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The effectiveness of child policies to boost child quality and quantity in the PAYG pension system

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This paper constructs an overlapping generations model to examine the effects of child policies and explore the policy design regarding policy support and the policy mix. We find that child allowances and educational subsidies have opposing effects on fertility and human capital growth, but they all have an inverted *U*-shaped relationship with pay-as-you-go pensions. With an appropriate adjustment in the policy mix, raising policy support could boost child quality and quantity and increase pensions in a competitive economy. Additionally, we contrast public and private educational subsidies and find that public educational subsidies are more effective when the policy mix emphasizes child allowances, whereas private educational subsidies perform better when the government increases policy support. We further extend our analysis to a social planner's allocation and find that social welfare can be maximized by appropriately setting the two policy instruments, whereas the policy design in a competitive economy cannot achieve maximum social welfare.

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Introduction

Many countries have had or are now experiencing a rapid fall in fertility due to socioeconomic progress, and changing marriage beliefs and preferences. The global average total fertility rate (TFR) has dramatically decreased over the past six decades, going from 5.28 in 1963 to 2.24 in 2023, according to the United Nations World Population Prospects 2024. Among the 237 recorded countries and areas in 2023, there are 130, 65, and 26 countries where TFRs fall below 2.1, 1.5, and 1.3, respectively (United Nations 2024). If the TFR remains below 1.5 for a long time, the country might fall in a "low fertility trap" that is difficult to escape. According to Lutz and Shirbeek (2005), once the TFR in a country or region falls below 1.5, it falls into the "low fertility trap" due to the self-reinforcing mechanisms of the very low birth rate. How to boost birth rates is a common concern faced by the entire globe.

To address this concern, various child policies have been adopted by developed countries facing the "low fertility trap." Two typical policies among them are child allowances and educational subsidies, which are effective in reducing child-raising costs and thus may encourage adults to have more children and invest more in their education (Gauthier 2007; Haan and Wrohlich 2011; Luci-Greulich and Thévenon 2013; Andersen et al. 2018). We call it the cost reduction effect. However, this conjecture is valid only if child policies do not alter individual budget constraints. Let's consider a case in which the government imposes wage taxes to finance child policies. Individuals' disposable income would decrease, which discourages them from giving birth and investing in education. The income loss effect would undermine the effectiveness of child policies in boosting child quality and quantity. Thus, raising policy support is not always effective, depending on the relative strength of the cost reduction effect and the income loss effect.

Child allowances and educational subsidies have different policy objectives. The former aims to reduce childcare costs and raise fertility, while the latter aims to reduce education costs and promote human capital accumulation. Some research suggests that adults determine the number of children and their human capital by doing a quality–quantity trade-off (Becker and Lewis 1973; Becker et al. 1990; de la Croix and Doepke 2003). In such a sense, the resource distribution structure between child allowances and educational subsidies, or the policy mix we will refer to as hereafter, would give rise to adaptive adjustment in individual life-cycle behaviors (e.g., fertility and private education investment), and subsequently impact child quality and quantity. Furthermore, pension benefits would also be affected. In the PAYG (pay-as-you-go) system, pension benefits are associated with wage income and the size of the labor force (Stauvermann and Kumar 2018), with the former depending on the human capital and the latter on fertility. Therefore, it is essential to investigate the effects of the child policy mix.

Existing literature generally examines the effects of child allowances and educational subsidies separately. Regarding child allowances, their impacts on fertility and human capital are ambiguous and depend on several parameters, such as the preference for having children (Fanti and Gori 2009), the output elasticity of physical capital (Yasuoka and Goto 2015), the provisions of grandparental childcare (Miyazawa 2016), and gender wage discrimination (Wang and Xu 2020). Some studies discuss the effects of child allowances in the PAYG pension system, claiming that the introduction of child allowances could raise fertility and improve social welfare (Groezen et al. 2003; Yasuoka and Goto 2011). Regarding educational subsidies, the policy incentivizes parents to invest more in children's education and accelerates human capital accumulation thereby (Yew and Zhang 2013; Morimoto and Tabata 2020). However, their impact on fertility is inconclusive, which might be negative (Yew and Zhang 2013), neutral (Zhang and Casagrande 1998), and positive (Yasuoka and Miyake 2014).

Some theoretical studies incorporate child allowances and educational subsidies into the overlapping generations (OLG) model and analyze the effects of multiple policies. These studies generally claim the opposite impacts of these two policies. For example, in the scenario without the pension system, Zhang (1997) shows that educational subsidies could not only directly promote economic growth but also indirectly raise it by reducing fertility, while child allowances yielded the opposite effects. Yasuoka and Miyake (2014) underscore the importance of the PAYG pension system and find that the pension system might reverse the effects of child policies if the pension benefit is associated with human capital. Chen (2015) illustrates that child allowances would raise the quantity of children, but lower the quality of children while educational subsidies would increase both the quantity and quality of children. Stauvermann and Kumar (2018) conclude that child allowances would raise fertility but decrease human capital growth, pension benefits, and life expectancy while educational subsidies lead to the opposite effects. However, the beforementioned literature treats one policy as given when analyzing the impact of the other and does not investigate an optimal policy design.

We assess the economic consequences of child policies and identify viable policy mixes that simultaneously boost child quality and quantity and raise pension benefits. Our research is highly related to Yasuoka and Miyake (2014) and Stauvermann and Kumar (2018) but differs in several aspects. First, rather than a small open economy, we construct the model in a closed one, which can show how child policies affect individual decision-making through factor pricing. Second, in addition to analyzing the effects of child allowances and educational subsidies independently, we introduce a new parameter that measures the percentage of child support resources allocated to child allowances into the model, allowing us to examine the effects of the policy mix. Third, we evaluate not only subsidies for private education but also subsidies for public education and compare the two approaches. Lastly, in addition to examining the effects of child policies within a competitive economy, we also explored the optimal design of child policies in a social planner's economy.

Our study makes three contributions to the literature. First, we provide an alternative explanation for why child policies cannot always improve child quality and quantity in terms of resource allocation. Previous research confirms the opposite impacts of child allowances and educational subsidies and demonstrates that they are complementary (Zhang 1997; Stauvermann and Kumar 2018). However, implementing these two policies together does not guarantee an increase in fertility and human capital growth. We find that increasing policy supports can only improve the quality and quantity of children when the policy mix is within a specified range. It emphasizes the importance of establishing a dynamic linkage mechanism between policy support and the policy mix.

Second, we differentiate the effects of two forms of educational subsidies. The public educational subsidy increases the public educational service provision while the private educational subsidy changes the relative costs of private education. Prior research concentrates on subsidies for private education (Zhang and Casagrande 1998; Yasuoka and Miyake 2014; Chen 2015) but neglects public education. We broaden the discussion and find the public educational subsidy is more effective when the government spends more money on child allowances. It has significant implications for policymakers to choose the appropriate educational subsidies in different situations.

Third, we explore the optimal child policy design that maximizes social welfare. Unlike existing studies (Yasuoka and Miyake 2014; Chen 2015; Stauvermann and Kumar 2018), which primarily focus on the effects of child policies within a competitive economy, our analysis extends to deriving the optimal relationship between policy support and policy mix in a social planner's economy. We find that policy design in a competitive economy cannot achieve maximum social welfare.

The remainder of the paper is organized as follows. Section "The model" constructs an OLG model with child allowances and private educational subsidies. Section "Comparative statics" evaluates the effects of child policies in the steady state. Section "Numerical simulation" conducts a numerical simulation and explores the relationship between policy support and the policy mix. Section "Further analysis" extends discussions on subsidies for public education and a social planner's allocation. The final section "Conclusion and discussion" concludes the paper.

The model

In this section, we incorporate child allowances and private educational subsidies into a three-period OLG model. The economy is a closed system which consists of homogenous agents, identical firms, and the government. In specific, agents maximize their lifetime utility by choosing consumption, saving, fertility, and education investment. Firms maximize their profits by employing physical capital and efficient labor for production. The government runs a child support scheme, a PAYG pension scheme, and a public education scheme based on a balanced budget. Noted that the closed-economy model offers the advantage of endogenizing wage rates and returns on physical capital. This feature allows for a more nuanced analysis of the dynamic effects of child policy on key economic variables and helps elucidate the intrinsic relationships between different policy instruments. (We choose a closed-economy model over a small open-economy model based on three key considerations. First, in a closed-economy model, wage rates and returns on physical capital are endogenous outcomes of economic equilibrium rather than exogenously determined constants. This endogeneity enables an analysis of how child policies affect individual life-cycle resource allocation by altering factor prices. Second, while small open-economy models offer simplicity in solving complex models, their results heavily rely on exogenous assumptions, making it challenging to isolate the effects of policies. Third, from a practical perspective, for countries with strong economic autonomy or limited dependence on external markets, such as China, a closed-economy model better reflects their economic characteristics.) These dynamics are difficult to capture in a small open-economy model.

Agents. We assume time is discrete and continues forever. Agents are identical within a generation and live for three periods, namely childhood, adulthood, and old age, corresponding to periods $t-1$, t, and $t+1$, respectively. In childhood, agents embody human capital and make no decisions. In adulthood, agents allocate their disposable income to consumption, savings, and child-raising. In old age, they retire and live upon the gross returns of savings and pension benefits.

