{\overline{\tilde{\upsilon}}} = \begin{pmatrix} \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \begin{pmatrix} \frac{1+C}{C}\left(1 - \left(1 - \ddot{\alpha}_1\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{\upsilon}}}_1}\right), \frac{1}{C}\left((1+C)\left(\frac{C\Psi-\Psi_1+1}{1+C}\right)^{\overline{\overline{\tilde{\upsilon}}}_1} - 1\right) \end{pmatrix} \oplus \\ \cup \begin{pmatrix} \ddot{\alpha}_1\left(x\right) \in \ddot{\alpha}\left(x\right), \\ \Psi_1\left(x\right) \in \Psi\left(x\right) \end{pmatrix} \begin{pmatrix} \frac{1+C}{C}\left(1 - \left(1 - \ddot{\alpha}_2\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{\upsilon}}}_2}\right), \frac{1}{C}\left((1+C)\left(\frac{C\Psi_2+1}{1+C}\right)^{\overline{\overline{\tilde{\upsilon}}}_2} - 1\right) \end{pmatrix} \end{pmatrix}$$

$$= \left(\begin{pmatrix} \frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_1\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}_1}\right)+\frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_2\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}_2}\right)- \\ \frac{C}{1+C}\left(\frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_1\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}_1}\right)\right)\cdot\left(\frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_2\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}_2}\right)\right) \end{pmatrix}, \right.$$
$$\left. \begin{pmatrix} \frac{1}{1+C}\begin{pmatrix} \left((1+C)\left(\frac{C\Psi_1+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}_1}-1\right)\frac{1}{C}+\left((1+C)\left(\frac{C\Psi_2+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}_2}-1\right)\frac{1}{C}- \\ C\begin{pmatrix}\left(\left((1+C)\left(\frac{C\Psi_2+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}_1}-1\right)\frac{1}{C}\right)\\ \cdot\left(\left((1+C)\left(\frac{C\Psi_2+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}_2}-1\right)\frac{1}{C}\right)\end{pmatrix} \end{pmatrix} \end{pmatrix} \right)$$

$$= \cup_{\left(\substack{\grave{\alpha}_\text{r}(\text{x})\in\grave{\alpha}(\text{x}),\\ \Psi_\text{r}(\text{x})\in\Psi(\text{x})}\right)}\left(\frac{1+C}{C}\left(1-\prod_{\underline{r}=1}^{2}\left(1-\grave{\alpha}_{\underline{r}}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}\underline{r}}\right),\left((1+C)\prod_{\underline{r}=1}^{2}\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}\underline{r}}-1\right)\frac{1}{C}\right)$$

Suppose that the statement is true for $n=k$, then we have

$$= \cup_{\left(\substack{\grave{\alpha}_\text{r}(\text{x})\in\grave{\alpha}(\text{x}),\\ \Psi_\text{r}(\text{x})\in\Psi(\text{x})}\right)}\left(\frac{1+C}{C}\left(1-\prod_{\underline{r}=1}^{k}\left(1-\grave{\alpha}_{\underline{r}}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}\underline{r}}\right),\left((1+C)\prod_{\underline{r}=1}^{k}\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}\underline{r}}-1\right)\frac{1}{C}\right)$$

Next, suppose that the statement is true for $n=k+1$, then we have;

$$IHFSWWA\left(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_k\right)=\cup_{\left(\substack{\grave{\alpha}_\text{r}(\text{x})\in\grave{\alpha}(\text{x}),\\ \Psi_\text{r}(\text{x})\in\Psi(\text{x})}\right)}\begin{pmatrix}\frac{1+C}{C}\left(1-\prod_{\underline{r}=1}^{k}\left(1-\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}\underline{r}}\right),\\ \left((1+C)\prod_{\underline{r}=1}^{k}\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}\underline{r}}-1\right)\frac{1}{C}\end{pmatrix}$$

$$\oplus\cup_{\left(\substack{\grave{\alpha}_\text{r+1}(\text{x})\in\grave{\alpha}(\text{x}),\\ \Psi_\text{r+1}(\text{x})\in\Psi(\text{x})}\right)}\begin{pmatrix}\frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_{k+1}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}_{k+1}}\right),\\ \left((1+C)\left(\frac{C\Psi_{k+1}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}_{k+1}}-1\right)\frac{1}{C}\end{pmatrix}$$

$$= \cup_{\left(\substack{\grave{\alpha}_\text{r+1}(\text{x})\in\grave{\alpha}(\text{x}),\\ \Psi_\text{r+1}(\text{x})\in\Psi(\text{x})}\right)}\left(\frac{1+C}{C}\left(1-\prod_{\underline{r}=1}^{k+1}\left(1-\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}}\underline{r}}\right),\left((1+C)\prod_{\underline{r}=1}^{k+1}\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}}\underline{r}}-1\right)\frac{1}{C}\right)$$

Hence proved.

