Entropy and Thermal Dynamics Motivated by Ternary Nanocomposites and Geometric Influence of Oblique Channel

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Abstract: Thermal transport in converging/diverging channels finds applications in various engineering fields, including heat exchangers, microfluidics, and biomedical devices, due to their ability to enhance heat transfer and control fluid flow. The use of nanoparticles cannot be circumvented because of their promising characteristics. These materials widely used in applied thermal engineering, to enhance or control thermal transport in machinery, transformer, chemical engineering. The flow in tank designed for oblique walls is common in practical situations and the dynamics of fluids are essential to maintain it for desired purposes. Therefore, the concept of traditional nanoliquid models is extended for advanced ternary fluids. To acquire the beneficial model results, the momentum slip, viscous dissipative phenomena and elastic walls conditions along with new innovative ternary nanoliquid characteristics are adopted for the model formulation. Then, the results with special emphasis on thermal, entropy optimization, shear drag and heat transport rate are obtained via RK scheme and a comprehensive physical description

is provided. The ternary nanoliquid possesses remarkable thermal transport and dominant behaviour is inspected for entropy, shear drag and heat transfer rate at the elastic walls under Ec, Re, α and γ. These fluids are recommended for different application especially in applied thermal engineering.

1   Introduction

The inherent properties of conventional fluids are imperative for practical applications in many applied research disciplines. However, heating or cooling of machinery is one of the important issues. Therefore, it was thought that the inherent characteristics of common solvents can be improved by adding some nanomaterials [1]. Thus, the concept of Nanofluids arrived in the end of 19th century. Primarily, these fluids consist of characteristically nanometer sized particles of multiple metals [2], their oxides [3], alloys [4], CNTs, ferromagnetic materials [5] and many other such materials [6]. This class of fluids effectively overcome many problems faced by the engineers because of their changed properties. The nanoparticles consistently dissolved in the common solvent to make the resultant mixture more efficient for practical interests. New inventions in nanofluids [7] provided a broad map to the generation of 21st century and large scale applications can be found in applied thermal, aerodynamics, mechanical engineering, paints, lubricants, medical sciences, nanotechnology, and electronics etc.

After, successful development and implementation of nanofluids [8]; another innovative fluid class introduced namely hybrid nanofluids. These fluids comprise two distinct sort of nanoparticles [9] in conventional liquids. The concentration of hybrid nanoparticles [10] is taken in such a way that these particles dispersed consistently, possesses effective characteristics and cannot settled down. Dispersion of hybrid nanoparticles improved thermal conductivity of simple nanofluids. Thus, these liquids are observed more effective for practical uses in the field of nanotechnology. The dynamics of hybrid nanoliquids are completely different than simple nano and common fluids [11], [12]. Therefore, researchers put their focus towards the investigation of these fluids and their applications. Therefore, various efficient fluids have been reported in the literature and found advantageous

outcomes. Hybrid nanofluids [13] study gained concrete attention of the fluid dynamists and they introduced new models in various physical structures and examined their characteristics like temperature performance [14], shear drag, heat transport rate [15], entropy [16] capacity and stability analysis.

Ternary nanofluids becomes much familiar due to promising performance and their applications in multiple fields. Pattnaik et al. [17] reported statistical investigation of a model for solar aircraft applications. The analysis provided in detail output of the problem with increasing quantities. Dey et al. [18] examined the importance of structures influence of NPs on the Dracy flow using Ag/MoS_4 hybrid type nanoparticles [19]. The numerical treatment performed and concluded that shape of used NPs has significant contribution due to their surface area and characteristics. Another study presented in [20] reveals that blood interaction with ternary nanoparticles behave differently and enhance the heat. The output accuracy observed through ADM and its comparison with numerical scheme. In 2024, Panda et al. [21] examined the joint contribution of $CoFe_2O_4$, and Fe_3O_4 materials through shrinkable surface and found that variant Lorentz and heating effects are crucial factors in enhancing the temperature and make the functional fluid more efficient. Ali et al. [22] reported the importance of hybrid nanoparticles and their properties influence on the thermal progress using Laplace scheme. The MoS_2 and Al_2O_3 are taken to improve the efficiency of primary functional liquid. The furnished results shows that permeability and reaction effects declines the movement.

Analysis of an MHD phenomena and its applications for KKL model has been examined in the study Puneet et al. [23]. Numerical results of the model shows that strong MHD drops the movement while enhanced thermal conductivity favors the temperature. Dey [24], and Mishra et al. [25] explore the convective heat and radiative phenomenon on the ternary and traditional fluid models for tapered walls channel and curved expanding surface with additional influence of physical constraints. It is scrutinized that the geometry configuration with physical influence enhances the thermal distribution prominently in which the role of nanoparticles concentration is important. Another significant investigation regarding the contribution of reactive species when interacting with hybrid nanoparticles have been reported by Shah et al. [26]. Based on furnished results, it is determined that hybrid type fluids provide good heat transmission than simple case. Analytical treatment of model for

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channel flow and its stability performance under the action of parameters has been studied by Rana et al. [27]. The study predicted critical values to achieve the potential and effective results for practical applications. Ahmed et al. [28] extended the concept of Walter/B fluid towards the influence of nanoparticles. Use of such material in the presence of radiation and convection provided considerable enhancement in the thermal profile included in the study's findings. Temporal stability of ternary nanoliquid inspired by the influence of multi parameters has been determined by Rana et al. [29]. The study focuses on entropy trends in channel made by unfirm slope walls and the output reveals that nanofluids are excellent to achieve the better performance of the system by minimizing the entropy. Rauf et al. [30] explored the applications of ferrofluid with joint effects of Lorentz forces for rotating disk model. It is scrutinized that increase in ferromagnetic number leads to declines in the temperature.

In recent times, another heat transfer fluids class is introduced known as ternary nanofluids [31], which result the homogenous dispersion of ternary nanoparticles [32], and basic common liquids. This class is observed as potential heat transport fluids which overcome the heat transfer deficiencies of hybrid and common nanofluids. The flow dynamics through a channel designed for oblique walls is a topic of key interest due to their versatile applications. Such flow setup has numerous applications in biomedical engineering, industries and various engineering disciplines. The opening and narrowing ends of the channel highly alter the model characteristics. In 2022, Zahan et al. [33] inspected the thermal performance of TNL through convergent/divergent ends of nozzle. The mixture components were considered as W/EW and Co-Ag-Zn. They noted that heat transport rate is high for all fluids when NPs ratio is taken 1/6:2/3:1/6. Another imperative attempt has been done by Faizan et al. [34]. The conducted study covered predicted ranges of parameters in convergent duct for steady convective flow and discussed the decorated results in detailed manner. They reported that $Re = 600$ and $\phi = 5\%$ are better for the fluid dynamics.

In 2023, Boujelbene et al. [35] designed a model for generalized newtonian liquid for oblique physical setup with oblique walls. They studied entropy minimization and thermal transmission with increasing MHD and slip effects. The impacts of multiple slips on the performance of NFs [36] for obliquely designed model described by Beg et al. [37]. The

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authors preferred FDKBM for mathematical investigation and suggested that the results would be contributed effectively nanocooling devices and drug delivery. In 2021, Laila and Marwat [38] focused on the numerical study of nanofluid in convergent/divergent physical domain with heated walls. They adopted 2phase model and deduced that mass transport increased when Nt, Pr, and Nb uphill's. In 2020, Saifi et al. [39] provided ADM analysis for the flow domain of oblique channel. The authors concentrated on the heating transport properties of nanoliquid and validated the ADM results. Some good studies in the field of nanoliquids by considering the key effects of physical parameters like elasticity of the walls, Lorentz force phenomena, thermal generation, darcy influences and Joule heating have been reported by various researchers (for instance see Refs. [40], [41], [42],).

The cited literature revealed that numerous attempts have been reported and extended the traditional model for nano and hybrid nanofluids for the flow situation usually termed as Jeffery Hamel which is of remarkable significance in the applied research areas. The important concept of dynamical study using new innovative ternary nanofluids is still a big gap in the research domain which needs to be filled. Thus, the previous hybrid model is extended to ternary nanofluids concept with additional physical constraints. The key purpose and research question of this study are:

- Extension of the traditional nanoliquid for enhanced thermal transport model using ternary nanoparticles and their influential characteristics.