Education and genius are two key factors affecting human capital accumulation. The former refers to formal education that young agents receive from school, and the latter refers to the cognitive ability and intelligence quotient inherited from parents. Following Blankenau and Simpson (2004), the human capital accumulation function is formulated as:

$$h_t = Be_{t-1}^{\gamma}E_{t-1}^{\eta}h_{t-1}^{1-\gamma-\eta},\ B>0,\ 0<\gamma<1,\ 0<\eta<1 \quad (1)$$

where e_{t-1} represents parents' private education investment in each child, E_{t-1} stands for the government's public education spending on each child, h_{t-1} denotes parents' human capital. B is a human capital scale factor, and γ and η are the elasticity of human capital technology concerning private education investment and public education spending, respectively.

In period t, agents in adulthood are endowed with h_t units of human capital and supply it inelastically in the labor market to obtain wage income w_th_t, with w_t being the wage rate. After paying various taxes, adults distribute their disposable income $(1-\tau-\theta-\sigma)w_th_t$ into consumption, savings, and child-raising, where τ, θ, and σ are the wage tax rate, the pension contribution rate, and the public education tax rate, respectively. Assuming caring a child takes ν percent of wage income (Boldrin and Jones 2002; Fanti and Gori 2012; Stauvermann and Kumar 2018), parents need to pay $\nu w_th_tn_t$ to care n_t children and extra e_tn_t to educate them. Following Schindler and Yang (2015), we assume child policies are funded by wage income taxes. The greater the support for childbirth, the higher the wage income tax, and the lower the individual disposable income. This is referred to as the income loss effect. The government shares χ_t percent of childcare costs by child allowances and ε_t percent of private education expenses by educational subsidies. Thus, parents' actual child-raising costs are $(1-\chi_t)\nu w_th_tn_t + (1-\varepsilon_t)e_tn_t$. It is evident that actual child-raising costs are negatively correlated with child support, which is termed as the cost reduction effect. When agents step into their old age in period $t+1$, their old-age consumption is financed by savings plus returns and pension benefits. Thus, the budget constraints of agents in adulthood and old age are:

$$c_t = (1-\tau-\theta-\sigma)w_th_t - (1-\chi_t)\nu w_th_tn_t - (1-\varepsilon_t)e_tn_t - s_t \quad (2)$$

$$d_{t+1} = R_{t+1}s_t + p_{t+1} \quad (3)$$

where c_t and d_{t+1} represent consumption in adulthood and old age, respectively, s_t denotes savings with R_{t+1} being the rate of return on savings, and p_{t+1} stands for pension benefits.

Following Ehrlich and Lui (1991), de la Croix and Doepke (2003), and Stauvermann and Kumar (2018), agents care about not only their consumption in adulthood and old age but also the quantity and quality of their children. The agents' lifetime utility function is expressed as:

$$U_t = \ln c_t + \beta \ln d_{t+1} + \varphi \ln(n_th_{t+1}),\ 0<\beta<1,\ \varphi>0 \quad (4)$$

where β is the discount factor, and φ is the altruism factor.

By maximizing agents' lifetime utility subject to their budget constraints, we derive the optimal saving, fertility, and private education investment per child as follows:

$$s_t = \frac{\beta}{1+\varphi+\beta}(1-\tau-\theta-\sigma)w_th_t - \frac{1+\varphi}{1+\varphi+\beta}\frac{p_{t+1}}{R_{t+1}} \quad (5)$$

$$n_t = \frac{\varphi(1-\gamma)}{(1+\varphi+\beta)(1-\chi_t)\nu}\left[(1-\tau-\theta-\sigma) + \frac{p_{t+1}}{R_{t+1}w_th_t}\right] \quad (6)$$

$$e_t = \frac{\gamma}{1-\gamma}\frac{(1-\chi_t)\nu}{1-\varepsilon_t}w_th_t \quad (7)$$

Equations (6) and (7) show that given other parameters fixed, higher wage income would decrease fertility and increase private education expenses in each child and thus human capital, which is in line with the substitution between child quantity and quality found in Becker and Lewis (1973).

Firms. We assume a continuum of firms produce a homogenous good in a competitive market by hiring physical capital and efficient labor with the Cobb–Douglas production function of:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha},\ A>0,\ 0<\alpha<1 \tag{8}$$

where Y_t, K_t, and L_t stand for the aggregate output, the physical capital input, and the efficient labor input in period t, respectively. A denotes the productivity of technology, and α represents the output elasticity of physical capital. Suppose N_t is the number of adults in period t and satisfies $N_t = N_{t-1}n_{t-1}$, and then the efficient labor is $L_t = N_t h_t$.

Assuming that output is sold at one unit price and physical capital fully depreciates at the end of each period (Yew and Zhang 2009; Fanti and Gori 2012; Cipriani and Pascucci 2020), profit maximization leads to the optimal wage rate w_t and the rate of return on physical capital R_t as:

$$w_t = (1-\alpha)A(k_t/h_t)^{\alpha},\ R_t = \alpha A(k_t/h_t)^{\alpha-1} \tag{9}$$

with $k_t = K_t/N_t$ denoting physical capital per worker. Moreover, the output per worker can be expressed as $y_t = Ak_t^{\alpha}h_t^{1-\alpha}$.

Government. The government levies a wage tax on adults' wage income to channel child support resources and distributes μ percent on child allowances and $1-\mu$ percent on educational subsidies. Specifically, μ describes the child policy mix, with a larger μ indicating that more resources are devoted to reducing childcare costs, otherwise private education costs. Recall from Eq. (2) that childcare costs and private education expenses afforded by the government are $\chi_t \nu w_t h_t n_t$ and $\varepsilon_t e_t n_t$, respectively. Then we have $\mu\tau w_t h_t = \chi_t \nu w_t h_t n_t$ and $(1-\mu)\tau w_t h_t = \varepsilon_t e_t n_t$, which could be simplified to:

$$\chi_t = \mu\tau/(\nu n_t),\ \varepsilon_t = (1-\mu)\tau w_t h_t/(e_t n_t) \tag{10}$$

Additionally, the government operates a PAYG pension scheme where pension benefits paid to retirees are financed by adults' pension contributions, and a public education scheme where public education spending is supported by public education taxes. The two schemes are operated separately and in budget balance so we have $\theta w_t h_t N_t = p_t N_{t-1}$ and $\sigma w_t h_t N_t = E_t N_{t+1}$. Pension benefits and public education spending can be expressed as:

$$p_t = \theta w_t h_t n_{t-1},\ E_t = \sigma w_t h_t / n_t \tag{11}$$

The pension replacement rate is an important criterion to measure the relative level of pension benefits (Buyse et al. 2017), which can be calculated as the ratio of pensions to wage income:

$$\delta_{t+1} = \frac{P_{t+1}}{w_t h_t} = \frac{\theta w_{t+1}h_{t+1}n_t}{w_t h_t} \tag{12}$$

General equilibrium. Given the initial physical capital per worker k_0 and efficient labor per worker h_0, the equilibrium based on the OLG model is a sequence of individual decision variables $\{c_t, d_{t+1}, s_t, n_t, e_t\}_{t=0}^{\infty}$, production factor variables $\{K_t, L_t\}_{t=0}^{\infty}$, factor price variables $\{w_t, R_t\}_{t=0}^{\infty}$, and policy variables $\{\varepsilon_t, \chi_t, p_{t+1}, E_t\}_{t=0}^{\infty}$ such that:

(a) Given $\{w_t, R_t, \varepsilon_t, \chi_t, p_{t+1}, E_t\}$, agents maximize their lifetime utility by choosing consumption, savings, fertility, and education investment.
(b) Given $\{w_t, R_t\}$, firms maximize profits by allocating physical capital and efficient labor.
(c) The government runs a child support scheme, a PAYG pension scheme, and a public education scheme, and maintains a balanced budget for each scheme.
(d) The labor market is clear. Labor needed for firm production is equivalent to labor supply from adults; that is, $L_t = N_t h_t$.
(e) The physical capital market is clear. Physical capital needed for firm production is financed by savings of adults from the last period; that is, $K_{t+1} = N_t s_t$. Dividing both sides by N_{t+1}, we obtain:

$$k_{t+1} = s_t / n_t \tag{13}$$

In the stationary equilibrium with perfect foresight, y_t, k_t, and h_t grow at the same rate g, while k_t/h_t, w_t, R_t, n_t, and δ_t are constant over time. Defining physical capital per efficient labor as $\tilde{k}_t = k_t/h_t$, the saving rate as $\hat{s}_t = s_t/(w_t h_t)$, the private education investment rate as $\hat{e}_t = e_t/(w_t h_t)$, and the public education spending rate as $\hat{E}_t = E_t/(w_t h_t)$, we derive the equilibrium as follows:

$$\hat{s}^* = \beta(1-\tau-\theta-\sigma)/m \tag{14}$$

$$n^* = \frac{1}{\nu}\left[\frac{\phi(1-\gamma)(1-\tau-\theta-\sigma)}{\alpha m} + \mu\tau\right] \tag{15}$$

$$\hat{e}^* = \frac{1}{n^*}\left[\frac{\phi\gamma(1-\tau-\theta-\sigma)}{\alpha m} + (1-\mu)\tau\right] \tag{16}$$

$$\hat{E}^* = \sigma / n^* \tag{17}$$

$$\tilde{k}^* = \{[(1-\alpha)A]^{1-\gamma-\eta}B^{-1}\hat{s}^*(n^*)^{-1}(\hat{e}^*)^{-\gamma}\cdot(\hat{E}^*)^{-\eta}\}^{\frac{1}{\alpha(\gamma+\eta)-\alpha+1}} \tag{18}$$

where $m = (1+\varphi+\beta) + (1+\varphi)\theta(1-\alpha)/\alpha$, $\phi = \varphi[\alpha+\theta(1-\alpha)]$. Apart from the trivial steady state where physical capital per efficient labor is zero, $\tilde{k}^*$ is the unique positive solution to the dynamic equation $\tilde{k}_{t+1} = f(\tilde{k}_t)$, derived from Eq. (13). Moreover, given that the function $f(.)$ is non-decreasing and concave, $\tilde{k}^*$ is globally stable as demonstrated in Appendix A.

