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Enhancing secondary-school mathematics teachers' ability to notice argumentation through an argumentation classroom situations-based course

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This study explores changes in secondary-school mathematics teachers' noticing of argumentation during a university course designed for this purpose. Thirty-one teachers analyzed written argumentation classroom situations (ACSs) at four points, focusing on two sub-skills: attending to structural and dialogic aspects of argumentation and interpreting the argumentation through various lenses (task characteristics, teaching strategies, cognitive and affective student characteristics, and socio-cultural characteristics). Analysis of the teachers' ACS reports revealed changes in teachers' noticing of various dimensions associated with argumentation. Analysis of teachers' written reflections and semi-structured interviews provided insights into the experience that supported the change in their noticing of argumentation. The study provides evidence of the potential of the course activities for teachers' learning to notice key aspects of argumentation.

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Introduction

In recent years, there has been increasing awareness of the importance of engaging students in argumentation in mathematics classrooms. This is reflected in recent educational reform documents around the world (e.g., CCSSI, 2021; Ministry of Education, 2021). Studies show that participation in argumentation fosters mathematical understanding and thinking (Francisco, 2013; Weber et al. 2008). Despite the importance of argumentation, it remains scarce in mathematics classrooms (Staples, 2014; Sriraman and Umland, 2020). Research has shown that teachers struggle to incorporate argumentation in their practice (Ayalon and Even, 2016; Hähkiöniemi, 2022; Kosko et al. 2014; Zhuang and Conner, 2022). For example, tensions often arise between encouraging student participation and adhering to mathematical standards for valid arguments (Ayalon et al. 2022; McClain and Cobb, 2001; Nathan and Knuth, 2003; Staples, 2007). Additionally, teachers' perceptions of their role in facilitating argumentation sometimes conflict with reform expectations. Some assume argumentation requires little scaffolding (Kosko et al. 2014). Moreover, teacher preparation programs often lack a focus on the specific skills needed to teach argumentation effectively, leaving teachers underprepared to foster it in their classrooms (Kazemi et al. 2021; Magiera et al. 2022).

Noticing skills are central to teachers' work in the classroom, and are considered a key factor in determining teachers' proficiency and in encouraging their practices (e.g., Dindyal et al. 2021; Jacobs et al. 2010). Our premise is that teachers who are better trained and equipped to notice argumentation will possess important capabilities and skills required to practice and cultivate argumentation in the mathematics classroom. This premise is based on the literature, which highlights noticing skills as part of a teacher's expertise that influences the quality of her instruction in the classroom (e.g., Dindyal et al. 2021; Jacobs et al. 2010).

Building on teacher-noticing research and with the goal of supporting teachers' proficiency in promoting argumentation in their classrooms, we investigate secondary-school mathematics teachers' (SMTs) noticing of argumentation within the context of a professional development course that incorporates argumentation classroom situations (ACSs). We use noticing as a framework to explore which aspects of argumentation teachers are more or less attuned to, based on the research-based assumption that this either supports or limits their classroom practice. Additionally, we examine changes in their noticing, focusing on the extent to which they attended to argumentation and their ability to interpret it. These changes are analyzed through their participation in the ACS-based course, alongside an exploration of the factors that SMTs identify as contributing to these changes.

Theoretical background

Argumentation. The literature offers various definitions of argumentation (Schwarz and Baker, 2017). The theoretical perspective for the approach taken in this paper views argumentation as a social process, situated, in our case, in the social norms of the classroom. We therefore follow van Eemeren's and Grootendorst's (2004) definition according to which argumentation is "a verbal, social, and rational activity aimed at convincing a reasonable critic of the acceptability of a standpoint by putting forward a constellation of propositions justifying or refuting the proposition expressed in the standpoint" (p 1). This definition posits argumentation in a social space and lays the groundwork for common descriptions of "deliberative argumentation" which have been shown to be notably 'fruitful' for learning (Felton et al. 2009). Learners' collaborative construction of arguments, listening critically yet respectfully to their peers' ideas, identifying the advantages and drawbacks in each idea, and striving toward a general consensus are the key features of this form of argumentation, according to Asterhan and Schwarz (2016, p 167).

Following González-Howard and McNeill (2020), in this study we consider argumentation as having two prominent and interrelated aspects: (1) the *structural aspects*, which consist of building a claim and suggesting an appropriate justification for it and (2) the *dialogic aspects*, which focus on individuals' interactions when listening to one another, building upon their peers' proposed ideas, and respectfully critiquing those ideas as the group works together toward attaining a consensus. In our opinion, teachers will attain greater success in incorporating argumentation into their classrooms by attending to both aspects of argumentation.