- The acquired model will be analyzed via numerical approach to furnish and discuss the model results.

- How the convergent/divergent ends influenced the velocity, entropy and thermal distributions of ternary nanofluids at different levels of angle?

- How thermal transport rate can be further improved in ternary nanoliquids, entropy minimization and comparative computational results for shear drag and heat transport rate varied?

- In which parametric ranges the model possesses more efficient results for entropy and thermal transport?

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2 Model Design for Oblique Tank

In this study, an oblique tank physical configuration is considered in polar coordinates. The tank by two walls which possesses the characteristics of stretching/shrinking ($u = u_w = s/r$) in the presence of momentum slip and apart from 2α. The flow ternary nanoliquid is taken inside the tank and the flow region is $r > 0$ along increasing radial direction. The fluid is obtained typically dispersion of ternary nanoparticles (Al_2O_3-TiO_2-CuO) and water (H_2O) are the components of flowing ternary fluid. Further, the tank has two ends one is converging end ($\alpha < 0$) and other is diverging corresponds to $\alpha > 0$. The flow is steady, incompressible, dissipative and ensure the absence of chemical reaction. The tank design and used nanoparticles properties are decorated in Fig. 1. Further, detailed estimated characteristics for ternary nanoliquids are enlisted in Table 1.
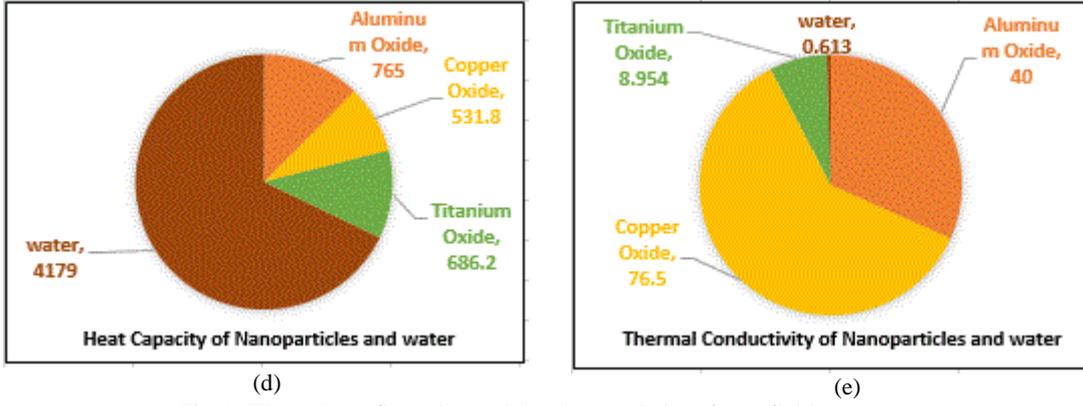

(a)

(b)

(c)

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(d)

(e)
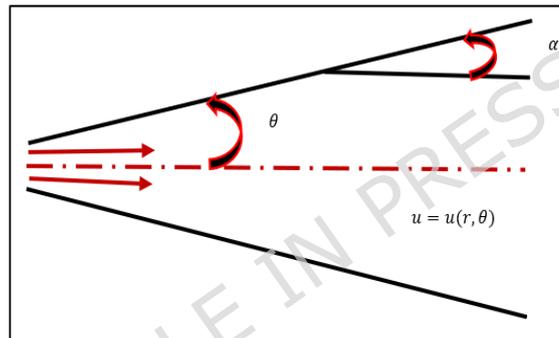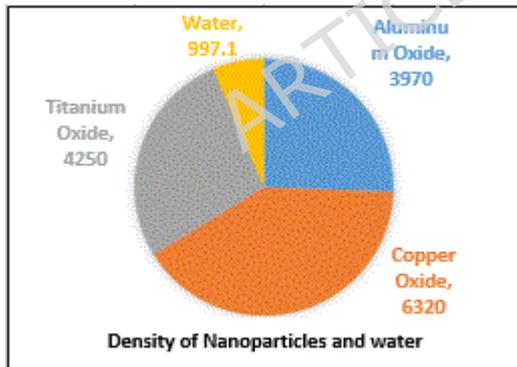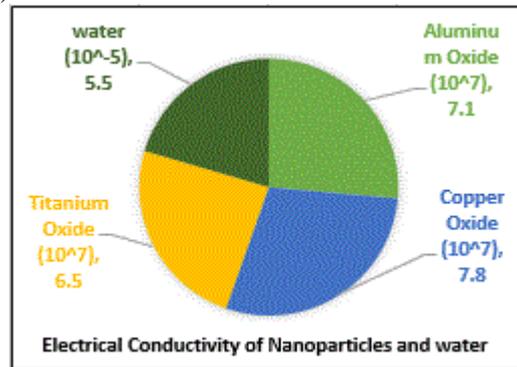
Fig. 1: The tank configuration and the characteristics of nanofluid components

To proceed with the mathematical model which describes the ternary nanoliquid flow inside an oblique tank with elastic walls and momentum slip is given below.

$$\rho_{TNFs}\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{u}) = 0 \tag{1}$$

$$\tilde{u}\frac{\partial}{\partial r}(\tilde{u}) = -\frac{1}{\rho_{TNFs}}\frac{\partial p}{\partial r} + \frac{\mu_{TNFs}}{\rho_{TNFs}}\left[\frac{\partial^2\tilde{u}}{\partial r^2} + \frac{1}{r}\frac{\partial\tilde{u}}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2\tilde{u}}{\partial\theta^2}\right) - \frac{\tilde{u}}{r^2}\right] \tag{2}$$

$$\frac{1}{r\rho_{TNFs}}\frac{\partial p}{\partial\theta} - \frac{\mu_{TNFs}}{r^2\rho_{TNFs}}\frac{\partial\tilde{u}}{\partial\theta} = 0 \tag{3}$$
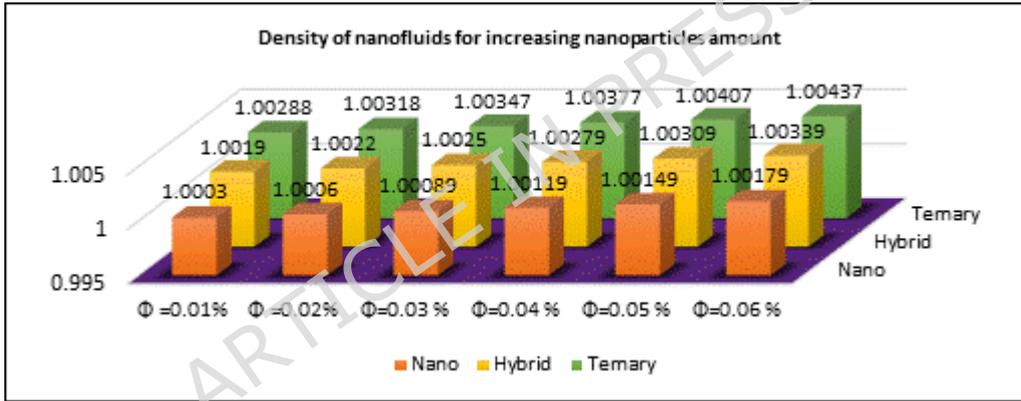
$$\tilde{u}\frac{\partial T}{\partial r} = \frac{k_{TNFs}}{(\rho c_p)_{TNFs}}\begin{bmatrix}\frac{\partial^2 T}{\partial r^2}\\+\frac{1}{r}\frac{\partial T}{\partial r}\\+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\end{bmatrix} + \frac{\mu_{TNFs}}{(\rho c_p)_{TNFs}}[\left(2\left(\frac{\partial}{\partial r}\hat{u}\right)^2\right) + \frac{2\tilde{u}^2}{r^2} + \frac{1}{r^2}\left(\frac{\partial}{\partial\theta}\tilde{u}\right)^2] \tag{4}$$