The corresponding steady-state level of the human capital growth rate and the pension replacement rate are:

$$g^* = \left\{B^{1-\alpha}[(1-\alpha)A]^{\gamma+\eta}(\hat{s}^*)^{\alpha(\gamma+\eta)}(n^*)^{-\alpha(\gamma+\eta)}(\hat{e}^*)^{\gamma(1-\alpha)}(\hat{E}^*)^{\eta(1-\alpha)}\right\}^{\frac{1}{\alpha(\gamma+\eta)-\alpha+1}} - 1 \tag{19}$$

$$\delta^* = \theta n^*(1+g^*) \tag{20}$$

According to the above equations, we can analyze the effects of child support policies in the stationary equilibrium.

Comparative statics

In this section, we conduct a comparative static analysis of the effects of child policies. To figure out the economic consequences of each child policy, we investigate the economies with child allowances only, with educational subsidies only, and with both.

Child allowances. If the government provides only child allowances, all child support resources aim to reduce childcare costs. In this case, $\mu = 1$. We obtain the derivative of $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* with respect to τ:

$$\frac{d\hat{s}^*}{d\tau}<0,\ \frac{dn^*}{d\tau}>0,\ \frac{d\hat{e}^*}{d\tau}<0,\ \frac{d\hat{E}^*}{d\tau}<0,\ \frac{dg^*}{d\tau}<0 \tag{21}$$

$$\frac{d\delta^*}{d\tau} = \frac{\delta^*}{\alpha(\gamma+\eta)-\alpha+1}\left[\frac{-\gamma-\alpha\eta}{1-\tau-\theta-\sigma} + (1-\alpha)(1-\gamma-\eta)\frac{\alpha m-\phi(1-\gamma)}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\tau\alpha m}\right] \tag{22}$$

When $\tau<(1-\theta-\sigma)\frac{(1-\alpha)(1-\gamma-\eta)[\alpha m-\phi(1-\gamma)]-(\gamma+\alpha\eta)\phi(1-\gamma)}{[(1-\alpha)(1-\gamma-\eta)+(\gamma+\alpha\eta)][\alpha m-\phi(1-\gamma)]}\equiv\tau_1$, we have $\frac{d\delta^*}{d\tau}>0$, and otherwise $\frac{d\delta^*}{d\tau}\leq 0$. Then the following proposition holds:

Proposition 1. Child allowances raise fertility but depress human capital growth. Moreover, there is an inverted *U*-shaped relationship between child allowances and the pension replacement rate.

In general, child allowances affect fertility through the cost reduction effect and the income loss effect. On one hand, child allowances encourage adults to have more children by sharing part of childcare costs. On the other hand, more child allowances entail a higher wage tax rate, resulting in lower disposable income, which restricts adults to give birth. The result of comparative static analysis shows that fertility eventually rises since the cost reduction effect is stronger than the income loss effect, driving down the public education spending on each child according to Eq. (17) and thus lowering human capital growth. Based on the trade-off between child quality and quantity, child allowances raise the relative cost of private education and thus reduce education investment in each child, which also hurts human capital accumulation. This finding is consistent with Stauvermann and Kumar (2018).

According to Eq. (20), the pension replacement rate is jointly determined by fertility and human capital growth. Equation (22) shows that when child allowances are insufficient ($\tau<\tau_1$), the increase in fertility outweighs the decrease in human capital growth and eventually promotes the replacement rate. When child allowances are adequate ($\tau\geq\tau_1$), the fall in human capital growth dominates and reduces the replacement rate. Therefore, child allowances first increase the replacement rate before reaching the inflection point and decrease it thereafter.

Educational subsidies. If the government only subsidizes parents' education investment, we have $\mu=0$. In this case, by taking the derivative of $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* with respect to τ, we obtain:

$$\frac{d\hat{s}^*}{d\tau}<0,\ \frac{dn^*}{d\tau}<0,\ \frac{d\hat{e}^*}{d\tau}>0,\ \frac{d\hat{E}^*}{d\tau}>0,\ \frac{dg^*}{d\tau}>0 \tag{23}$$

$$\frac{d\delta^*}{d\tau}=\frac{\delta^*}{\alpha(\gamma+\eta)-\alpha+1}\left[\frac{-(1-\gamma-\eta)(1-\alpha)-\alpha(\gamma+\eta)}{1-\tau-\theta-\sigma}+\gamma(1-\alpha)\frac{\alpha m-\phi\gamma}{\phi\gamma(1-\tau-\theta-\sigma)+\tau\alpha m}\right] \tag{24}$$

When $\tau<\frac{(1-\theta-\sigma)\gamma\{(1-\alpha)\alpha m-\phi[\alpha(\gamma+\eta)+(1-\alpha)(1-\eta)]\}}{[\alpha(\gamma+\eta)+(1-\alpha)(1-\eta)](\alpha m-\phi\gamma)}\equiv\tau_2$, we have $\frac{d\delta^*}{d\tau}>0$, and otherwise $\frac{d\delta^*}{d\tau}\leq 0$. Then the following proposition holds:

Proposition 2. Educational subsidies raise human capital growth but decrease fertility. Moreover, there is an inverted *U*-shaped relationship between educational subsidies and the pension replacement rate.

Like child allowances, educational subsidies affect fertility in two ways. On the one hand, an education subsidy reduces parents' education expenses and encourages adults to have more children. On the other hand, less wage income and disposable income induced by a higher tax burden restrict adults to give birth. Unlike child allowances, the income loss effect in this case plays a leading role, so the net impact of educational subsidies on fertility is negative. Lower fertility increases public education spending on each child. Based on the substitution between child quality and quantity, educational subsidies reduce the relative cost of private education and stimulate parents to increase education investment in each child. An increase in both public education spending and private education investment promotes human capital growth. Our finding is consistent with Yasuoka and Miyake (2014) and Stauvermann and Kumar (2018). Moreover, if the education subsidy is relatively small (large), the increase in human capital growth is greater (smaller) than the decrease in fertility, which raises (decreases) the replacement rate.

Policy combination. According to Propositions 1 and 2, child allowances and educational subsidies have opposite effects on fertility and human capital growth, and they all have an inverted *U*-shaped effect on the pension replacement rate. To investigate the aggregate effects of implementing child allowances and educational subsidies simultaneously, we analyze in a framework incorporating both child policies.

The policy mix and policy support are two primary dimensions of child policies. We first examine the impact of the child policy mix by taking the derivative of $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* concerning μ:

$$\frac{d\hat{s}^*}{d\mu}=0,\ \frac{dn^*}{d\mu}>0,\ \frac{d\hat{e}^*}{d\mu}<0,\ \frac{d\hat{E}^*}{d\mu}<0,\ \frac{dg^*}{d\mu}<0 \tag{25}$$

$$\frac{d\delta^*}{d\mu}=\frac{\delta^*(1-\alpha)\tau\alpha m}{\alpha(\gamma+\eta)-\alpha+1}\left[\frac{1-\gamma-\eta}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m}-\frac{\gamma}{\phi\gamma(1-\tau-\theta-\sigma)+(1-\mu)\tau\alpha m}\right] \tag{26}$$

When $\mu<\frac{\tau\alpha m(1-\gamma-\eta)-\phi\gamma\eta(1-\tau-\theta-\sigma)}{\tau\alpha m(1-\eta)}\equiv\mu_1$, we have $\frac{d\delta^*}{d\mu}>0$, and otherwise $\frac{d\delta^*}{d\mu}\leq 0$. Then the following proposition holds:

Proposition 3. In a policy mix involving child allowances and educational subsidies, an increasing proportion of child allowances leads to higher fertility, a decline in human capital growth, and an inverted U-shaped pattern in the pension replacement rate.

Given the policy support, a higher proportion of resources allocated to child allowance would raise fertility and reduce the human capital growth rate as the effect of child allowances is amplified. A policy mix that emphasizes educational subsidies does the opposite. It suggests that adjusting the policy mix alone cannot simultaneously boost child quality and quantity, which underscores the importance of considering both the policy mix and policy support in the policy design. In addition, Eq. (26) shows that the replacement rate increases with the policy mix when child allowances account for a smaller ratio ($\lambda<\lambda_1$) but decreases otherwise. Therefore, given the policy support, there is an optimal child policy mix that maximizes the replacement rate.