Teaching for argumentation. Argumentation has been highlighted as a crucial part of mathematical classroom discourse in recent studies on mathematics education (e.g., Ayalon and Hershkowitz, 2018; Conner et al. 2014; Staples, 2014). Several scholars have identified a variety of factors that enable or inhibit students' engagement in argumentation (e.g., Ayalon and Nama, 2024; Mueller et al. 2014; Staples, 2014; Yackel and Cobb, 1996). We focus on five prominent factors: the nature of the mathematical tasks, the teaching strategies used, the students' cognitive characteristics, the students' affective characteristics, and the socio-cultural characteristics (Ayalon, 2024).

The term *task characteristics* refers to the task's enabling the use of multiple solution-strategies and representations (Mueller et al. 2014; Solar et al. 2020); encouraging multiple (correct and incorrect) answers (Solar et al. 2020); constructing conjectures, raising justifications and assessing them (Bieda et al. 2014); and encouraging students to collaboratively seek optional ideas, discuss dissimilarities in perspectives, and critique the ideas that have been raised (Ayalon and Nama, 2024; Mueller et al. 2012).

Teaching strategies refer to teacher support of students engaging in argumentation, including encouraging students to raise different ideas and to justify them (Conner et al. 2014; Staples, 2014); discussing with students which justifications are deemed acceptable in the mathematics classroom (e.g., Staples, 2014; Yackel and Cobb, 1996); allowing students sufficient time and space to generate ideas (Solar et al. 2020); giving prompts to students who have difficulty proposing justifications (e.g., Conner et al. 2014); and elucidating the underlying ideas of mathematical argumentation; for example, posing a counterexample to challenge a student's argument (Francisco, 2022). The teacher could encourage an argumentative dialog between the students that promotes co-construction of arguments and critique of others' ideas in a way that cultivates reciprocal respect (Conner et al. 2014; Kosko et al. 2014; Mueller et al. 2014).

Students' characteristics focus on both cognitive and affective characteristics (Slakmon and Schwarz, 2019). In *cognitive* terms, previous studies underscore the importance of building upon students' mathematical reasoning in support of argumentation (e.g., Francisco, 2022). The teacher should appreciate student thinking, prior knowledge that facilitates generating and critiquing arguments (Knuth and Sutherland, 2004), and an array of argumentation skills that include working together to construct arguments, critiquing peers' ideas, offering justifications, and potentially revising their own suggestions in light of the classroom discourse (Mueller et al. 2012; Stein and Albro, 2001). In *affective* terms, the teacher should be sensitive to her students' motivations, emotions, and self-confidence (Ayalon et al. 2022; Slakmon and Schwarz, 2019), especially since argumentation has the power to evoke both positive and negative emotions among students (e.g., Ayalon et al. 2022).

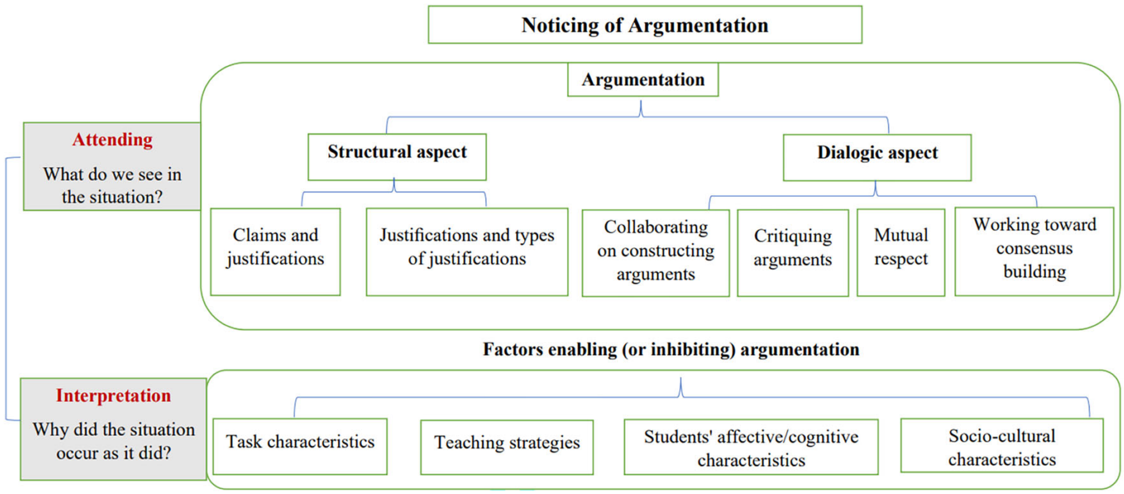


Fig. 1 Our conceptualization of argumentation in the mathematics classroom and the components of noticing of argumentation (Second Author, 2024).