Further, the tank associated to $\frac{\partial\tilde{u}}{\partial\theta} = 0, \frac{\partial T}{\partial\theta} = 0, \tilde{u} = u_w$ along the middle line and $\tilde{u} = \tilde{u}_w - \gamma_1\frac{\partial\tilde{u}}{\partial\theta}, T = r^{-2}T_w$ at the tank walls when it approaches to α and $-\alpha$. Now, for further simplification of the model, we used the subsequent transformations and enhanced characteristics given in Table 1 [43], [44]. The empirical formulas for the properties in Table 1 are estimated in Fig. 2 by changing the concentration level in the basic fluid. The 3D bar charts reveal that ternary nanoliquid has dominant properties in all the cases followed by hybrid and common nanoliquids. The concentration level is taken from 0.01% to 0.06% for better model outcomes. It is worth highlighting that, in Table 1 $\hat{\mathcal{E}}_1 = \phi_1, \hat{\mathcal{E}}_2 = \phi_2$ and $\hat{\mathcal{E}}_3 = \phi_3$ are the nanoparticles concentration for Al$_2$O$_3$, TiO$_2$ and CuO, respectively.
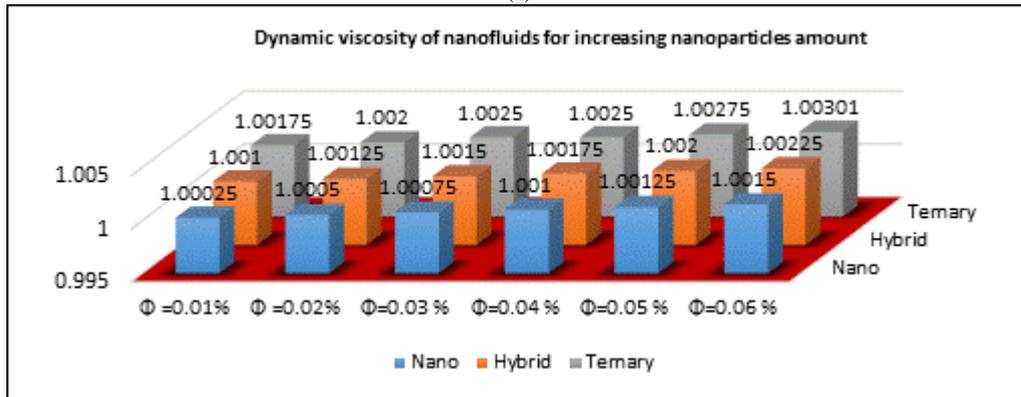
$$F(\eta) = \frac{\breve{f}(\theta)}{\breve{f}_{max}}, \eta = \alpha^{-1}\theta, \beta(\eta) = \frac{Tr^2}{T_w} \tag{5}$$

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Table 1: Influential characteristics of ternary nanofluids for the current designed model

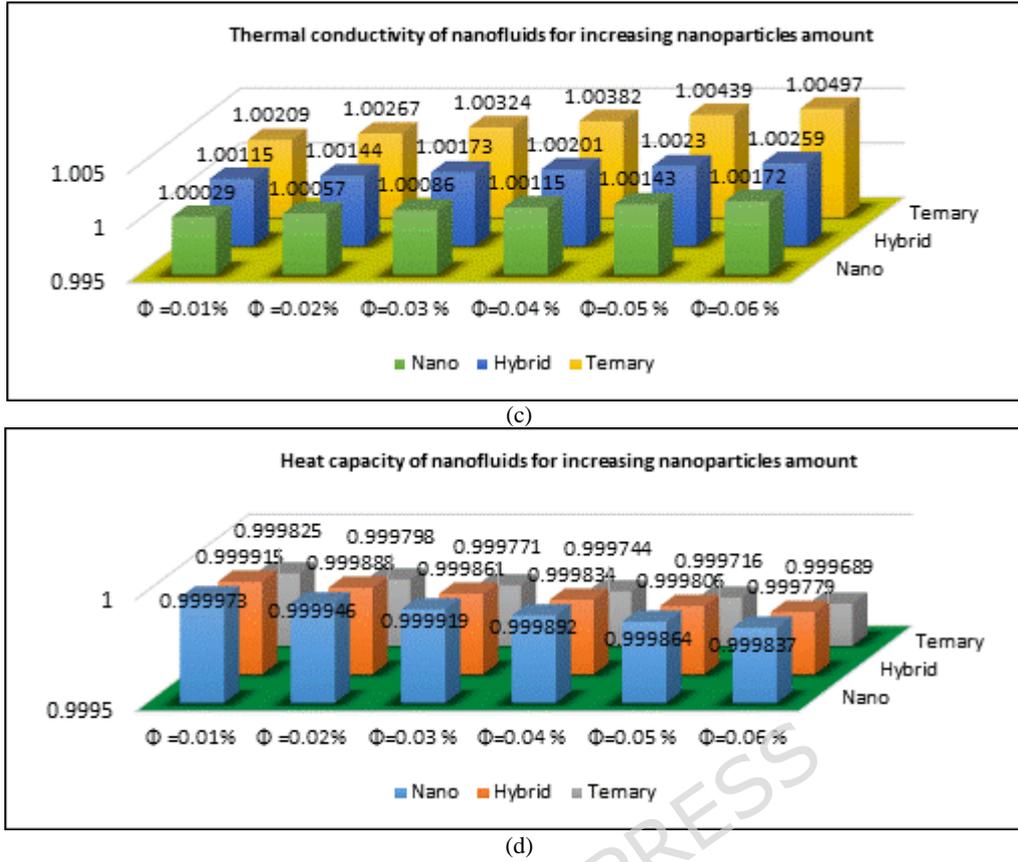| Nanofluids Property	Notation	Formulas
Heat Capacity (HC)	$\dfrac{(\rho c_p)_{trinano}}{(\rho c_p)_w}$	$(1-\hat{\mathcal{E}}_3)\left[(1-\hat{\mathcal{E}}_2)\left(1-\hat{\mathcal{E}}_1+\dfrac{\hat{\mathcal{E}}_1(\rho c_p)_{Al_2O_3}}{(\rho c_p)_w}\right)+\dfrac{\hat{\mathcal{E}}_2(\rho c_p)_{TiO_2}}{(\rho c_p)_w}\right]$ $+\dfrac{\hat{\mathcal{E}}_3(\rho c_p)_{CuO}}{(\rho c_p)_w}$
Density	$\dfrac{\rho_{trinano}}{\rho_w}$	$(1-\hat{\mathcal{E}}_3)\left[(1-\hat{\mathcal{E}}_2)\left[\left(1-\hat{\mathcal{E}}_1+\dfrac{\hat{\mathcal{E}}_1\rho_{Al_2O_3}}{\rho_w}\right)\right]+\dfrac{\hat{\mathcal{E}}_2\rho_{TiO_2}}{\rho_w}\right]+\dfrac{\hat{\mathcal{E}}_3\rho_{CuO}}{\rho_w}$
Dynamic Viscosity (DV)	$\dfrac{\mu_{trinano}}{\mu_w}$	$\dfrac{1}{\left(1-\hat{\mathcal{E}}_1\right)^{2.5}\left(1-\hat{\mathcal{E}}_2\right)^{2.5}\left(1-\hat{\mathcal{E}}_3\right)^{2.5}}$
Thermal Conductivity (TC)	$\dfrac{k_{trinano}}{k_w}$	$\dfrac{\breve{k}_{CuO}+2\breve{k}_{(Al_2O_3-TiO_2)w}-2\hat{\mathcal{E}}_3(\breve{k}_{(Al_2O_3-TiO_2)w}-\breve{k}_{CuO})}{\breve{k}_{CuO}+2\breve{k}_{(Al_2O_3-TiO_2)w}+\hat{\mathcal{E}}_3(\breve{k}_{(Al_2O_3-TiO_2)w}-\breve{k}_{CuO})}$ $k_{(Al_2O_3-TiO_2)w}=k_w\left[\dfrac{\breve{k}_{TiO_2}+2\breve{k}_{(Al_2O_3)w}-2\hat{\mathcal{E}}_2(\breve{k}_{(Al_2O_3)w}-\breve{k}_{TiO_2})}{\breve{k}_{TiO_2}+2\breve{k}_{(Al_2O_3)w}+\hat{\mathcal{E}}_2(\breve{k}_{(Al_2O_3)w}-\breve{k}_{TiO_2})}\right]$ $\breve{k}_{(Al_2O_3)w}=k_w\left[\dfrac{\breve{k}_{Al_2O_3}+2\breve{k}_w-2\hat{\mathcal{E}}_1(\breve{k}_w-\breve{k}_{Al_2O_3})}{\breve{k}_{Al_2O_3}+2\breve{k}_w+\hat{\mathcal{E}}_1(\breve{k}_w-\breve{k}_{Al_2O_3})}\right]$