We then analyze the effects of the child policy support. Taking the derivative of $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* with respect to τ, respectively, we obtain:

$$\frac{d\hat{s}^*}{d\tau}=\frac{-\beta}{m} \tag{27}$$

$$\frac{dn^*}{d\tau}=\frac{1}{\nu}\left[\mu-\frac{\phi(1-\gamma)}{\alpha m}\right] \tag{28}$$

$$\frac{d\hat{e}^*}{d\tau}=\nu\frac{\alpha m\phi(1-\gamma-\mu)(1-\theta-\sigma)}{[\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m]^2} \tag{29}$$

$$\frac{d\hat{E}^*}{d\tau}=\frac{-\sigma}{\nu(n^*)^2}\left[\mu-\frac{\phi(1-\gamma)}{\alpha m}\right] \tag{30}$$

$$\frac{dg^*}{d\tau}=\frac{1+g^*}{\alpha(\gamma+\eta)-\alpha+1}\left\{\frac{\gamma(1-\alpha)[(1-\mu)\alpha m-\phi\gamma]}{\phi\gamma(1-\tau-\theta-\sigma)+(1-\mu)\tau\alpha m}-\frac{(\gamma+\eta)[\mu\alpha m-\phi(1-\gamma)]}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m}-\frac{\alpha(\gamma+\eta)}{1-\tau-\theta-\sigma}\right\} \tag{31}$$

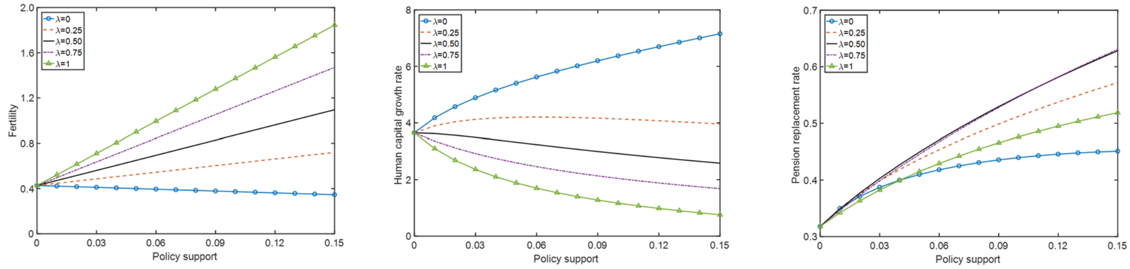

Fig. 1 The effects of child policy support. Note: These figures simulate the effects of changing the child policy support level on fertility, the human capital gowth rate, and the pension replacement rate in cases where the policy mix is 0, 0.25, 0.5, 0.75, and 1, respectively.

$$\frac{d\delta^*}{d\tau} = \frac{\delta^*}{\alpha(\gamma+\eta)-\alpha+1}\left\{\frac{(1-\alpha)\gamma[(1-\mu)\alpha m-\phi\gamma]}{\phi\gamma(1-\tau-\theta-\sigma)+(1-\mu)\tau\alpha m} + \frac{(1-\alpha)(1-\gamma-\eta)[\mu\alpha m-\phi(1-\gamma)]}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m} - \frac{\alpha(\gamma+\eta)}{1-\tau-\theta-\sigma}\right\} \tag{32}$$

Regarding Eqs. (28) and (30), when $\mu > \frac{\phi(1-\gamma)}{\alpha m} \equiv \mu_2$, we have $\frac{dn^*}{d\tau} > 0$ and $\frac{d\hat{E}^*}{d\tau} < 0$. As for Eq. (29), $\frac{d\hat{e}^*}{d\tau} < 0$ if $\mu > 1-\gamma$. Due to the complexity of Eqs. (31) and (32), it is hard to identify the sign of $\frac{dg^*}{d\tau}$ and $\frac{d\delta^*}{d\tau}$. (The sign of $dg^*/d\tau$ and $d\delta^*/d\tau$ are all determined by quadratic functions of τ.) But in particular, $\frac{dg^*}{d\tau} < 0$ if $\frac{dn^*}{d\tau} > 0$, $\frac{d\hat{e}^*}{d\tau} < 0$, and $\frac{d\hat{E}^*}{d\tau} < 0$, when $\mu > \max\{1-\gamma, \mu_2\}$. Then the following proposition holds:

Proposition 4. The effect of child policy support on fertility depends on the policy mix, while that on human capital growth and the pension replacement rate is ambiguous.

As mentioned above, raising policy support would have the cost reduction effect and the income loss effect. Its net impact depends on the policy mix. On the one hand, Propositions 1 and 2 imply that increasing policy support would enlarge both the positive effect of child allowances and the negative effect of educational subsidies on fertility. On the other hand, increasing the proportion of child allowances would amplify the effects of child allowances. When the proportion gets above the critical value ($\lambda > \lambda_2$), the positive effect outweighs the negative. Increasing the policy support at this point would raise fertility and thus reduce the public education spending rate. As for human capital growth, a higher focus on child allowance strengthens the negative effect of child allowances and diminishes the positive effect of educational subsidies. Theoretically, there exists a threshold of policy mix above which the human capital growth rate declines with the increase of policy support but we can not determine its feasibility. Thus, the effects of raising policy effort on the pension replacement rate are ambiguous.

Numerical simulation

Parameterization. The values of β, ν, α, γ, and η are in line with those in literature. Previous studies (Yew and Zhang 2009; Mizuno and Yakita 2013; Cipriani and Pascucci 2020) usually set the quarterly discount factor as 0.99. Taking one period of the OLG model as 30 years, then the discount factor over a period is $\beta = 0.99^{120} \approx 0.3$. According to the statistical result of Banerjee et al. (2014), we assume rearing a child costs 10% of wage income, that is, $\nu = 0.1$, which is consistent with the setting in Yew and Zhang (2009). Following Fanti (2014) and Liu and Thøgersen (2020), we set the output elasticity of physical capital $\alpha = 0.35$. Regarding the elasticity of human capital technology, we set $\gamma + \eta = 0.5$ based on the empirical results of Qin et al (2016) and Yuan (2017). We further assume $\gamma = \eta = 0.25$.

We then determine the values of θ, σ, φ, A, and B based on Chinese data. The contribution rate of China's basic old-age insurance for urban employees is 16% since May 2019, so we set $\theta = 0.16$. According to the data of the China Statistical Yearbook, the average ratio of public education spending to GDP is 3.58%, then $\sigma w_t L_t / Y_t = 3.58\%$. From Eqs. (8) and (9), we obtain $w_t L_t = (1-\alpha) Y_t$ and calculate $\sigma = 0.055$. The 2020 Chinese census shows that the TFR in China is 1.3. Noting that the OLG model does not distinguish genders, the fertility in the model is half of the TFR, that is, $n = 0.65$. We assume $\tau = 0.05$ and $\mu = 0.5$ in the base case due to the lack of practical data. We plug relevant parameter values into Eq. (15) and get $\varphi = 0.096$. Furthermore, the annual economic growth rate is assumed to be 5% so $g = 1.05^{30} - 1 = 3.322$, and the pension replacement rate δ is calculated as 0.449 from Eq. (20), which is close to the current average in China. By calibrating the productivity of technology and human capital scale factor with Eq. (19), we get $A = B = 9.333$.

Simulation results. Figure 1 shows the effects of increasing policy support given different policy mixes. We find when the government allocates all resources to educational subsidies ($\mu = 0$), increasing policy support would decrease fertility and increase human capital growth, which is consistent with Proposition 2. On the other extreme case ($\mu = 1$), the effect of increasing policy support gets reversed, confirming Proposition 1. When implementing both child policies ($0 < \mu < 1$), the effects of policy support depend on the policy mix as stated in Proposition 4. We notice that if the policy mix satisfies certain conditions, intensifying policy support within a reasonable range can improve both fertility and human capital growth (e.g., $\mu = 0.25$ and $\tau \le 0.07$). It suggests that a careful policy design in both the policy mix and policy support is needed to boost the quality and quantity of children. In addition, raising policy support appropriately could increase the replacement rate regardless of the policy mix, (By plugging in relevant parameters, we calculate $\tau_1 = 0.362$, $\tau_2 = 0.182$. According to Proposition 1, when the government allocates all resources to child allowances, raising policy support could increase the replacement rate if $\tau < 0.362$. According to Proposition 2, when the government allocates all resources to educational subsidies, raising policy support could increase the replacement rate if $\tau < 0.182$.) but the specific degree of influence is slightly different.

Figure 2 illustrates the effects of the child policy mix under different support levels. Consistent with Proposition 3, a policy mix putting more focus on child allowances (increasing μ) would yield higher fertility and lower human capital growth, and an inverted U-shaped change in the pension replacement rate. In particular, the optimal policy mix achieving the maximum replacement rate is $\mu = 0.578$ when $\tau = 0.05$, which means if the government taxes 5% of wage to channel funds into child support, devoting 57.8% of policy resources to child allowances yields the highest replacement rate. Additionally, the higher the policy support, the more significant the effects of the policy mix would be.