Theorem 2. (Idempotency) Let a collection of IHFVs $\tilde{v}_{\underline{r}}=\left(\grave{\alpha}_{\underline{r}},\Psi_{\underline{r}}\right)$, where $(\underline{r}=1,2,\ldots,n)$. If $\tilde{v}_1=\tilde{v}_2=\cdots=\tilde{v}_\text{i}=\tilde{v}$. So,

$$IHFSWWA\left(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_\text{i}\right)=\tilde{v}$$

Proof: Let the $\tilde{v}_{\underline{r}}=\left(\grave{\alpha}_{\underline{r}},\Psi_{\underline{r}}\right)$, where $(\underline{r}=1,2,\ldots,n)$. We have.

$$IHFSWWA\left(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_n\right)=\begin{pmatrix}\frac{1+C}{C}\left(1-\prod_{\underline{r}=1}^{n}\left(1-\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}\underline{r}}}\right),\\ \frac{1}{C}\left((1+C)\prod_{\underline{r}=1}^{n}\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}\underline{r}}}-1\right)\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1+C}{C}\left(1-\left(1-\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right)^{\overline{\overline{\tilde{v}}\underline{r}}}\right),\\ \frac{1}{C}\left((1+C)\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)^{\overline{\overline{\tilde{v}}\underline{r}}}-1\right)\end{pmatrix}=\begin{pmatrix}\frac{1+C}{C}\left(1-\left(\left(1-\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right)\right)\right),\\ \frac{1}{C}\left((1+C)\left(\left(\frac{C\Psi_{\underline{r}}+1}{1+C}\right)\right)-1\right)\end{pmatrix}$$

$$= \left(\frac{1+C}{C}\left(1-1+\grave{\alpha}_\text{r}\left(\frac{C}{1+C}\right)\right),\frac{1}{C}\left(1+C\Psi_{\underline{r}}-1\right)\right)$$

$$= \left(\frac{1+C}{C} \left(\grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right), \frac{1}{C} \left(C\Psi_{\text{r}} \right) \right) = \left(\grave{\alpha}_{\text{r}}, \Psi_{\text{r}} \right) = \tilde{v}_{\text{r}}$$

Theorem 3. (Boundedness) Let a collection of IHFVs. $\tilde{v}_{\text{r}} = \left(\grave{\alpha}_{\text{r}}, \Psi_{\text{r}} \right)$, where $(\text{r} = 1, 2, \ldots, n)$. If $\tilde{v}^{-} = \left(\min \left[\grave{\alpha}_{\text{r}} \right], \max \left[\Psi_{\text{r}} \right] \right)$ and $\tilde{v}^{+} = \left(\max \left[\grave{\alpha}_{\text{r}} \right], \min \left[\Psi_{\text{r}} \right] \right)$, then;

$$\tilde{v}^{-} \leq \text{IHFSWWA} \left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_{\text{i}} \right) \leq \tilde{v}^{+}$$

Proof: Let the $\tilde{v}_{\text{r}} = \left(\grave{\alpha}_{\text{r}}, \Psi_{\text{r}} \right)$, where $(\text{r} = 1, 2, \ldots, n)$. We have.

$$IHFSWWA \left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n \right) =$$

$$\min \grave{\alpha}_{\text{r}} \leq \max \Psi_{\text{r}}$$

$$\min \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \leq \max \Psi_{\text{r}} \left(\frac{C}{1+C} \right)$$

$$1 - \min \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \geq 1 - \max \Psi_{\text{r}} \left(\frac{C}{1+C} \right)$$

$$\left(1 - \min \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}} \geq \left(1 - \max \Psi_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}}$$

$$\left(1 - \min \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\sum_{\text{r}=1}^{n} \overline{\overline{\tilde{v}_{\text{r}}}}} \geq \left(1 - \max \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\sum_{\text{r}=1}^{n} \overline{\overline{\tilde{v}_{\text{r}}}}},$$

$$\left(1 - \min \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right) \geq \prod_{\text{r}=1}^{n} \left(1 - \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}} \geq \left(1 - \max \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right),$$

$$\left(\min \left(\frac{C}{1+C} \right) \right) \leq 1 \leq \prod_{\text{r}=1}^{n} \left(1 - \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}} \leq \max \left(\frac{C}{1+C} \right),$$

$$\min \grave{\alpha}_{\text{r}} \leq \sqrt{ \frac{1+C}{C} \left(1 - \prod_{\text{r}=1}^{n} \left(1 - \grave{\alpha}_{\text{r}} \left(\frac{C}{1+C} \right) \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}} \right) } \leq \max \grave{\alpha}_{\text{r}}$$

and

$$\min \grave{\alpha}_{\text{r}} \geq \sqrt{ \frac{1}{C} \left((1+C) \prod_{\text{r}=1}^{n} \left(\frac{C\Psi_{\text{r}}+1}{1+C} \right)^{\overline{\overline{\tilde{v}_{\text{r}}}}} - 1 \right) } \geq \max \grave{\alpha}_{\text{r}}$$