(a)

(b)

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(c)

(d)

Fig. 2: Estimated values of ternary nanoliquids using formulas described in Table 1

By exercising the above supportive nanomaterials formulas, similarity invariants and flow conditions on the middle and sides of the tank, the subsequent model is obtained.

$$F''' + \frac{Re2\alpha\frac{\rho_{trinano}}{\rho_w}}{\frac{\mu_{trinano}}{\mu_w}}[FF'] + 4\alpha^2 F' = 0 \tag{6}$$

$$\beta'' + \alpha^2 4\beta + \frac{\frac{EcPr}{Re}\alpha\left[\frac{\mu_{trinano}}{\mu_w}*\frac{(\rho c p)_{trinano}}{(\rho c p)_w}\right][\alpha^2 4(F'+\beta'^2)]}{\frac{k_{trinano}}{k_w}} + \left[\frac{\frac{(\rho c p)_{trinano}}{(\rho c p)_w}}{\frac{k_{trinano}}{k_w}}\right]Pr\alpha^2\beta F = 0 \tag{7}$$

Further, the reduced flow conditions to investigate the model are in the following appropriate form. The physical constituents embedded in the new model are $Pr = Prandtl\ number, Ec = Eckert\ number, Re = Reynolds\ number, c_1\ is\ the\ stretching\ or\ shrinking\ parameter\ and\ \gamma = momentum\ slip$.

$$F(0) = 1, F'(0) = 0, F(1) = c_1 - \gamma F'(1), \beta'(0) = 0\ and\ \beta(1) = 1 \tag{8}$$

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Investigation of entropy in nanofluid flow confined to slanted walls channel is imperative. Integrated parameters potentially alter the entropy performance. For the current problem, the entropy is formulated as below.

$$S_g = \frac{k_{trinano}}{T_w^2} r^4 \left(\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial \theta} r^{-1}\right)^2 \right) + \frac{\mu_{trinano}}{T_w} r^2 \left(4\left(\frac{\partial u}{\partial r}\right)^2 + r^{-2}\left(\frac{\partial u}{\partial \theta}\right)^2 \right), \qquad (9)$$

Implementing the similarity variables and then simplification of Eq. (9), yields the following form for entropy generation.

$$N_g = \frac{k_{trinano}}{k_w}(4\beta^2 + \alpha^{-2}\beta'^2) + \left(\frac{PrEc}{Re}\right) \alpha \frac{\mu_{trinano}}{\mu_w}(4F^2 + \alpha^{-2}F'^2), \qquad (10)$$

3 Model Analysis

Mathematical analysis is an essential part of the study which explore the impacts of integrated physical constituents on the model properties. The flow problems are modeled in the form of mathematical model [45] which comprises the nonlinear terms and ODEs [46], [47]. Thus, it not easy to investigate the model in exact solution. Therefore, numerical approach is a reliable mathematical tool under such scenarios. The current problem is then analyzed via numeric approach termed as RK-scheme, [48]. In this technique, the given nonlinear model [49], transformed into the corresponding IVP [50], which then solved further to acquire the desired output. The scheme accuracy is considered up 10^{-6} and tackled the model successfully. The whole working rules [51], [52], for this technique are deeply elaborated in Fig. 3.

For implementation of the scheme, it is obligatory to transform the problem in appropriate form. For this, make the below transformations.

$$\check{\mathcal{G}}_1 = F, \check{\mathcal{G}}_2 = F', \check{\mathcal{G}}_3 = F'', \check{\mathcal{G}}'_3 = F''', \qquad (11)$$

$$\check{\mathcal{G}}_4 = \beta, \check{\mathcal{G}}_5 = \beta', \check{\mathcal{G}}'_5 = \beta'', \qquad (12)$$

Now, reorganized the problem in the following version.

$$F''' = -\left[\frac{Re2\alpha\frac{\rho_{trinano}}{\rho_w}}{\frac{\mu_{trinano}}{\mu_w}}[FF'] + 4\alpha^2 F'\right], \qquad (13)$$

$$\beta'' = -\left[\alpha^2 4\beta + \frac{\frac{EcPr}{Re}\alpha\left[\frac{\mu_{trinano}}{\mu_w} * \frac{(\rho c_p)_{trinano}}{(\rho c_p)_w}\right][\alpha^2 4(F' + \beta'^2)]}{\frac{k_{trinano}}{k_w}} + \left[\frac{\frac{(\rho c_p)_{trinano}}{(\rho c_p)_w}}{\frac{k_{trinano}}{k_w}}\right] Pr\alpha^2\beta F\right], (14)$$

Eqs. (13-14) further simplified using the Eqs. (11-12).

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$$\check{\mathcal{G}}'_3 = -\left[\frac{Re2\alpha\frac{\rho_{trinano}}{\rho_w}}{\frac{\mu_{trinano}}{\mu_w}}\left[\check{\mathcal{G}}_1\check{\mathcal{G}}_2\right] + 4\alpha^2\check{\mathcal{G}}_2\right], \tag{15}$$

$$\check{\mathcal{G}}'_5 = -\left[\alpha^24\check{\mathcal{G}}_4 + \frac{\frac{EcPr}{Re}\alpha\left[\frac{\mu_{trinano}}{\mu_w}*\frac{(\rho c_p)_{trinano}}{(\rho c_p)_w}\right]\left[\alpha^24(\check{\mathcal{G}}_2+\check{\mathcal{G}}_5^2)\right]}{\frac{k_{trinano}}{k_w}} + \left[\frac{\frac{(\rho c_p)_{trinano}}{(\rho c_p)_w}}{\frac{k_{trinano}}{k_w}}\right]Pr\alpha^2\check{\mathcal{G}}_4\check{\mathcal{G}}_1\right], \tag{16}$$

The Eqs. (15-16) are now in well suitable form. The conditions reduced in the as given below.

$$\check{\mathcal{G}}_1(a) = 1, \check{\mathcal{G}}_2(a) = 0, \check{\mathcal{G}}_1(b) = c_1 - \gamma\check{\mathcal{G}}_2(b), \check{\mathcal{G}}_5(a) = 0 \text{ and } \check{\mathcal{G}}_4(b) = 1 \tag{17}$$

The algorithm is now implemented and achieved the output with good accuracy. Further, the computational estimation of the velocity and temperature are given in Table 2 and Table 3 for convergent/divergent ends under stretching and shrinking cases.