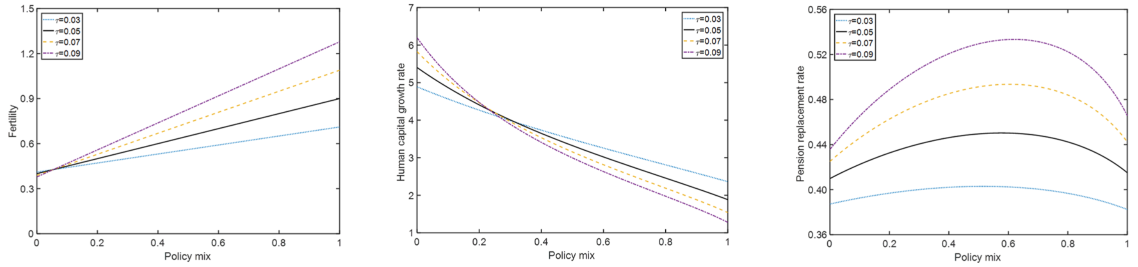

Fig. 2 The effects of the child policy mix. Note: These figures simulate the effects of changing the child policy mix on fertility, the human capital growth rate, and the pension replacement rate in cases where the support level is 0.03, 0.05, 0.07, and 0.09, respectively.

Table 1 Corresponding critical values of policy mix to policy support.

τ	0.055	0.06	0.065	0.07	0.075	0.08	0.085	0.09	0.095	0.10
μ_1	0.586	0.594	0.600	0.605	0.609	0.613	0.617	0.620	0.623	0.625
μ_2	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054
μ_a	0.460	0.426	0.397	0.373	0.352	0.333	0.317	0.302	0.289	0.277
μ_b	0.487	0.475	0.464	0.454	0.445	0.436	0.428	0.420	0.412	0.405
μ_c	0.264	0.251	0.239	0.228	0.218	0.209	0.200	0.193	0.185	0.178
μ_d	0.297	0.211	0.157	0.120	0.092	0.072	0.056	0.044	0.034	0.026

Desired policy mixes are those in which raising policy support simultaneously increases the fertility rate, the human capital growth rate, and the pension replacement rate above the baseline values (n>0.65, g>3.322, δ>0.449). As stated in Proposition 3, raising μ would increase n and decrease g. We subsequently suppose that μ_a is the minimum value that ensures n>0.65 and μ_b is the maximum value that ensures g>3.322. According to Proposition 4 and Fig. 1, raising τ would always increase n if $\mu>\mu_2$ and decrease g if $\mu>\mu_c$. Thus, increasing policy support would raise fertility above the baseline if $\mu>\max\{\mu_2,\mu_a\}$ and improve human capital growth above the baseline if $\mu_c<\mu<\mu_b$. As for δ, recall that when $\mu<\mu_1$, raising either μ or τ would increase δ. Assuming that μ_d is the minimum value that ensures δ>0.449, raising τ would increase the pension replacement rate above the baseline if $\mu_d<\mu<\mu_1$. In summary, when the policy mix satisfies the condition of $\max\{\mu_2,\mu_a,\mu_c,\mu_d\}<\mu<\min\{\mu_1,\mu_b\}$, increasing policy support would simultaneously boost the quality and quantity of children and raise pension benefits.

Table 1 shows the corresponding critical values of the policy mix given some policy support levels. When τ ranges from 0.055 to 0.10, we have $\max\{\mu_2,\mu_c,\mu_d\}<\mu_a<\mu_b<\mu_1$, which satisfies the constraint condition of $\max\{\mu_2,\mu_a,\mu_c,\mu_d\}<\mu<\min\{\mu_1,\mu_b\}$ and indicates there exists a range of policy mixes with which the government devotes more policy support would boost child quality and quantity and raise pension benefits simultaneously. For example, when $\tau=0.08$, μ should be in the range of (0.333, 0.436). It suggests that the percentage of child support resources devoted to child allowances should fall between 33.3% and 43.6% when policy support (the wage tax rate) is 0.08. In addition, if the government prefers greater improvements in the fertility rate and the pension replacement rate, the policy mix could be closer to the upper limit; if the government is more interested in promoting the human capital growth rate, the policy mix could be closer to the lower limit. Therefore, it is important to establish a dynamic adjusting mechanism that relates policy mix with policy support when designing child policies.

Further analysis

Subsidies for public education. There are basically two types of educational subsidies. One is targeting private education to share parents' educational investment; the other is to subsidize the provision of public education. For example, Arizona's school voucher program is a typical private educational subsidy but in China, Germany, and France, the public educational subsidy is more common. The two kinds of educational subsidies have different influences on child-raising decisions, so we evaluate and compare the effects of child policies involving the two kinds of educational subsidies in this section.

Since the public educational subsidies are used to increase public education provision instead of sharing private education costs, the budget constraint of adults becomes:

$$c_t = (1-\tau-\theta-\sigma)w_t h_t - (1-\chi_t)\nu w_t h_t n_t - e_t n_t - s_t \quad (33)$$

With agents' lifetime utility function and the human capital accumulation function unchanged, we solve the utility maximization problem subject to budget constraints. The optimal savings and fertility are consistent with Eqs. (5) and (6), while the optimal private education investment in each child becomes:

$$e_t = \frac{\gamma}{1-\gamma}(1-\chi_t)\nu w_t h_t \quad (34)$$

The setup of firms and the government is the same as in Section "The model" except that $1-\mu$ percent of child support resources are to subsidize public education provision, not private education investment. The balanced budget of the child support scheme run by the government becomes:

$$\chi_t = \frac{\mu\tau}{\nu n_t},\ E_t = \frac{(1-\mu)\tau+\sigma}{n_t} w_t h_t \quad (35)$$

According to the market clearing conditions, we can derive the endogenous variables in the steady state. The saving rate $\hat{s}^*$, the fertility rate n^*, the human capital growth rate g^*, and the pension replacement rate δ^* are the same as Eqs. (14), (15), (19), and (20), respectively. The private education investment rate per child $\hat{e}^*$ and the public education spending rate per child $\hat{E}^*$ are expressed as:

$$\hat{e}^* = \frac{\phi\gamma(1-\tau-\theta-\sigma)}{\alpha m n^*} \quad (36)$$

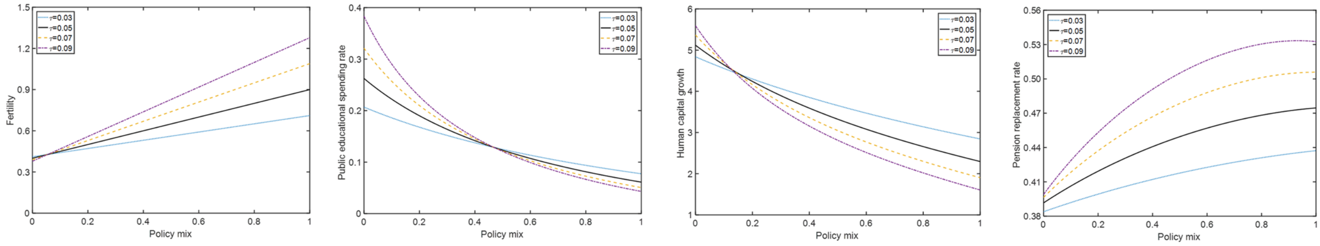

Fig. 3 The effects of child policy mixes with public educational subsidies implemented. Note: These figures simulate the effects of changing the child policy mix invloving public educational subsidies on fertility, the public education spending rate, the human capital gowth rate, and the pension replacement rate in cases where the support level is 0.03, 0.05, 0.07, and 0.09, respectively.

$$\hat{E}^* = \frac{(1-\mu)\tau + \sigma}{n^*} \tag{37}$$

We first investigate the effects of the policy mix by taking the derivative of $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* with respect to μ. We get $\frac{d\hat{s}^*}{d\mu} = 0$, $\frac{dn^*}{d\mu} > 0$, $\frac{d\hat{e}^*}{d\mu} < 0$, $\frac{d\hat{E}^*}{d\mu} < 0$, $\frac{dg^*}{d\mu} < 0$, while

$$\frac{d\delta^*}{d\mu} = \frac{\delta^*(1-\alpha)\tau\alpha m}{\alpha(\gamma+\eta)-\alpha+1}\left\{\frac{1-\gamma-\eta}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m} - \frac{\eta}{[(1-\mu)\tau+\sigma]\alpha m}\right\} \tag{38}$$

From Eq. (38), when $\mu < \frac{(1-\gamma-\eta)(\tau+\sigma)\alpha m - \eta\phi(1-\gamma)(1-\tau-\theta-\sigma)}{(1-\gamma)\tau\alpha m} \equiv \lambda_3$, we have $\frac{d\delta^*}{d\mu} > 0$, and $\frac{d\delta^*}{d\mu} \le 0$ otherwise. Then the following proposition holds:

Proposition 5. In a policy mix involving child allowances and public educational subsidies, an increasing proportion of child allowances leads to higher fertility, a decline in human capital growth, and an inverted U-shaped pattern in the pension replacement rate.

When the government focuses on child allowances, childcare costs drop but private education costs remain unchanged, which leads to higher fertility and lower private educational investment per child. Relatively insufficient public educational subsidies further reduce public education provision, which results in less public education received by each child and slower human capital growth. In addition, when the proportion of policy resources on child allowances is relatively small ($\mu < \mu_3$), the increase in fertility plays a leading role and the replacement rate rises according to Eq. (38). If the government devotes more resources to child allowances ($\mu \ge \mu_3$), the decrease in the human capital growth rate dominates and drives the replacement rate down.