Hence proved

$$\tilde{v}^{-} \leq \text{IHFSWWA} \left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_{\text{i}} \right) \leq \tilde{v}^{+}$$

Theorem 4. (Monotonicity) Let a collection of IHFVs. $\tilde{v}_{\text{r}} = \left(\grave{\alpha}_{\text{r}}, \Psi_{\text{r}} \right)$ and $\tilde{v}_{\text{r}}' = \left(\grave{\alpha}_{\text{r}}', \Psi_{\text{r}}' \right)$, where $(\text{r} = 1, 2, \ldots, n)$ and hold the following conditions:

$$\text{IHFSWWA} \left(\tilde{v}_1, \tilde{v}_2, \ldots \tilde{v}_i \right) \leq \text{IHFSWWA} \left(\tilde{v}_1', \tilde{v}_2', \ldots, \tilde{v}_2' \right)$$

Proof: The proof is straightforward.

Definition 8. Let a collection of IHFVs $\tilde{v}_{\text{r}} = \left(\grave{\alpha}_{\text{r}}, \Psi_{\text{r}} \right)$, where $(\text{r} = 1, 2, \ldots, n)$. And $\overline{\overline{\tilde{v}_{\text{r}}}}$ be the WV, $\sum_{\text{r}=1}^{n} \overline{\overline{\tilde{v}_{\text{r}}}} = 1$, the parameter $C \geq 1$.

$$IHFSWWG \left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n \right) = \overset{n}{\underset{\text{r}=1}{\otimes}} \overline{\overline{\tilde{v}_{\text{r}}}} \tilde{v}_{\text{r}}$$

Theorem 5. Let a collection of IHFVs $\tilde{\tilde{v}}_{\underset{.}{r}} = \left(\hat{\alpha}_{\underset{.}{r}}, \Psi_{\underset{.}{r}}\right)$, where $(\underset{.}{r} = 1, 2, \ldots, n)$, $\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}$ be the WV, the sum of WVs is one, and $C \geq 1$. Then, the aggregation result is also an IHFV. The obtained outcomes are given as follows:

$$IHFSWWG\left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n\right) = \cup_{\left(\begin{array}{c} \alpha_{\underset{.}{r}}(x) \in \alpha(x), \\ \Psi_{\underset{.}{r}}(x) \in \Psi(x) \end{array}\right)} \left(\begin{array}{c} \frac{1}{\Phi}\left((1+\Phi)\prod_{\underset{.}{r}=1}^{n}\left(\frac{\Phi\alpha_{\underset{.}{r}}^2+1}{1+\Phi}\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}} - 1\right), \\ \frac{1+\Phi}{\Phi}\left(1 - \prod_{\underset{.}{r}=1}^{n}\left(1 - \Psi_{\underset{.}{r}}^2\left(\frac{\Phi}{1+\Phi}\right)\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}}\right) \end{array}\right) \quad (9)$$

The proof is straightforward.

Remark 2. The suggested IHFSWWG operator satisfies the three main requirements of AOs: Monotonicity, boundedness, and idempotency.

MADM algorithm based on the developed approach

The detailed steps of the MADM algorithm are based on the IHFSWWA and IHFSWWG theory. Some steps are given as follows:

Step 1. Firstly, we have to collect information in the form of IHFVs and construct the decision matrix. Where columns represent the alternatives and rows represent the attributes.

$$M\left(I_j\right) = \left(\begin{array}{cccc} I_{11} & I_{12} & \cdots & I_{1n} \\ I_{21} & I_{22} & \cdots & I_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1} & I_{m2} & \cdots & I_{mn} \end{array}\right)$$

The sum of the weights of attributes should be 1.

Step 2. Evaluate the decision matrix using proposed IHFSWWA and IHFSWWG operators as discussed in Eqs. 8 and 9.

$$IHFSWWA\left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n\right) = \cup_{\left(\begin{array}{c} \alpha_{\underset{.}{r}}(x) \in \alpha(x), \\ \Psi_{\underset{.}{r}}(x) \in \Psi(x) \end{array}\right)} \left(\begin{array}{c} \frac{1+\Phi}{\Phi}\left(1 - \prod_{\underset{.}{r}=1}^{n}\left(1 - \alpha_{\underset{.}{r}}\left(\frac{\Phi}{1+\Phi}\right)\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}}\right), \\ \frac{1}{\Phi}\left((1+\Phi)\prod_{\underset{.}{r}=1}^{n}\left(\frac{\Phi\Psi_{\underset{.}{r}}+1}{1+\Phi}\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}} - 1\right) \end{array}\right)$$

and

$$IHFSWWG\left(\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n\right) = \cup_{\left(\begin{array}{c} \alpha_{\underset{.}{r}}(x) \in \alpha(x), \\ \Psi_{\underset{.}{r}}(x) \in \Psi(x) \end{array}\right)} \left(\begin{array}{c} \frac{1}{\Phi}\left((1+\Phi)\prod_{\underset{.}{r}=1}^{n}\left(\frac{\Phi\alpha_{\underset{.}{r}}^2+1}{1+\Phi}\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}} - 1\right), \\ \frac{1+\Phi}{\Phi}\left(1 - \prod_{\underset{.}{r}=1}^{n}\left(1 - \Psi_{\underset{.}{r}}^2\left(\frac{\Phi}{1+\Phi}\right)\right)^{\overline{\overline{\tilde{\tilde{v}}}}_{\underset{.}{r}}}\right) \end{array}\right)$$

Step 3. Utilizing the score function formula for the IHFVs, determine the total views regarding the alternatives concerning each factor. The following Eq. 3 is used to view the IHFV scores.