Table 2: The computational values of the velocity variations $F(\eta)$ for different ranges of the parameters

η	Convergent end $\alpha = -10^o$		Divergent end $\alpha = 10^o$	
	Shrinking	Stretching	Shrinking	Stretching
0.0	1	1	1	1
0.1	0.992318	0.994998	0.986745	0.990877
0.2	0.968999	0.979813	0.947476	0.963851
0.3	0.929229	0.953912	0.883629	0.919921
0.4	0.871668	0.916401	0.797417	0.860657
0.5	0.794477	0.866034	0.691593	0.788063
0.6	0.695391	0.801219	0.569169	0.704420
0.7	0.571821	0.720046	0.433115	0.612119
0.8	0.421056	0.620346	0.286093	0.513502
0.9	0.240554	0.499789	0.130215	0.410728
1.0	0.028383	0.356052	0.033142	0.305667

Table 3: The computational values of the temperature variations $\beta(\eta)$ for different ranges of the parameters

η	Convergent end $\alpha = -10^o$		Divergent end $\alpha = 10^o$	
	Shrinking	Stretching	Shrinking	Stretching
0.0	1.63404	1.67854	1.58862	1.64381
0.1	1.62636	1.67065	1.58113	1.63606
0.2	1.60352	1.64713	1.55891	1.61300
0.3	1.56602	1.60842	1.52266	1.57520
0.4	1.51472	1.55520	1.47350	1.52357
0.5	1.45080	1.48844	1.41289	1.45929
0.6	1.37570	1.40934	1.34251	1.38377
0.7	1.29112	1.31931	1.26419	1.29852
0.8	1.19895	1.21993	1.17980	1.20512
0.9	1.10122	1.11290	1.09117	1.10512

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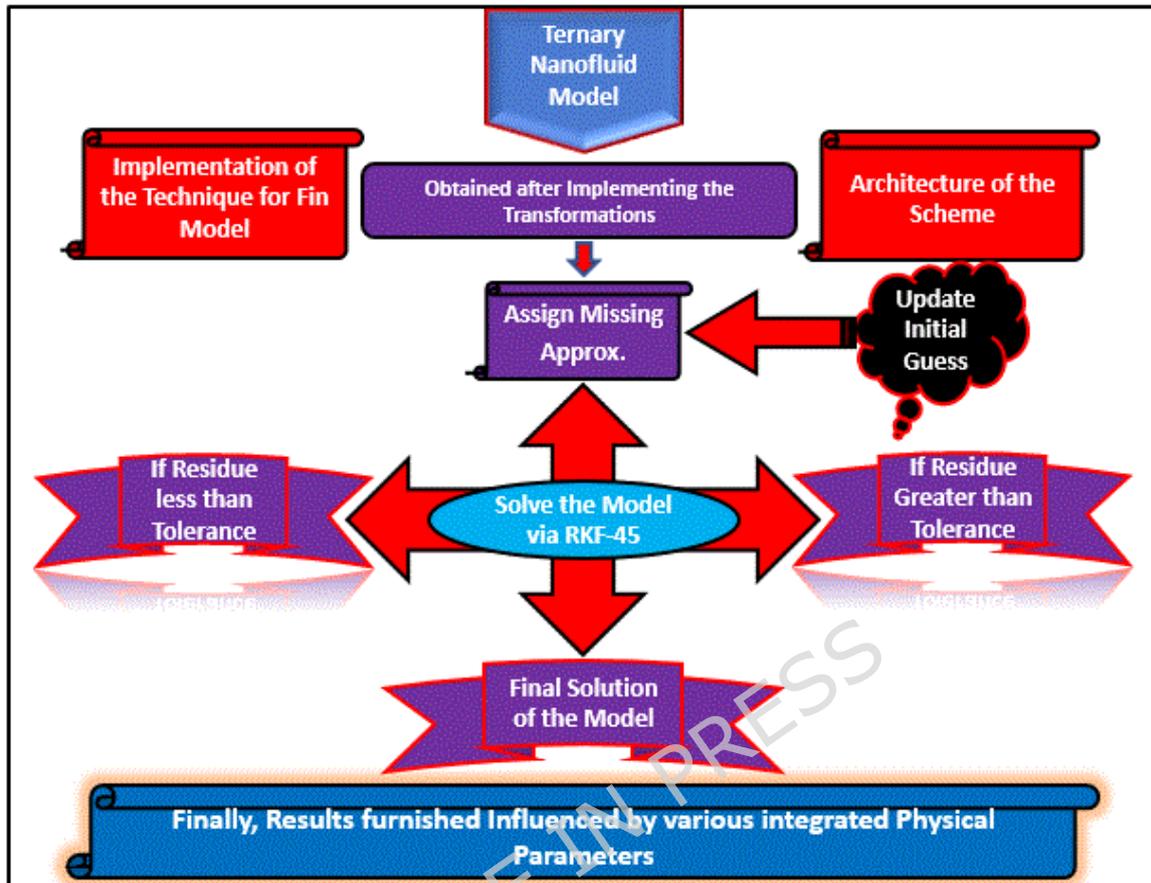| **1. 0**	1	1	1	1

Fig. 3: The computational procedure of the scheme for the current ternary model

4   Results and discussion

The physical constituents appeared in the model highly contribute in the problem dynamics under certain feasible ranges. To examine this model behaviour, Fig. 4, Fig. 5, and Fig. 6 are organized for both convergent and divergent ends of the domain. Further, the results are demonstrated for stretching and shrinking of the walls.

Fig. 4a-h illustrated the velocity changes against stretching and shrinking cases for both convergent and divergent ends. It is imperative to highlight that the angle α is taken in degree measurement in which $\alpha > 0^o$ and $\alpha < 0^o$ correspond to divenrgent and convergent ends. The results illustrated in Figs. 4a-d reveal that the velocity of ternary nanoliquid rises through convergent end and rapid changes are observed for stretching case. Physically, at this end the pressure increases at the inlet position and meanwhile stretching walls effects drag the fluid particles attached there; as a consequence, the ternary nanoliquid

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particles move rapidly. However, at the middle domain the velocity of optimum and obey the flow conditions. At divergent end, the motion of ternary nanoliquid declines in both cases. Physically, when the channel end opens it releases the pressure there and fluid molecules disperse inside the domain and follow parabolic path. This time, back flow of ternary nanoliquid is examined when the channel end opens at larger angle. Thus, the back flow is inversely related to the channel opening.

Fig. 4e-h highlights the results for ternary nanoliquid motion for increasing values of Re and different levels of stretching/shrinking, respectively. For convergent and divergent ends reverse fluid movement is investigated. The desired quick movement of the particles can be achieved by keeping the channel end at $\alpha = -10^o$ while the slow movement corresponds to divergent end $\alpha = 10^o$. The higher Re boosts the motion for convergent while opposes in divergent case (Fig. 4e-f). Further, for more declines is seen for at divergent end. A fascinating change in the nanoliquid movement are seen from Fig. 4g-h when the walls are kept at different stretching/shrinking levels. In both conditions, the reverse changes in the movement are obvious. Thus, the elastic property of the tank walls is good to control and boost the movement inside the domain.
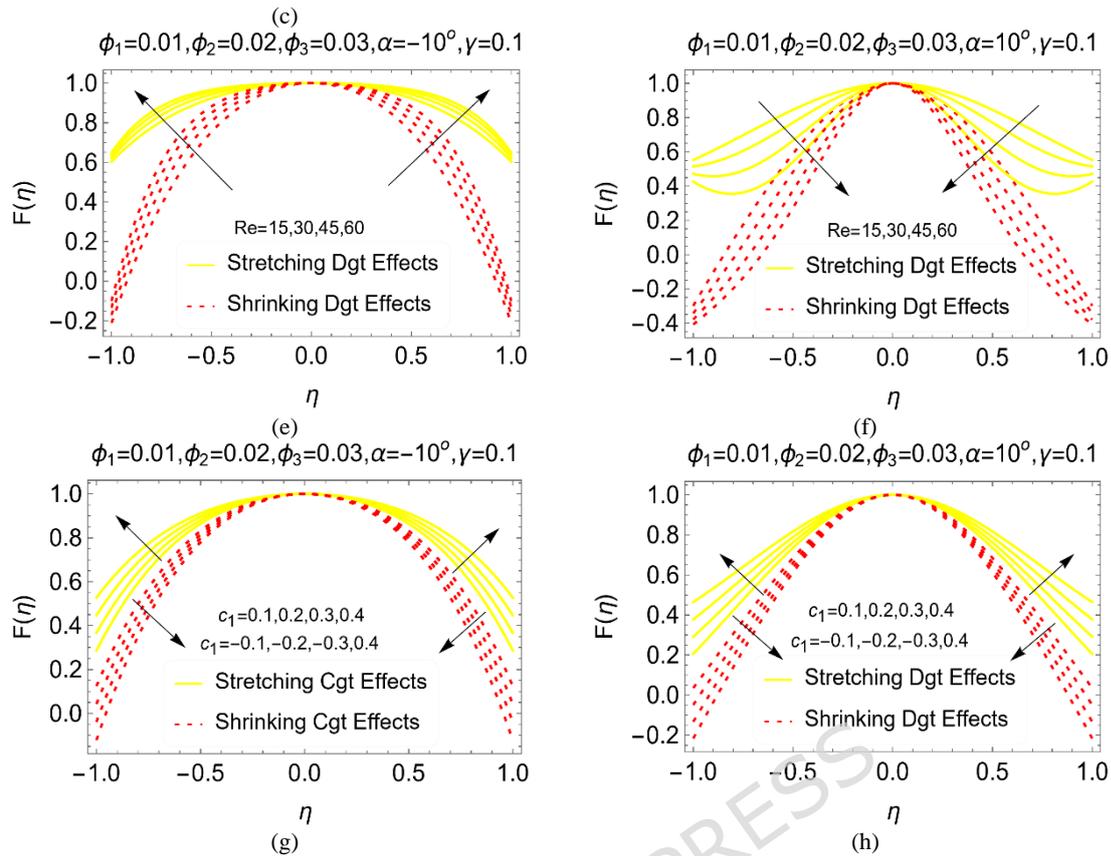