We then differentiate $\hat{s}^*$, n^*, $\hat{e}^*$, $\hat{E}^*$, g^*, and δ^* with respect to τ to examine the effects of changing policy support. We obtain that $\frac{d\hat{s}^*}{d\tau} < 0$, $\frac{d\hat{e}^*}{d\tau} < 0$, and $\frac{dn^*}{d\tau} > 0$ when $\mu > \mu_2$. Furthermore,

$$\frac{d\hat{E}^*}{d\tau} = \nu\alpha m \frac{\phi(1-\gamma)(1-\theta) - \mu[\phi(1-\gamma)(1-\theta-\sigma)+\sigma\alpha m]}{[\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m]^2} \tag{39}$$

$$\frac{dg^*}{d\tau} = \frac{1+g^*}{\alpha(\gamma+\eta)-\alpha+1}\left\{\frac{-(\gamma+\eta)[\lambda\alpha m - \phi(1-\gamma)]}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m} - \frac{\gamma+\alpha\eta}{1-\tau-\theta-\sigma} + \frac{\eta(1-\alpha)(1-\mu)}{(1-\mu)\tau+\sigma}\right\} \tag{40}$$

$$\frac{d\delta^*}{d\tau} = \frac{\delta^*}{\alpha(\gamma+\eta)-\alpha+1}\left\{\frac{(1-\gamma-\eta)(1-\alpha)[\mu\alpha m - \phi(1-\gamma)]}{\phi(1-\gamma)(1-\tau-\theta-\sigma)+\mu\tau\alpha m} - \frac{\gamma+\alpha\eta}{1-\tau-\theta-\sigma} + \frac{\eta(1-\alpha)(1-\mu)}{(1-\mu)\tau+\sigma}\right\} \tag{41}$$

Regarding Eq. (39), when $\mu < \frac{\phi(1-\gamma)(1-\theta)}{\phi(1-\gamma)(1-\theta)+\sigma[\alpha m - \phi(1-\gamma)]} \equiv \mu_4$, we have $\frac{d\hat{E}^*}{d\tau} > 0$. Although we can not fully identify the sign of $\frac{dg^*}{d\tau}$ and $\frac{d\delta^*}{d\tau}$, $\frac{dg^*}{d\tau} < 0$ can be derived when $\mu > \mu_4$. Then the following proposition holds:

Proposition 6. With public educational subsidies involved, the effect of child policy support on fertility still depends on the policy mix, and its influences on human capital growth and the pension replacement rate are also ambiguous.

Child allowances and public educational subsidies still have opposite effects on fertility and human capital growth. When implemented together, raising policy efforts also has the income loss effect and the cost reduction effect. The former restricts agents to give birth and reduces private education investment in each child, while the latter reduces childcare costs but no longer private education costs, leading to higher fertility and less private education investment in a child. Both mechanisms discourage private education investment in a child but the effects on fertility and human capital growth depend on the policy mix. A policy mix putting more focus on child allowances (increasing μ) enlarges the cost reduction effect and raises fertility. Although raising policy efforts would increase total public education spending, public education received by each child may decrease if fertility increases more. Accompany with less private education investment, the human capital growth rate declines. If fertility drops, public education received by each child increases. The changes in human capital growth become ambiguous and so is the pension replacement rate.

Next, we conduct a numerical simulation to confirm the above two propositions. Letting $g^* = 3.322$ when $\tau = 0.05$ and $\mu = 0.5$ as the benchmark parameterization, we calibrate $A = B = 10.276$. Figure 3 demonstrates the effects of changing the policy mix with public educational subsidies implemented, which is consistent with Proposition 5. (For the effects on the replacement rate, we find the inflection point is $\mu_3 > 1$ (which is out of feasibility) when $\tau \le 0.07$. Thus raising μ could always increase the replacement rate according to Proposition 5.) Figure 4 illustrates the effects of increasing policy support with public educational subsidies implemented. It shows that the changes in all three variables (fertility, human capital growth, and replacement rate) are subject to the policy mix.

We then simulate the corresponding critical values of the policy mix that ensures key variables above the baseline values given some policy support levels. Table 2 shows that we have $\mu_2 < \mu_c < \mu_d < \mu_a < \mu_b < \mu_3$ when τ ranges from 0.055 to 0.1. Thus, there exists a policy mix satisfying the constraint condition of $\max\{\mu_2, \mu_a, \mu_c, \mu_d\} < \mu < \min\{\mu_3, \mu_b\}$ such that raising policy support would boost child quality and quantity as well as pension benefits.

Lastly, we compare the effects of child policies between the two types of educational subsidies. The simulation results are shown in Fig. 5. First, the effects on fertility coincide. Second, given the policy mix ($\mu = 0.5$), subsidies for private education achieve a higher human capital growth rate and pension replacement rate

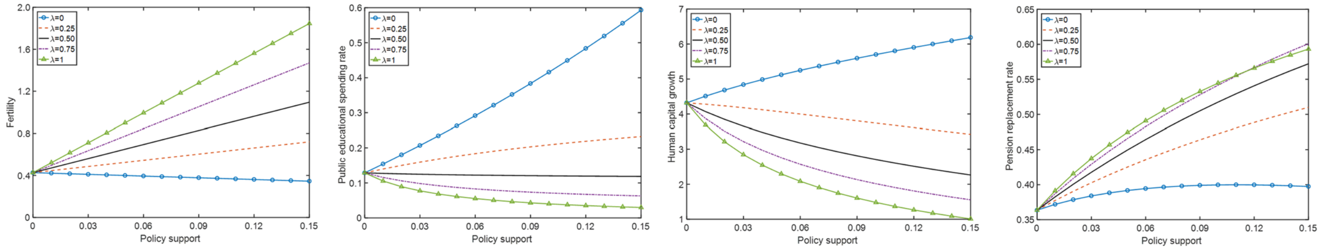

Fig. 4 The effects of child policy support with public educational subsidies implemented. Note: These figures simulate the effects of changing the child policy support level on fertility, the public educational spending rate, the human capital gowth rate, and the pension replacement rate in cases where the policy mix involving public educational subsidies is 0, 0.25, 0.5, 0.75, and 1, respectively.

Table 2 Corresponding critical values of policy mix to policy support with public educational subsidies implemented.

τ	**0.055**	**0.06**	**0.065**	**0.07**	**0.075**	**0.08**	**0.085**	**0.09**	**0.095**	**0.10**
μ_2	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054	0.054
μ_3	1.093	1.059	1.030	1.005	0.984	0.965	0.949	0.934	0.921	0.909
μ_a	0.460	0.426	0.397	0.373	0.352	0.333	0.317	0.302	0.289	0.277
μ_b	0.473	0.450	0.430	0.412	0.396	0.382	0.369	0.357	0.347	0.337
μ_c	0.140	0.135	0.131	0.126	0.122	0.118	0.114	0.110	0.107	0.103
μ_d	0.420	0.359	0.312	0.275	0.246	0.221	0.201	0.184	0.170	0.157

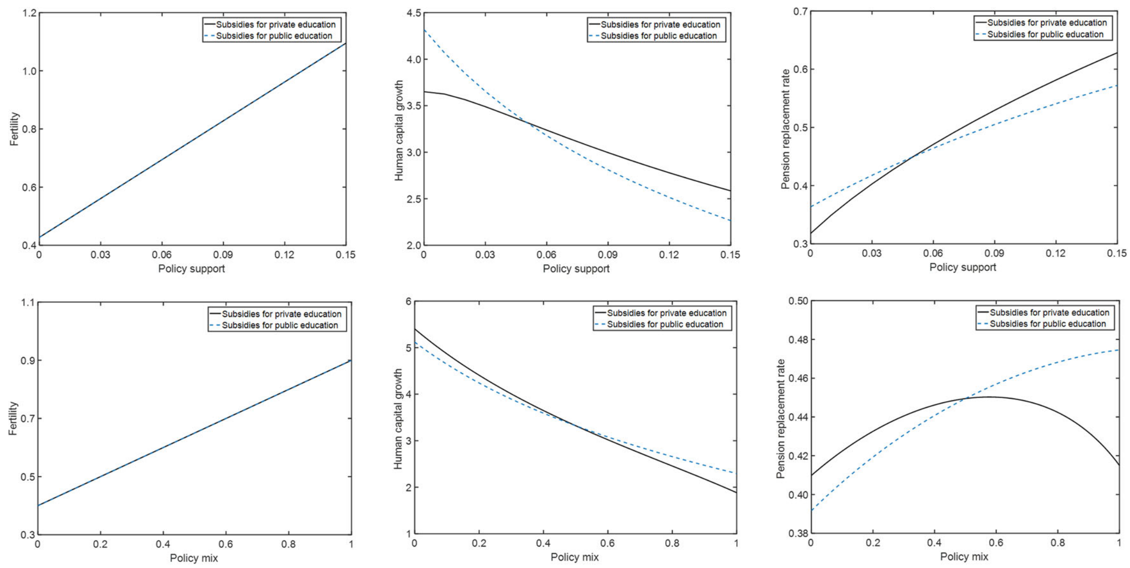

Fig. 5 The effects of child policies under the two kinds of educational subsidies. Note: The black line represents the scenario where the government subsidizes private education, while the blue dashed line represents the scenario where the government subsidizes public education. The three figures in the first row illustrate the effects of changing the policy support level on fertility, the human capital growth rate, and the pension replacement rate respectively when the policy mix is 0.5. The three figures in the second row depict the effects of changing the policy mix on fertility, the human capital growth rate, and the pension replacement rate respectively when the policy support is 0.05.

when the government increases the policy support (τ>0.05). On the other hand, given the policy support (τ= 0.05), subsidies for private education yield a greater human capital growth rate and replacement rate when the government favors educational subsidies (μ<0.5). Therefore, the policy combination involving private educational subsidies performs better when the policy support is raised or educational subsidies are preferred.