To find the cumulative opinions about the alternatives concerning each factor by using the SV formula for the IHFVs. We see the scores of the IHFVs using the following Eq. 3.

$$SV\left(I_j\right) = \left(\frac{1 + \alpha_j(x) - \Psi_j(x)}{2}\right)$$

Step 4. To determine which choice is better, rank the total results in descending order.

Step 5. Choose the best option in the final phase.

For a better understanding, the MADM algorithm is presented in Fig. 2, as given below:

Case study

The internet is the combination of millions of devices that are connected and also share their resources. The use of the internet has several pros and cons. The media plays a key role in delivering information to the people. The traditional way of information transfer is prolonged, like newspapers, radio, etc. The traditional media was not very inactive and attractive, but now, in the modern age, the information transfer process is speedy and it's also beautiful and interactive. The delivery of information is reliable. The new media technologies also improve the thinking power of humans. The flow of information is smooth in digital media. The information is reliable on digital media. Now the people are connected, whether they are living or in the village. The whole world converts into a global village. It is possible to share the information in a small unit of time from one corner of the world to another. The primary purpose of the internet is the sharing of information, and digital media works as a spinal
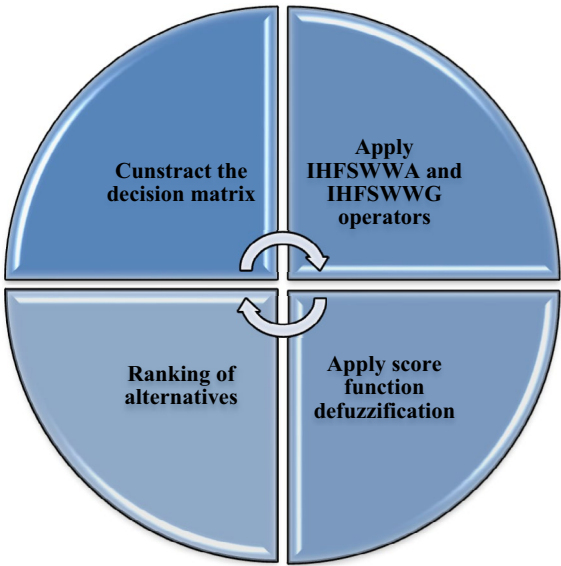
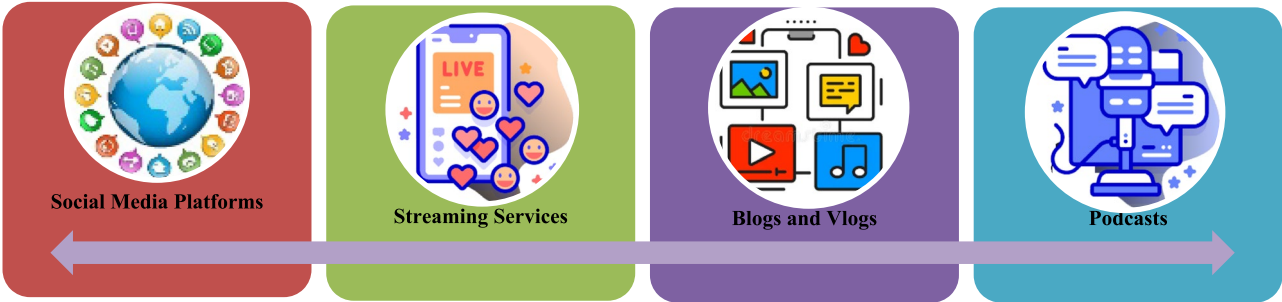**Fig. 2**. Shows the MADM algorithm based on the proposed AOs.

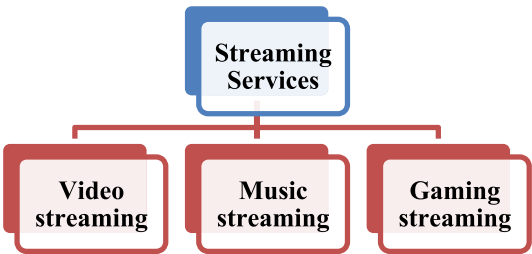**Fig. 3**. Some trending social media in the modern age.