(a)

(b)

(d)

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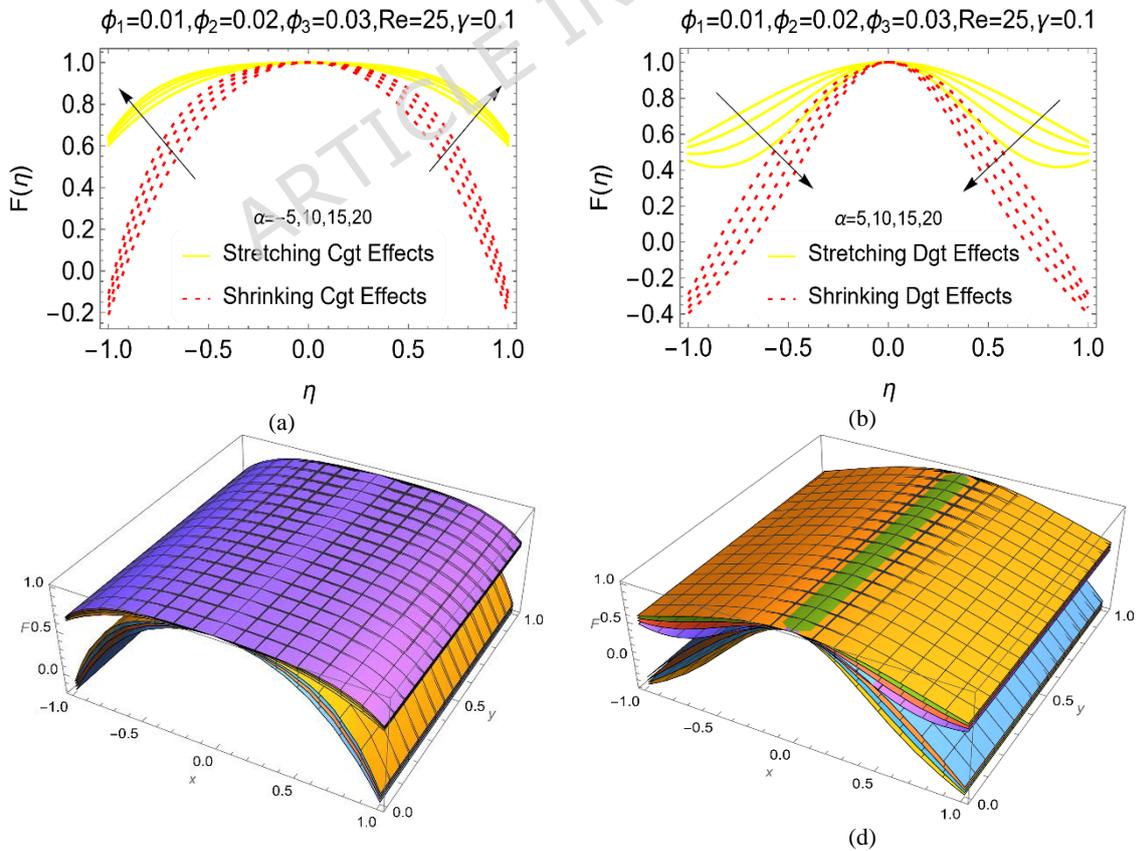
Fig. 4: The velocity characteristics for convergent and divergent ends (a-d) α, (e-f) Re and (g-h) c_1

The next, Fig. 5a-f portrayed for the analysis of ternary nanoliquid temperature characteristics when the physical quantities vary. The cases for stretching/shrinking and convergent and divergent openings are furnished. It is noticeable that when the elastic parameter changes $c_1 = 0.1\ to\ 0.4$ and $c_1 = -0.1\ to -0.4$ along $\alpha = -10^o$ (convergent end) and $\alpha = 10^o$ (divergent end), then the liquid temperature heightened for stretching walls case and drop when the walls shrinked. Physically, the liquid particles present in the surrounding of mid of the flow domain attained optimum temperature; while, rest of the particles have low temperature. Similarly, when the strength of Re increased, the temperature shown prominent variations in both cases. However, at converging end remarkable thermal enhancement is noted while it reduced towards the diverging end. These influences are illustrated in Fig. 5c-d.

Addition of viscous dissipative phenomena in the ternary nanoliquid model highly affect the temperature. Physically, due to heightened Ec values, the internal energy of the liquid got increased and transformation process occur from higher temperature to the particles at low temperature region. Thus, the viscous dissipation effects are noted good for the heating

property of TNL. These are demonstrated in Fig. 5e-f for both ends for elastic walls. Further, the TNPs concentration is kept at 1%, 2% and 3%, respectively to acquire the homogeneity properties of TNL.
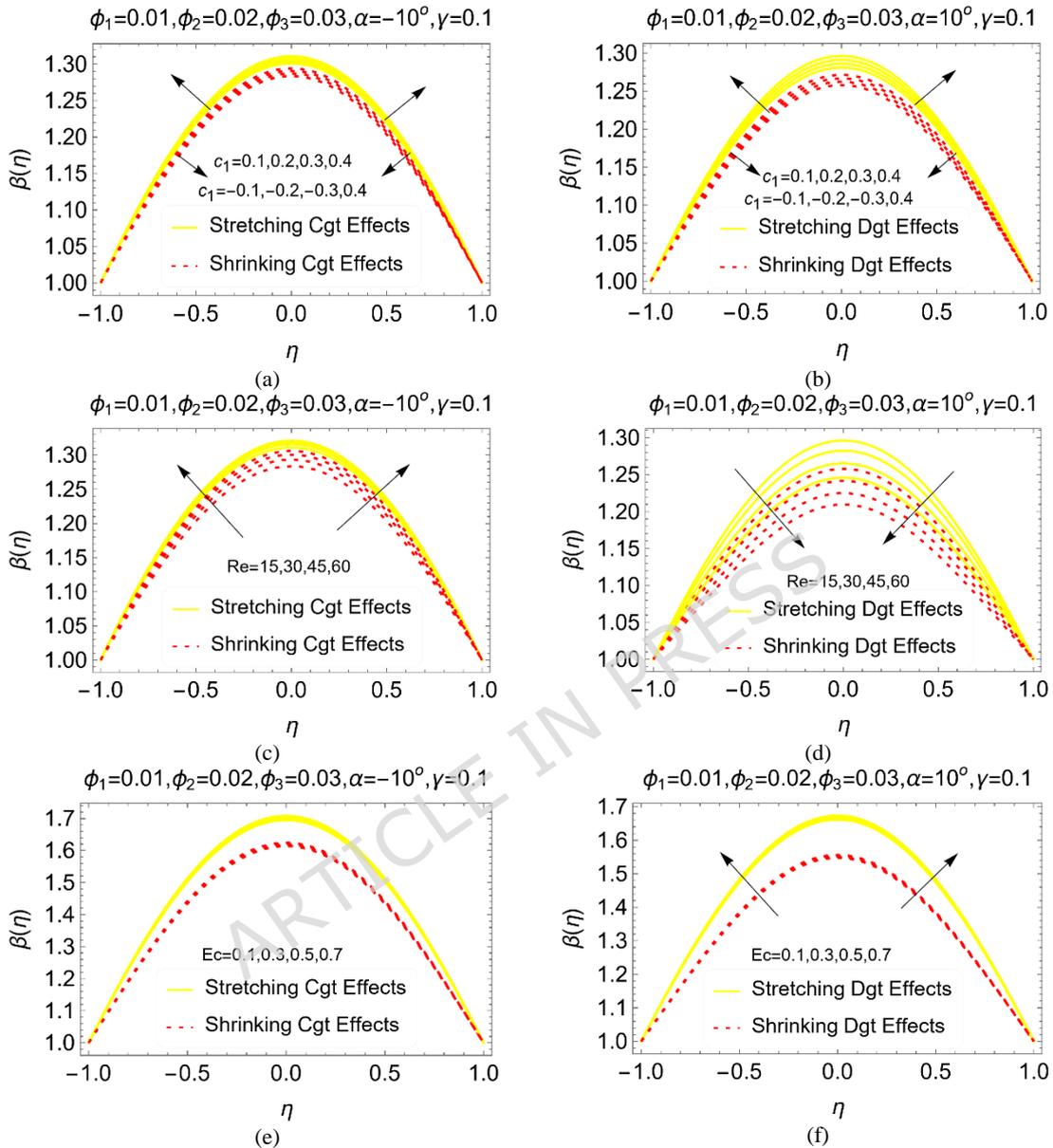