Social planner's allocation. Previous analysis has evaluated the economic consequences of altering child policy support and the policy mix within a competitive economic framework but have largely overlooked their welfare implications. Since enhancing social welfare is a core objective of public policies, this sub-section illustrates the allocation decisions of a benevolent social planner. The planner considers the welfare of all generations and commits to decisions at the beginning of a period, subject to the resource constraints.

Following Ono and Uchida (2021), the social welfare function is defined as the discounted sum of the lifetime utility of all current and future generations: $SW = \sum_{t=-1}^{\infty} \rho^t U_t$. The parameter 0<$\rho$<1 denotes the social discount factor, capturing the weight the social planner assigns to future generations relative to the current generation. Combined with Eq. (4), the social welfare function can be specified as:

$$SW = \frac{\beta}{\rho} \ln \mathrm{d}_0 + \sum_{\mathrm{t}=0}^{\infty} \rho^{\mathrm{t}} [\ln \mathrm{c}_\mathrm{t} + \beta \ln \mathrm{d}_{\mathrm{t}+1} + \varphi \ln(\mathrm{n}_\mathrm{t} \mathrm{h}_{\mathrm{t}+1})] \quad (42)$$

According to Section "The model," the resource constraint faced by the social planner is $Y_t = N_t c_t + N_{t-1} d_t + N_{t+1}$

$(e_t + E_t) + N_{t+1}\nu w_t h_t + K_{t+1}$, which can be rewritten as:

$$Ak_t^{\alpha}h_t^{1-\alpha} = c_t + d_t/n_{t-1} + (e_t + E_t)n_t + \nu n_t(1-\alpha)Ak_t^{\alpha}h_t^{1-\alpha} + k_{t+1}n_t \tag{43}$$

Furthermore, to ensure the validity of the social planner's problem, we impose the transversality condition, which guarantees that resources are neither over-accumulated nor inefficiently utilized over time. This condition is expressed as:

$$\lim_{t\to\infty} \rho^t \lambda_t k_t = 0 \tag{44}$$

where λ_t represents the Lagrange multiplier. As time approaches infinity, the discounted marginal utility of future physical capital, multiplied by the capital stock, converges to zero, ensuring dynamic consistency and rationality in resource allocation.

We construct the Lagrangian function to solve the social welfare maximization problem, which is given by:

$$\begin{aligned} \mathrm{L} = \tfrac{\beta}{\rho}\ln \mathrm{d}_0 + \sum_{t=0}^{\infty}\rho^t\left[\ln \mathrm{c}_t + \beta \ln \mathrm{d}_{t+1} + \varphi \ln(\mathrm{n}_t\mathrm{h}_{t+1})\right] \\ + \sum_{t=0}^{\infty}\lambda_t\Big[Ak_t^{\alpha}h_t^{1-\alpha} - c_t - \tfrac{d_t}{n_{t-1}} - (e_t + E_t)n_t \\ - \nu n_t(1-\alpha)Ak_t^{\alpha}h_t^{1-\alpha} - k_{t+1}n_t\Big] \end{aligned} \tag{45}$$

Given k_0 and h_0, the social planner maximizes social welfare by selecting the consumption levels of adults and the elderly, along with private and public educational investments, fertility rates, and physical capital per efficient labor for the next period. From this, we derive the following optimality conditions:

$$\lambda_t = \rho^t/c_t \tag{46}$$

$$d_t = \beta c_t n_{t-1}/\rho \tag{47}$$

$$\varphi\gamma\rho^t = \lambda_t n_t e_t - \lambda_{t+1}(1-\alpha)Ak_{t+1}^{\alpha}h_{t+1}^{1-\alpha}\gamma\left[1 - \nu n_{t+1}(1-\alpha)\right] \tag{48}$$

$$\varphi\eta\rho^t = \lambda_t n_t E_t - \lambda_{t+1}(1-\alpha)Ak_{t+1}^{\alpha}h_{t+1}^{1-\alpha}\eta\left[1 - \nu n_{t+1}(1-\alpha)\right] \tag{49}$$

$$\varphi\rho^t = \lambda_t n_t\left[e_t + E_t + \nu(1-\alpha)Ak_t^{\alpha}h_t^{1-\alpha} + k_{t+1}\right] - \lambda_{t+1}d_{t+1}/n_t \tag{50}$$

$$\lambda_t n_t = \lambda_{t+1}\alpha Ak_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}[1 - \nu n_{t+1}(1-\alpha)] \tag{51}$$

Based on the above equations, we can derive the steady-state fertility rate and physical capital per efficient labor in the social planner's economy:

$$\hat{n} = \frac{1}{\nu(1-\alpha)}\frac{[\beta + \rho(1+\varphi+\beta)][\alpha + (1-\alpha)(\gamma+\eta)] - [\beta + \varphi(1-\gamma-\eta)]}{[\beta + \rho(1+\varphi+\beta)][\alpha + (1-\alpha)(\gamma+\eta) - 1/\rho]} \tag{52}$$

$$\hat{k} = \left\{\frac{\rho^{1-\gamma-\eta}\alpha A[\beta + \rho(1+\varphi+\beta)]^{\gamma+\eta}[1 - \nu\hat{n}(1-\alpha)]}{B\hat{n}^{1-\gamma-\eta}(\eta/\gamma)^{\eta}(\gamma A)^{\gamma+\eta}\{\varphi + (1-\alpha)[\beta + \rho(1+\varphi+\beta)][1 - \nu\hat{n}(1-\alpha)]\}^{\gamma+\eta}}\right\}^{\frac{1}{\alpha(\gamma+\eta)-\alpha+1}} \tag{53}$$

It is easy to observe that $\hat{n}$ and $\hat{k}$ are both functions of ρ. For convenience, we define the expression inside the curly brackets for $\hat{k}$ as $\Psi(\rho)$. By equating physical capital per efficient labor in the social planner's economy with that in the competitive economy—specifically, by solving Eqs. (18) and (53) simultaneously—we obtain the optimal design of the child policy under social welfare maximization. The optimal relationship between policy support and policy mix is then expressed as: (The specific derivation process is provided in Appendix B.)

$$\tau^* = \frac{\ln\Delta + \gamma\ln[\phi\gamma(1-\theta-\sigma)] + (1-\gamma-\eta)\ln[\phi(1-\gamma)(1-\theta-\sigma)] - \ln(1-\theta-\sigma) - (1-\eta)\ln(\alpha \mathrm{m})}{q_1 + q_2\mu} \tag{54}$$

where $\Delta = \frac{\Psi(\rho)Bm}{\beta}[\nu(1-\alpha)A]^{\gamma+\eta-1}\sigma^{\eta}$, $q_1 = \frac{-\alpha m - \phi\eta}{\phi(1-\theta-\sigma)}$, $q_2 = \frac{\alpha m\eta}{\phi(1-\gamma)(1-\theta-\sigma)}$.

According to Eq. (54), given the policy mix, the social discount factor, and other parameter values, the optimal policy support that maximizes social welfare can be determined. It is important to note that this analysis assumes the economy operates under a dynamically inefficient state, which aligns with the empirical evidence of China's economic characteristics and the model's parameter settings.

Next, we proceed with numerical simulations. On the one hand, to ensure that $\hat{n}>0$, the social discount factor must satisfy $\rho<\frac{[\varphi+\beta(1-\alpha)](1-\gamma-\eta)}{(1+\varphi+\beta)[\alpha+(1-\alpha)(\gamma+\eta)]}$. On the other hand, to ensure that $\tau^*>0$, the policy mix must satisfy $\mu>\frac{(\alpha m+\phi\eta)(1-\gamma)}{\alpha m\eta}$. Thus, we set $\rho=0.14$ and $\mu=0.5$, with other parameter values consistent with the subsection "Parameterization." Substituting these values into Eq. (54) yields $\tau^*=0.085$. This result suggests that, with a social discount factor of 0.14 and an equal allocation of fertility support resources, the policy support that maximizes social welfare should be 0.085. Moreover, as shown in Table 1, within a competitive economy, when policy support is 0.085, the policy mix that simultaneously boosts both the quality and quantity of children while raising pension benefits, ranges between 0.317 and 0.428, which is < 0.5. It indicates that child policy design in a competitive economy cannot achieve the maximum social welfare. Therefore, to enhance social welfare and improve both the quantity and quality of children, the government should carefully coordinate the two policy instruments. Otherwise, efforts to prioritize one aspect will come at the expense of the other.

Conclusions and discussions

We analyze the impacts of child policies on child quality and quantity as well as on pension benefits by incorporating child allowances and educational subsidies into an OLG model. We also explore the policy design regarding policy support and the policy mix within the competitive economy and the social planner's economy. In addition, we distinguish public educational subsidies from private educational subsidies and contrast the efficacy of the two.