Fig. 4. Types of streaming.

cord for information sharing. Some kinds of digital media have a substantial effect on human nature, thinking power, living strander, and flow of information. Some leading new trending media listed are given below in Fig. 3 as follows:

In this case study, we aim to investigate the effect of the presented media on our growing generation. The details of these presented media are given as follows which presents a concise overview of how platforms like social media, streaming services, blogs, and communication tools influence users.

Alternatives	Attributes
Social media platforms	Communication and relationships
Streaming services	Education and learning habits
Blogs and vlogs	Privacy and security concerns
Podcasts	Human health

Table 1. Shows the list of alternatives and attributes.

	$\mathcal{L}_1$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_4$
ς_1	$\left\{\begin{array}{c}\{0.2,0.1\}, \\ \{0.2,0.2\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.4,0.1\}, \\ \{0.2,0.1\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2,0.3\}, \\ \{0.3,0.2\}\end{array}\right\}$	$\{0.1,0.3\}$
ς_2	$\left\{\begin{array}{c}\{0.1\}, \\ \{0.1,0.2\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2\}, \\ \{0.2,0.2\}\end{array}\right\}$	$\{0.2,0.2\}$	$\left\{\begin{array}{c}\{0.4\}, \\ \{0.1,0.3\}\end{array}\right\}$
ς_3	$\{0.2,0.1\}$	$\left\{\begin{array}{c}\{0.1,0.1\}, \\ \{0.1,0.3\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.3,0.3\}, \\ \{0.2\}\end{array}\right\}$	$\left\{\begin{array}{c}\{0.2\}, \\ \{0.4,0.2\}\end{array}\right\}$
ς_4	$\{0.1,0.6\}$	$\{0.3,0.1\}$	$\{0.4,0.4\}$	$\left\{\begin{array}{c}\{0.1,0.5\}, \\ \{0.2\}\end{array}\right\}$

Table 1. IHFV-based decision matrix.

Trending social media	
Social media platforms	**i.** The connection of millions of users in such a way that they live in the same home. Communication, sharing content like videos, text messages, images, and so many kinds of files can be shared with social media. **ii.** In the modern era, so many social media platforms are available like Facebook, Instagram, Pinterest, and Snapchat. The social media network is also beneficial from a business point of view.
Streaming services	The central role of streaming services is entertainment. In streaming, the media user does not need to wait for content to download; the user can watch, read, or listen to the content in real time. So many users use this media for online business purposes Some types of streaming are given below in Fig. 4
Blogs and Vlogs	**i.** A special kind of media in the modern age. The users can share routine work in the form of text and images with detailed explanations. Some users share their experiences daily. However, the vlogs are advanced in that users share videos about particular topics daily. **ii.** The main difference between a Blog and a Vlog is that a blog is in the form of text, while a Vlog is created in the form of video. The vlogs are more comprehensive than blogs. Some vloggers do live sessions about different topics.
Podcasts	It is the alternative to old-fashioned radio technology and is popular for offering entertainment, education, and information flow. Each Podcast has a unique theme or topic. The Podcast may have a single episode or a series of episodes. Nowadays, it is accessible through the web, mobile apps, or any other means. The features of podcasts are storytelling, discussions, and news. So, the significant advantages of podcasts are accessibility, diverse content, and building communities

Moreover, some significant attributes like communication and relationships, education and learning habits, privacy and security concerns, and human health also become the reason for ranking the effect of social media on our new generations. For convenience, the list of alternatives and attributes is provided in Table 1.

Numerical example
To highlight the significance of the proposed IHFSWWA and IHFSWWG theory, we offered a numerical example of the ranking effect of media on our generation.

In this example, a hypothetical dataset was used to simulate a realistic decision-making environment involving the evaluation of social media and internet impacts on younger generations. The attributes and values were designed to reflect credible decision criteria and decision-maker evaluations. Although the data is hypothetical, it captures the complex nature of hesitation and uncertainty often encountered in real-world evaluations. The proposed IHFSWWA and IHFSWWG operators are well-suited for modelling such information and can be effectively extended to practical applications such as educational policy-making, digital well-being assessment, and media strategy planning. The details of the numerical example are given as follows:

Example 1. Consider a set of finite media that directly affect the new growing generation $\wp_i$, $(i = 1, 2, 3, 4)$. Considered a finite set of alternatives λ_i, $(i = 1, 2, 3, 4)$ with weightage are λ_1 with communication and relationships with, λ_2 are education and learning habits, λ_3 is privacy and security concerns with, and λ_4 is human health. By using the set of information in the shape of IHFVs, construct the decision matrix and aggregate the fuzzy data by applying the proposed IHFSWWA and theory.
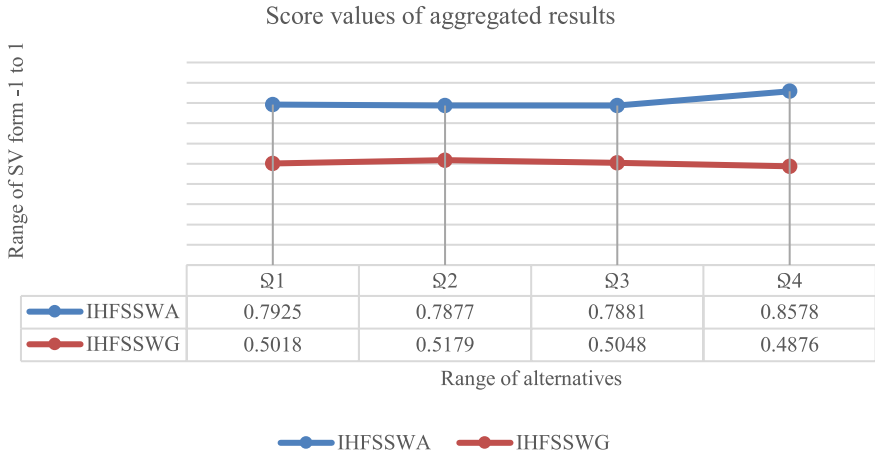
Step 1. Construct a decision matrix in the form of IHFVs by taking expert opinions and letting alternatives be based on selected attributes, as shown in Table 2. $\mathcal{T}_1$ to $\mathcal{T}_4$ presents the attribute and ς_1 to ς_4 presents the alternatives under observations using the IHFS environment.

	IHFSWWA	IHFSWWG
ς_1	$(0.2765, -0.3084)$	$(-0.3561., 0.3511)$
ς_2	$(0.2448., -0.3206)$	$(-0.3783, 0.3276)$
ς_3	$(0.2615, -0.3146)$	$(-0.3655, 0.3520)$
ς_4	$(0.3779, -0.3377)$	$(-0.3118, 0.3494)$

Table 3. Aggregated results using IHFSWWA and IHFSWWG operators.

	IHFSWWA	IHFSWWG
ς_1	0.7925	0.5018
ς_2	0.7877	0.5179
ς_3	0.7881	0.5048
ς_4	0.8578	0.4876

Table 4. Aggregated results using IHFSWWA and IHFSWWG operators.

	$\Omega1$	$\Omega2$	$\Omega3$	$\Omega4$
IHFSSWA	0.7925	0.7877	0.7881	0.8578
IHFSSWG	0.5018	0.5179	0.5048	0.4876

Fig. 5. Geometric abstract of SVs.

	Ordering of options
IHFSWWA	$\varsigma_4 > \varsigma_1 > \varsigma_3 > \varsigma_2$
IHFSWWG	$\varsigma_2 > \varsigma_3 > \varsigma_1 > \varsigma_4$

Table 5. Ranking of SVs.

Step 2. Apply the proposed theory of IHFSWWA and IHFSWWG operators for ranking the effect of media on a new generation. The obtained outcomes are given in Table 3.

Applied the diagnosed theory of IHFSWWA and IHFSWWG operators discussed in Eqs. 8 and 9, the obtained results can be seen in Table 3.

Step 3. Use the SF formula shown in Eq. 3 to defuzzifed data or compute data into a single number to get the best option. The results of SF are displayed in Table 4.

After utilizing the SF formula to compile the outputs of the IHFSWWA and IHFSWWG operators into single numbers for clarity, Table 4 displays the score values. Additionally, Fig. 5 provides a pictorial representation of the results collected in the preceding Table.

The results from IHFSWWA were shown by the blue line in the geometric representation of aggregated outcomes, whereas the orange line showed the results from IHFSWWG operators.

Step 4. To get the ranking option, conveniently arrange all aggregated results in descending order.

In Table 5. It is noticed that ς_4 is the most affected media on the new generation under the consideration of the discussed attributes using the IHFSWWA operator, while ς_2 be the most effective media using the IHFSWWG operator.

	Operator	Score value	Ranking
Proposed work	IHFSWWA	$SV\,(\varsigma_1) = 0.792,\ SV\,(\varsigma_2) = 0.787,\ SV\,(\varsigma_3) = 0.788,\ SV\,(\varsigma_4) = 0.858$	$\varsigma_4 > \varsigma_1 > \varsigma_3 > \varsigma_2$
	IHFSWWG	$SV\,(\varsigma_1) = 0.502,\ SV\,(\varsigma_2) = 0.518,\ SV\,(\varsigma_3) = 0.504,\ SV\,(\varsigma_4) = 0.487$	$\varsigma_2 > \varsigma_3 > \varsigma_1 > \varsigma_4$
Zhou et al.[41]	IHFHWA	$SV\,(\varsigma_1) = 0.188,\ SV\,(\varsigma_2) = 0.192,\ SV\,(\varsigma_3) = 0.191,\ SV\,(\varsigma_4) = 0.194$	$\varsigma_4 > \varsigma_2 > \varsigma_3 > \varsigma_1$
	IHFHWG	$SV\,(\varsigma_1) = 0.206,\ SV\,(\varsigma_2) = 0.197,\ SV\,(\varsigma_3) = 0.196,\ SV\,(\varsigma_4) = 0.188$	$\varsigma_1 > \varsigma_2 > \varsigma_3 > \varsigma_4$
Zhou and Li[42]	IHFEWA	$SV\,(\varsigma_1) = 0.471,\ SV\,(\varsigma_2) = 0.487,\ SV\,(\varsigma_3) = 0.489,\ SV\,(\varsigma_4) = 0.485$	$\varsigma_3 > \varsigma_2 > \varsigma_4 > \varsigma_1$
	IHFEWG	$SV\,(\varsigma_1) = 0.524,\ SV\,(\varsigma_2) = 0.513,\ SV\,(\varsigma_3) = 0.525,\ SV\,(\varsigma_4) = 0.602$	$\varsigma_4 > \varsigma_3 > \varsigma_1 > \varsigma_2$
Huang[43]	IFHWA	failed to aggregate	failed to aggregate
	IFHWG	failed to aggregate	failed to aggregate
Seikh and mandal[44]	IFDWA	failed to aggregate	failed to aggregate
	IFDWG	failed to aggregate	failed to aggregate
Senapati et al.[45]	IFAAWA	failed to aggregate	failed to aggregate
	IFAAWG	failed to aggregate	failed to aggregate