Fig. 5: Temperature characteristics for convergent and divergent ends (a-b) c_1, (c-d) Re, and (e-f) Ec

The entropy investigation in TNL is very important under certain physical aspects. Therefore, for the current designed model these effects are furnished in Fig. 6a-l for different physical constituents. Fig. 6a-d reveals opposite changes in entropy at the converging and diverging ends and are observed very clear for widening and diminution cases. For Cgt case, the entropy enhanced very clearly towards $\eta = 1.0$ and this enhancement gradually slowdown near $\eta = 0.0$. However, the entropy of TNL system can

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be minimized for widening walls of the channel. In similar way, when the TNL becomes highly dissipative it diminishes the entropy for Cgt case while favors it for Dgt case. Thus, the Ec for smaller values is better tool to minimize the entropy phenomena in TNL. These are portrayed in Fig. 6e-h. Further, the contribution of tank walls elastic characteristics possesses very weak impacts on the entropy phenomena and given in Fig. 6i-l.

(a)

(b)

(c)

(d)

(e)

(f)

(g)
$\phi_1=0.01,\phi_2=0.02,\phi_3=0.03,\alpha=-10^o,\gamma=0.1$

(h)
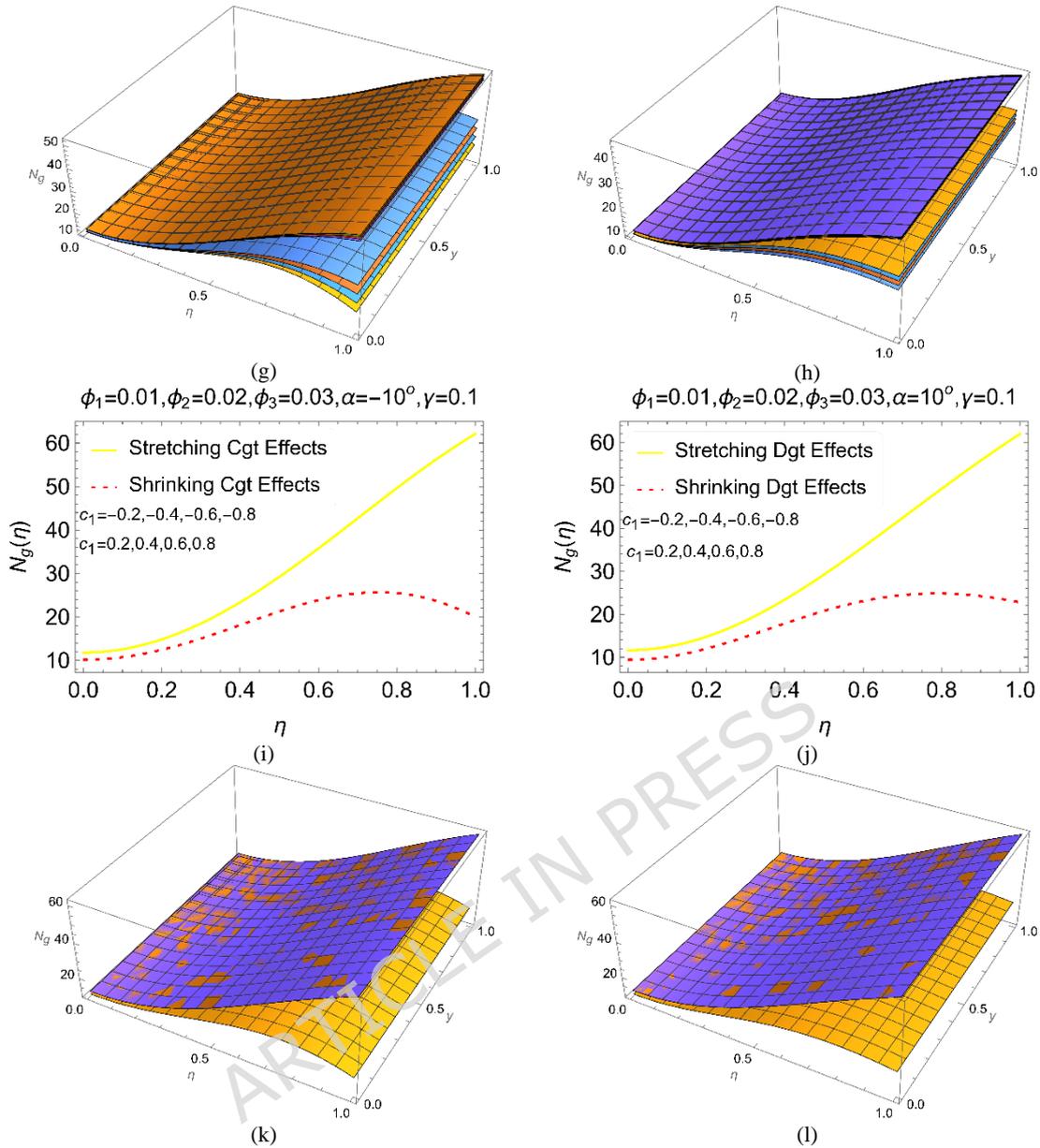$\phi_1=0.01,\phi_2=0.02,\phi_3=0.03,\alpha=10^o,\gamma=0.1$

(i)

(j)

(k)