Our four main findings are as follows. First, child allowances and educational subsidies have opposite effects on fertility and human capital growth, and they all have an inverted *U*-shaped effect on the pension replacement rate. It suggests that a single policy cannot coordinate all goals. Second, in a competitive economy that includes child allowances and educational subsidies, blindly channeling more fiscal support to child policies may not be desired. Only in conjunction with appropriate policy mixes, raising policy support could achieve the simultaneous promotion of child quality and quantity as well as pension benefits. It addresses the importance of building a dynamic adjusting mechanism that relates policy mix with policy support. Third, the effect of public educational subsidies is consistent with private ones but different economic mechanisms result in disparities in critical values and efficacy. When implemented with child allowances, private educational subsidies are more effective than public educational subsidies if the government increases the policy support or puts more effort into educational subsidies. This underscores the importance of selecting the appropriate form of educational subsidies under different policy environments. Lastly, we derive the optimal child policy design within the social planner's economy, demonstrating that social welfare can be maximized with appropriate child policy support and mix.

Our research highlights the coordination of different child policies and the linkage between policy support and the policy mix. Concerning the first, we suggest the government provides various child support measures, such as child allowances and education subsidies, and effectively combines them to create a synergy. In addition, a diversified fertility support system can target different populations and motivate them to give birth. As for the second, we suggest establishing a dynamic adjustment mechanism linking policy support and the policy mix, where the government flexibly adjusts the policy mix while changing policy support to achieve multiple goals and further promote the coordinated development of the child support system and the pension system.

There are several limitations to our study. First, we set childcare as goods rather than time. Since childcare takes time as well, childcare time or services need to be considered to capture the opportunity costs. In the future, we could incorporate formal or informal childcare services in the model to comprehensively examine the effects of child policies and provide a deeper insight into the construction of the child support system. Second, our model assumes agents are homogenous. However, individuals differ in fertility willingness due to disparities in economic strength, family characteristics, health status, and job category. Exploring the effects of child policies in a more all-encompassing framework with heterogeneous agents is a vital direction for improvement.

Data availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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References

Andersen S, Drange N, Lappegård T (2018) Can a cash transfer to families change fertility behaviour? Demographic Research 38:897–928. https://doi.org/10.4054/DemRes.2018.38.33

Banerjee A, Meng X, Porzio T, Qian N (2014) Aggregate fertility and household savings: a general equilibrium analysis using micro data (No. w20050). National Bureau of Economic Research, Cambridge, MA. https://doi.org/10.3386/w20050

Becker GS, Lewis HG (1973) On the interaction between the quantity and quality of children. J Political Econ 81:279–288. https://doi.org/10.1086/260166

Becker GS, Murphy KM, Tamura R (1990) Human capital, fertility, and economic growth. J Political Econ 98:S12–S37. https://doi.org/10.1086/261723

Blankenau WF, Simpson NB (2004) Public education expenditures and growth. J Dev Econ 73:583–605. https://doi.org/10.1016/j.jdeveco.2003.05.004

Boldrin M, Jones LE (2002) Mortality, fertility, and saving in a Malthusian economy. Rev Econ Dyn 5:775–814. https://doi.org/10.1006/redy.2002.0186

Buyse T, Heylen F, Van De Kerckhove R (2017) Pension reform in an OLG model with heterogeneous abilities. J Pension Econ Financ 16:144–172. https://doi.org/10.1017/S1474747215000281

Chen H-J (2015) Child allowances, educational subsidies and occupational choice. J Macroecon 44:327–342. https://doi.org/10.1016/j.jmacro.2015.03.004

Cipriani GP, Pascucci F (2020) Pension policies in a model with endogenous fertility. J Pension Econ Financ 19:109–125. https://doi.org/10.1017/S1474747218000148

de la Croix D, Doepke M (2003) Inequality and growth: why differential fertility matters. Am Econ Rev 93:1091–1113. https://doi.org/10.1257/000282803769206214

Ehrlich I, Lui FT (1991) Intergenerational trade, longevity, and economic growth. J Political Econ 99:1029–1059. https://doi.org/10.1086/261788

Fanti L (2014) Raising the mandatory retirement age and its effect on long-run income and pay-as-you-go (PAYG) pensions. Metroeconomica 65:619–645. https://doi.org/10.1111/meca.12055

Fanti L, Gori L (2012) A note on endogenous fertility, child allowances and poverty traps. Econ Lett 117:722–726. https://doi.org/10.1016/j.econlet.2011.12.091

Fanti L, Gori L (2009) Population and neoclassical economic growth: a new child policy perspective. Econ Lett 104:27–30. https://doi.org/10.1016/j.econlet.2009.03.014

Gauthier AH (2007) The impact of family policies on fertility in industrialized countries: a review of the literature. Popul Res Policy Rev 26:323–346. https://doi.org/10.1007/s11113-007-9033-x

Groezen B, Leers T, Meijdam L (2003) Social security and endogenous fertility: pensions and child allowances as siamese twins. J Public Econ 87:233–251. https://doi.org/10.1016/S0047-2727(01)00134-7

Haan P, Wrohlich K (2011) Can child care policy encourage employment and fertility? Labour Economics 18:498–512. https://doi.org/10.1016/j.labeco.2010.12.008

Liu P, Thøgersen J (2020) Pay-as-you-go pensions and endogenous retirement. Macroecon Dyn 24:1700–1719. https://doi.org/10.1017/S1365100518001013

Luci-Greulich A, Thévenon O (2013) The impact of family policies on fertility trends in developed countries. Eur J Popul 29:387–416. https://doi.org/10.1007/s10680-013-9295-4

Lutz W, Skirbekk V (2005) Policies addressing the tempo effect in low-fertility countries. Popul Dev Rev 31:699–720. https://doi.org/10.1111/j.1728-4457.2005.00094.x

Miyazawa K (2016) Grandparental child care, child allowances, and fertility. J Econ Ageing 7:53–60. https://doi.org/10.1016/j.jeoa.2016.03.002

Mizuno M, Yakita A (2013) Elderly labor supply and fertility decisions in aging-population economies. Econ Lett 121:395–399. https://doi.org/10.1016/j.econlet.2013.09.022

Morimoto T, Tabata K (2020) Higher education subsidy policy and R&D-based growth. Macroecon Dyn 24:2129–2168. https://doi.org/10.1017/S1365100519000142

Ono T, Uchida Y (2021) Political economy of taxation, debt ceilings, and growth. Eur J Political Econ 68:101996. https://doi.org/10.1016/j.ejpoleco.2020.101996

Qin X, Wang T, Zhuang CC (2016) Intergenerational transfer of human capital and its impact on income mobility: evidence from China. China Econ Rev 38:306–321

Schindler D, Yang H (2015) Catalysts for social insurance: education subsidies versus physical capital taxation. Int Tax Public Financ 22:274–310. https://doi.org/10.1007/s10797-014-9309-0

Stauvermann PJ, Kumar RR (2018) Demographic change, PAYG pensions and child policies. J Pension Econ Financ 17:469–487. https://doi.org/10.1017/S1474747216000299

United Nations (2024) World population prospects 2024. Department of Economic and Social Affairs, Population Division. https://population.un.org/wpp/Download/Standard/Fertility/

Wang R, Xu G (2020) Can child allowances improve fertility in a gender discrimination economy? Econ Model 93:162–174. https://doi.org/10.1016/j.econmod.2020.08.002

Yasuoka M, Goto N (2015) How is the child allowance to be financed? By income tax or consumption tax? Int Rev Econ 62:249–269. https://doi.org/10.1007/s12232-014-0200-1

Yasuoka M, Goto N (2011) Pension and child care policies with endogenous fertility. Econ Model 28:2478–2482. https://doi.org/10.1016/j.econmod.2011.06.003

Yasuoka M, Miyake A (2014) Fertility rate and child care policies in a pension system. Econ Anal Policy 44:122–127. https://doi.org/10.1016/j.eap.2014.01.007

Yew SL, Zhang J (2013) Socially optimal social security and education subsidization in a dynastic model with human capital externalities, fertility and endogenous growth. J Econ Dyn Control 37:154–175. https://doi.org/10.1016/j.jedc.2012.07.003

Yew SL, Zhang J (2009) Optimal social security in a dynastic model with human capital externalities, fertility and endogenous growth. J Public Econ 3–4:605–619. https://doi.org/10.1016/j.jpubeco.2008.10.005

Yuan W (2017) The sins of the fathers: intergenerational income mobility in China. Rev Income Wealth 63:219–233. https://doi.org/10.1111/roiw.12222

Zhang J (1997) Fertility, growth, and public investments in children. Can J Econ 30:835–842. https://doi.org/10.2307/136272

Zhang J, Casagrande R (1998) Fertility, growth, and flat-rate taxation for education subsidies. Econ Lett 60:209–216. https://doi.org/10.1016/S0165-1765(98)00097-4

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Conceptualization: Peng Jing and Xi Lin; methodology: Minglu Wang and Xi Lin; formal analysis and investigation: Minglu Wang and Peng Jing; visualization: Minglu Wang; writing—original draft preparation: Minglu Wang; writing—review and editing: Peng Jing and Xi Lin; funding acquisition: Peng Jing, and Xi Lin.

Competing interests
The authors declare no competing interests.

Ethical approval
Ethical approval was not required as the study did not involve human participants.

Informed consent
This article does not contain any studies with human participants performed by any of the authors.

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