Table 6. Ranking of score values.
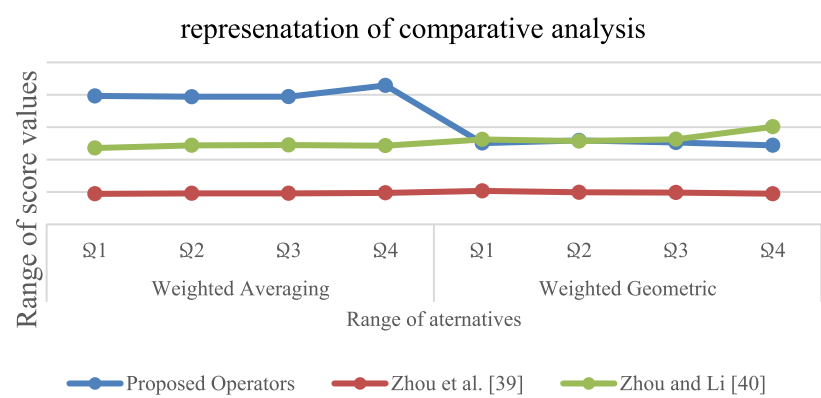
Fig. 6. Representation of the comparative analysis.

Comparative analysis with present AOs

This section will provide a thorough comparison with the current AOs shown in Table 6. Using the IHFV data in Table 2, we can verify the precision and accuracy of our combined results with those of the current AOs. The significance and relevance of the suggested work are demonstrated by a comparison with the intuitionistic hesitant fuzzy (IHF) Hamacher weighted average (WA) (IHFHWA) and IHF Hamacher weighted geometric (WG) (IHFHWG) operators proposed by Zhou et al.[41] and the IHF Einstein WG (IHFEWG) and the IHF Einstein WA (IHFEWA) operators developed by Zhou and Li[42].

It is also noticed that many AOs are unable to handle the IHFV-based information. Because of limitations in their structures, such as a simple fuzzy set has no concept of NMD, and a simple IFS has no concept of hesitancy degree. Due to such limitations, a considerable amount of information is lost. Few FS and IFS-based AOs are discussed as follows: The intuitionistic fuzzy Hamacher WA (IFHWA) and intuitionistic fuzzy Hamacher WG (IFHWG) operators discussed by Huang[43], Seikh and Mandal[44] developed the intuitionistic fuzzy domain WA (IFDWA) and intuitionistic fuzzy domain WG (IFDWG) operators, and Senapati et al.[45] developed the intuitionistic fuzzy Aczel-Alsina WA (IFAAWA) and intuitionistic fuzzy Aczel-Alsina WG (IFAAWG) operators.

Table 6 above shows ranking results of alternatives using the proposed IHFSWWA and IHFSWWG operators and other IHFHWA, IHFHWG, IHFEWA, and IHFEWG operators. It is noticeable that our results are more accurate and precise. It is also evident that our proposed theory framework is more reliable than existing theories. Furthermore, the graphical representation of Table 6 is provided in Fig. 6.

The combined results and comparison with other AO results are shown in Fig. 6. It is evident from numerical values that the suggested theory produces more accurate and exact outcomes. Our suggested findings are shown by the blue line in the diagram above. The grey line displays the aggregated outcome using the AOs provided by Zhou and Li[42], whereas the orange line shows the aggregated results suggested by Zhou et al.[41].