Socio-cultural characteristics relate to cultural classroom norms (e.g., appreciating the value of argumentation, expectations for peer collaboration, respectful critique, convincing peers of the reasonableness of one's proposed justifications, and suggesting alternatives (e.g., Makar et al. 2015; Mueller et al. 2014)); as well as the socio-mathematical norms that determine acceptable justifications in the classroom (Stephan, 2020; Yackel and Cobb, 1996).

Teacher noticing. Noticing as a core teaching skill has been highlighted in recent studies in mathematics education (e.g., Jacobs et al. 2010; van Es et al. 2017; Wei et al. 2023). Teachers' noticing has been demonstrated to be a crucial contributor to refining teachers' expertise and their students' academic achievements (Santagata and Yeh, 2016; van Es et al. 2017). Mathematics education currently comprises diverse conceptualizations of noticing (König et al. 2022). Similar to many other scholars, we too regard noticing as comprising three interrelated skills. These include: *attending* to noteworthy features of instruction, *interpreting* them while endorsing alternative perspectives to better understand what is being observed, and *deciding how to respond* (Jacobs et al. 2010). The focus of this paper is solely on attending and interpreting skills.

Noticing is deemed part of a complex and challenging teaching skillset that can be developed through teacher training programs (e.g., Jacobs et al. 2010; König et al. 2022; Santagata et al. 2021; van Es et al. 2017). Common noticing activities employed in such training programs require teachers to analyze artifacts, such as written classroom artifacts (e.g., Scherrer and Stein, 2013); video-clips of classrooms in action (see Santagata et al. 2021) and/or student written work (e.g., Jacobs et al. 2010; Magiera and Zambak, 2021). Most researchers have focused on mathematics teachers' noticing of their students' mathematical thinking (e.g., Bas-Ader et al. 2023; Jacobs et al. 2010; van Es and Sherin, 2008), while others have underlined specific issues, for instance, students' participation and equity (Louie et al. 2021) and curricular noticing (Amador et al. 2017). This study builds on and extends the existing literature on teachers' noticing by conceptualizing a specific type of noticing for mathematics teachers: *noticing of argumentation*. Teachers are invited to notice the argumentation in given argumentation classroom situations (ACSs) through the use of specific lenses derived from our approach to argumentation, as described below.

Conceptualization of teachers' noticing of argumentation. Following Jacobs et al. (2010) and the educational literature on argumentation, we conceptualize noticing of argumentation as comprising three key interrelated skills: *attending, interpreting*, and *deciding how to respond* (Ayalon, 2024; Nama and Ayalon, 2024). *Attending* refers to identifying both the structural and dialogic characteristics of argumentation in the classroom situation (González-Howard and McNeill, 2020). *Interpreting* refers to reasoning and making sense of argumentation in the classroom situation, weighing factors that might enable or inhibit argumentation (e.g., Ayalon and Nama, 2024; Staples, 2014; Yackel and Cobb, 1996). Finally, deciding how to respond relates to what one would do, assuming s/he were the teacher in that situation, toward fostering argumentation. In this study, we focus on the first two sub-skills: attending and interpreting.

Researchers have demonstrated the efficiency of structured frameworks or guided instruction in supporting teachers' ability to notice (Diamond et al. 2018; Mitchell and Marin, 2015; Stockero et al. 2017; Walkoe, 2015). In light of this, we chose to provide guidance to participants in identifying structural and dialogic characteristics of argumentation and interpreting the argumentation through distinctive lenses. This decision was primarily motivated by the intricate nature of teaching and learning associated with argumentation. By offering partial guidance, we aimed to reduce complexity while simultaneously allowing for independent thought surrounding identification and interpretation.

Figure 1 summarizes our conceptualization of argumentation in the mathematics classroom and of noticing of argumentation. We employed this model in designing the research tool and in analyzing the data to assess the SMTs' change in attending to and interpreting argumentation.