(l)
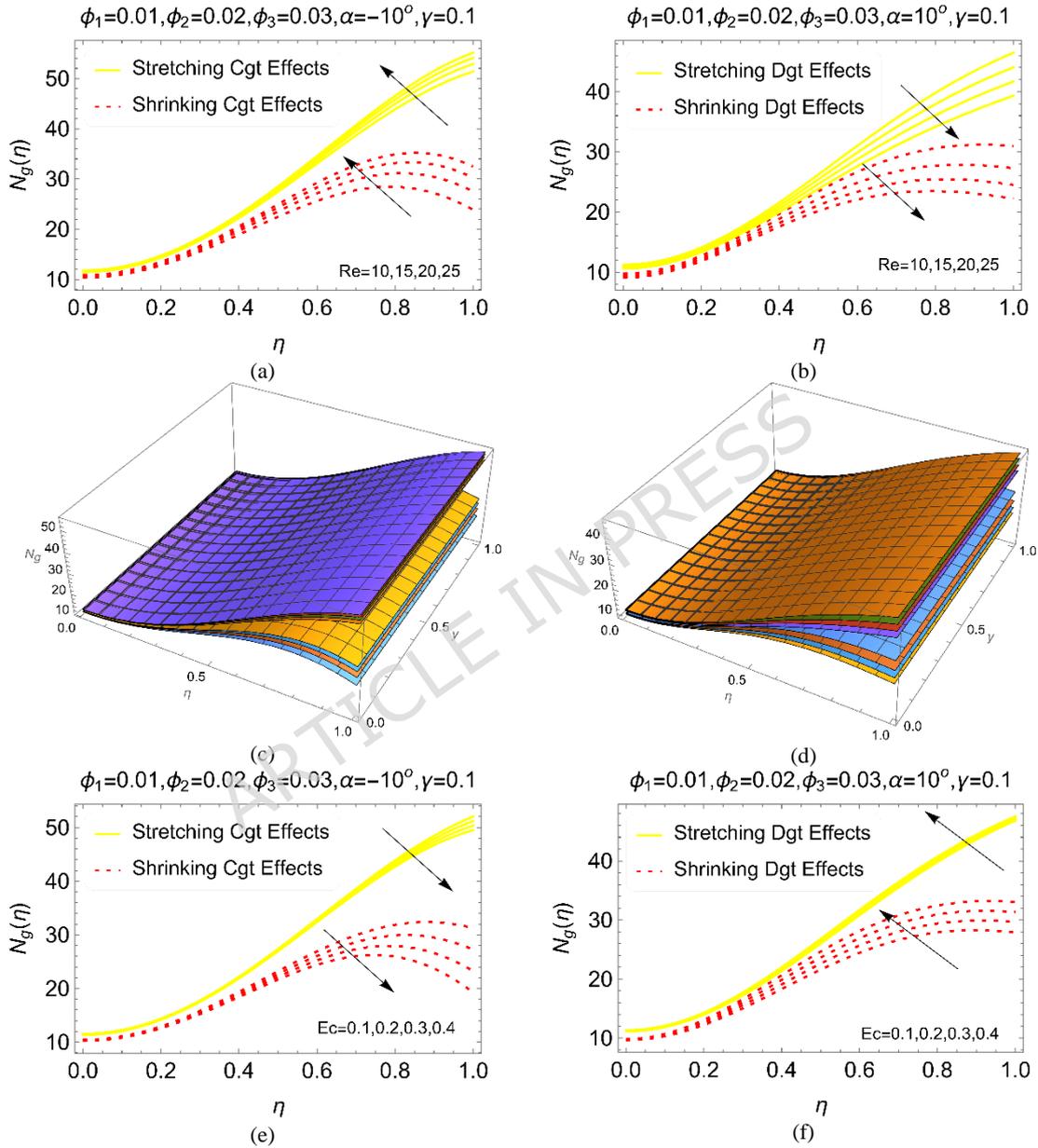
Fig. 6: Entropy characteristics for convergent and divergent ends (a-d) Re, (e-h) Ec, and (i-l) c_1

The shear drag and rate of thermal transport in TNL greatly influenced by the TNPs concentration, elastic property of the walls, dissipation energy and slippery surface. The computation of the quantities has been performed for multiple physical aspects with focus on the wall's elastic parameter and both convergent/divergent ends of the tank. These estimations are enlisted in Table 4 and Table 5 for SF and Nu, respectively. By adding high concentration of nanoparticles, the SF on the elastic walls increased while it is examined maximum for stretching case. For the same parameters, dominant SF is noted towards divergent end. The numerical computations for Nu under enlarging parametric ranges are organized in Table 5. The results against the nanoparticles amount shown that the thermal

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transport rate reduced when the nanoparticles increased. Thus, the used oxide nanoparticles have good coolant characteristics. Similarly, for Re in the range 15-35 and Ec from 0.2 to 0.8 reduced the transport of heat at the tank walls.

Table 2: Computation for skin friction under different constraints

Physical ranges				Convergent end $\alpha = -10^o$		Divergent end $\alpha = 10^o$	
ϕ_1	ϕ_2	ϕ_3	Re	Shrinking	Stretching	Shrinking	Stretching
0.01	0.02	0.03	10	−2.1615	−1.4769	−1.5792	−1.0000
0.02				−2.1607	−1.4763	−1.5799	−1.0007
0.03				−2.1598	−1.4756	−1.5809	−1.0016
0.04				−2.1587	−1.4747	−1.5822	−1.0027
0.05				−2.1574	−1.4737	−1.5836	−1.0039
0.01	0.01			−2.1567	−1.4732	−1.5843	−1.0046
	0.02			−2.1615	−1.4769	−1.5792	−1.0000
	0.03			−2.1657	−1.4802	−1.5745	−0.9959
	0.04			−2.1695	−1.4831	−1.5703	−0.9923
	0.05			−2.1729	−1.4857	−1.5667	−0.9891
	0.02	0.01		−2.1611	−1.4766	−1.5795	−1.0003
		0.02		−2.1614	−1.4769	−1.5793	−1.0001
		0.03		−2.1615	−1.4768	−1.5792	−1.0000
		0.04		−2.1612	−1.4767	−1.5794	−1.0002
		0.05		−2.1608	−1.4764	−1.5799	−1.0007
		0.03	15	−2.2971	−1.5804	−1.4283	−0.8655
			20	−2.4277	−1.6781	−1.2779	−0.7259
			25	−2.5532	−1.7702	−1.1294	−0.5822
			30	−2.6733	−1.8571	−0.9844	−0.4358
			35	−2.7884	−1.9392	−0.8442	−0.2882

Table 3: Computation for Nu under different constraints

Physical ranges					Convergent end $\alpha = -10^o$		Divergent end $\alpha = 10^o$	
ϕ_1	ϕ_2	ϕ_3	Re	Ec	Shrinking	Stretching	Shrinking	Stretching
0.01	0.02	0.03	10	0.1	1.2509	1.3991	1.0569	1.2437
0.02					1.1878	1.3173	0.9500	1.1240
0.03					1.1277	1.2411	0.8564	1.0179
0.04					1.0708	1.1704	0.7742	0.9236
0.05					1.0170	1.1049	0.7021	0.8396
0.01	0.01				1.2985	1.4542	1.0983	1.2937
	0.02				1.2780	1.4193	1.0164	1.2060
	0.03				1.2558	1.3839	0.9391	1.1219
	0.04				1.2324	1.3485	0.8667	1.0414
	0.05				1.2080	1.3132	0.7992	0.9648
	0.02	0.01			1.3395	1.5006	1.1279	1.3305
		0.02			1.3222	1.4695	1.0491	1.2464
		0.03			1.3026	1.4372	0.9753	1.1662
		0.04			1.2811	1.4042	0.9066	1.0900

		0.05			1.2583	1.3710	0.8428	1.0178
		0.03	15		1.2509	1.3991	1.0569	1.2437
			20		1.2780	1.4193	1.0164	1.2060
			25		1.3026	1.4372	0.9753	1.1662
			30		1.3251	1.4533	0.9341	1.1246
			35		1.3457	1.4677	0.8930	1.0816
			10	0.2	1.2148	1.3686	1.1022	1.2862
				0.3	1.2089	1.3616	1.1073	1.2925
				0.4	1.2030	1.3547	1.1124	1.2988
				0.5	1.1972	1.3478	1.1176	1.3052
				0.6	1.1914	1.3410	1.1228	1.3117

5 Conclusions

A comprehensive investigation of entropy and thermal performance of ternary nanoliquid in a tank with oblique walls which possesses the properties of stretching/shrinking and momentum slip is conducted. The designed model is analyzed via numerical approach and then the dynamical results are furnished for the model parameters. It is noticeable that:

- The flow of TNF in the tank enhanced at the convergent ($\alpha = -5^o, -10^o, -15^o, -20^o$) part and reduced at the divergent ($\alpha = 5^o, 10^o, 15^o, 20^o$) parts under elastic walls.

- For different stretching/shrinking levels of the tank walls possesses reverse variations in the TNF movement and more fully developed field observed for divergent end.

- The Eckert number influenced the temperature of TNF when varied in the range $0.1, 0.3, 0.5, 0.7$ and dual temperature behaviour is noticed for varying elastic number c_1 against $0.1, 0.2, 0.3, 0.4$ and $-0.1, -0.2, -0.3, -0.4$, respectively.

- The entropy of TNF can be optimized for Dgt case for $Re = 10, 15, 20, 25$ and more minimization is evident for shrinking case while it enhanced for Cgt end with same Re ranges.

- The SF enhanced for larger particles concentration and transport rate of heat declined against it which shows that the used oxide nanoparticles are better sources for the cooling of TNF in a tank.

Declaration of Competing Interest
There is no competing interest regarding this work.

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