To validate the effectiveness of the proposed operators, a comprehensive assessment is conducted by focusing on these aspects, including flexibility, efficiency, robustness, and the ability to handle uncertainty.

   i. **Flexibility:** The proposed operators for IHFSWWA and IHFSWWG are very flexible as they efficiently deal with input including membership, non-membership and hesitancy. However, the standard FS and IFS do not have the ability to consider such varied input, the proposed work can do this without losing the details

AOs	Handling uncertainty	Flexibility	Robustness to noise	Computational complexity	Suitability for decision-making
IHFSWWA	✓	✓	✓	✓	✓
IHFSWWG	✓	✓	✓	✓	✓
IHFHWA	✓	✓	✕	✕	✓
IHFHWG	✓	✓	✕	✕	✓
IHFEWA	✓	✓	✓	✕	✓
IHFEWG	✓	✓	✓	✕	✓
IFHWA	✕	✓	✕	✓	✕
IFHWG	✕	✓	✕	✓	✕
IFDWA	✓	✓	✕	✓	✓
IFDWG	✓	✓	✕	✓	✓
IFAAWA	✓	✓	✓	✓	✓
IFAAWG	✕	✕	✓	✕	✕

Table 7. Characteristic comparison.

of decision-maker preferences. Tables 6 show that the proposed work efficiently deal with IHFV data while the common AOs fail to do so.

ii. **Computational complexity and efficiency:** Even though the case study is hypothetical, the proposed model is more efficient, generates accurate rankings, and indicates better performance. As in Table 6, the IH-FSWWA and IHFSWWG operators show stable results for every alternative. However, some of the AOs failed to give the ranking outcomes. This means that the proposed approach is practical and stable for computation within the IHF environments.

iii. **Effectiveness and robustness:** The capability and reliability are proven through its performance when dealing with hesitation and uncertainty. Figure 6 shows that, compared to prior approaches, the proposed approach provides efficient score values and is more effective in making accurate decisions. The stability and reliability of the proposed operators hold up in different hesitant frameworks, while other approaches failed in defining the ranking outcomes.

Moreover, to highlight the strengths and weaknesses of each operator, the characteristics comparison analysis is shown in Table 7.

So, the above analysis showed that the proposed IFSWWA and IFSWWG operators demonstrate superior flexibility, robustness, and computational efficiency compared to existing methods and offer more effective handling of uncertainty and hesitancy in complex decision-making scenarios.

Conclusion

This work highlights the importance of a robust decision-making tool and discusses some key findings addressed in the proposed work for handling uncertainty and fuzzy information. In the age of technology, internet use has several advantages and disadvantages, such as concerns about human health, education and learning patterns, relationships and communication, and privacy and security. The purpose of this study is to investigate and examine how the internet and media usage influence critical aspects of the younger generation including communication, learning behaviours, privacy, and mental well-being. For this, the idea of IHFS is utilized as it is one of the best frameworks for the assessment of fuzzy data and its ability to handle hesitant information. Moreover, the SWO also provides a flexible environment for the aggregation of uncertain information. With motivation from IHFS and SWO, we constructed some new AOs called IHFSWWA and IHFSWWG operators, including an investigation of some desired properties of AOs. We developed a MADM algorithm based on the proposed approach and provided solutions to numerical examples for media that affect new generations under various significant attributes. Additionally, we investigate the authenticity and applicability of the proposed work by comparing it with existing MADM methodologies. For convenience, the graphical representation of the main proposed results and comparison is provided to visualize the ranking performance and reliability.

Limitations and future direction

Since the proposed IHFSWAO offers flexibility, robustness, and efficient results. However, it has some limitations such as the proposed approach mainly deals with theoretical frameworks together with mathematical properties even though it lacks comprehensive real-world practical examples or extensive experimental testing. The proposed operators operate with predefined parameter settings yet these settings might not lead to ideal results for all decision-making scenarios. The processing time increases when dealing with a high degree of hesitancy in large datasets because of the computational complexity. So, this study can be extended to investigate automatic system parameter adjustments to increase the versatility of these operators in varied applications. Machine learning (ML) and artificial intelligence (AI) systems should be integrated to automatically modify parameters and enable real-time decision functions which would make these operators more practical to use.

In the future, we hope to expand our suggested strategy in the following ways, such as quasirung orthopair fuzzy sets by Seikh et al.[46], bonferroni operator[47], Fuzzy soft-max AOs[48], Metric spaces[49], soft sets[50], Aczel-Alsina (AA) operations for q-rung ortho pair FS (q-ROFS) framework provided by Khan et al.[51], the idea of

q-ROFS structure-based power AOs developed by Khan et al.[52], Complex t-spherical FS (TSFS) based AOs given by Ullah et al.[53], Complex Picture fuzzy (PF) Dombi AOs[54], Zhang et al.[55] provided the generalized q-ROFS-based AOs, and Mahmood et al.[56] offered the concept of spherical FS (SFS). Moreover, it can be extended to develop the proposed model for applications involving complex decision environments such as multi-criteria group decision-making (MCGDM) and dynamic decision systems.

Data availability

The dataset generated and/or analyzed during the current study is not publicly available due to privacy concerns, but is available from the corresponding author on reasonable request.

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Author contributions

Yuxin Li conceived the idea. Yao Liu and Dan Zhao contributed to the validation and investigation of the results. Yuxin Li supervised the work. All authors contributed significantly to the main manuscript text.

Competing interests

The authors declare no competing interests.

Additional information

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