Argumentation classroom situation (ACS). It is broadly acknowledged that classroom scenarios, such as those captured through written transcripts or videos, can effectively illustrate instructional contexts aimed at promoting teacher noticing (e.g., Scherrer and Stein, 2013; van Es et al. 2017). Following Ayalon (2024), this study defines an argumentation classroom situation (ACS) as a genuine instructional event in a mathematics classroom, demonstrating significant potential for enhancing the skill of noticing argumentation. An ACS involves mathematically focused interactions between students and a teacher, where students present claims and justifications, collaborate on ideas, and engage in critique as they work towards building consensus. It encapsulates essential criteria, including structural components like claims and justifications, and dialogic elements such as co-constructing arguments, critiquing, fostering mutual respect, and striving for consensus. These elements are essential for argumentation, considered "productive" for enhancing mathematical understanding and learning, making it crucial for teachers to attend to them (Francisco and Maher, 2005; Weber et al. 2008). Additionally, an ACS enables offering rich interpretations for the argumentation from various angles, acknowledging different factors that may affect them and suggesting alternative instructional approaches.

The ACS proved to be a powerful tool for developing noticing skills, enabling teachers to focus on key aspects of argumentation and interpret these aspects meaningfully, as demonstrated in our previous studies (Nama and Ayalon, 2024; Nama et al. 2023). In these earlier studies, teachers engaged with an ACS as a one-time experience, analyzing a single classroom situation as part of a structured task. For example, in one study, ACS analysis within a peer assessment cycle revealed significant changes in teachers' attention to and interpretation of argumentation (Nama and Ayalon, 2024). In another study (Nama et al. 2023), a comparative analysis of preservice and in-service teachers' engagement with an ACS highlighted notable differences in how these groups notice and interpret argumentative practices in the classroom.

Building on these findings, the current study extends the use of ACS by integrating it as a central and recurring tool throughout a professional development course. Rather than a single exposure, teachers in this study engaged with multiple ACSs over an extended period, allowing us to examine how their noticing of argumentation evolved over time. This shift enables a deeper exploration of the processes underlying teachers' professional growth and the sustained impact of structured ACS-based learning experiences.

Research objectives and questions

Building on previous findings regarding teachers' noticing of argumentation (Nama and Ayalon, 2024; Nama et al. 2023), this study investigates the effectiveness of using ACSs as a recurring tool throughout a professional development course. Unlike earlier studies that explored ACS analysis in a single-instance setting, this study examines how sustained engagement with multiple ACSs over time impacts teachers' noticing of argumentation. Specifically, our research aims to explore the change in SMTs' noticing of argumentation in relation to two components: attending to, and interpreting of, argumentation during their participation in an ACS-based course. In addition to tracking changes in teachers' noticing, we seek to understand the factors that contributed to these changes from the teachers' perspectives.

The research questions are as follows:

RQ1. What changes occur in SMTs' noticing of argumentation in terms of their attention to and interpretation of argumentation through participation in the ACS-based course over time?

RQ2.What factors contributed to the change in SMTs' noticing of argumentation, from their point of view?

Methods

We used a mixed-methods design (Creswell and Plano Clark, 2007) to analyze changes in SMTs' noticing of argumentation (attention and interpretation) over the duration of a 14-week course focused on argumentation in mathematics teaching. This section describes the research context and participants as well as the methods for data collection and analysis.

Research context and participants. This study took place during 14 weekly 4 h-long sessions in a university course focused on argumentation in mathematics teaching. A cohort of 31 SMTs participated in this study as part of their fulfillment of a master's degree in mathematics education. All participants held a B.Ed. in mathematics education or a B.Sc., majoring in mathematics or a mathematics-related subject. Their teaching experience ranged from 1–27 years, averaging 7 years; 17 participants had 1–5 years of experience, and 14 had more than five years of experience; thirteen teachers taught students aged 12–15, and 18 teachers taught students aged 15–18. The teachers in this cohort had not been explicitly exposed to argumentation in their formal academic education. Participants were informed that the course facilitator's assessments of their ACS written reports were intended for professional learning only and would not be used in determining their final course grade. As suggested by the ethics committee, at the end of the course, after publishing the participants' grades, we requested their consent to use their ACS written reports for our research. All of them consented to our request.

Throughout the course, participants did not receive any feedback from the facilitator (the second author) regarding their ACS reports and were not exposed to the rubric format (see the ACS report format in the research tools section). Consequently, it appears that participants were unaware of our implicit objective to examine changes in their noticing throughout the course. This suggests that in their ACS reports, participants did not enhance their analysis to meet a perceived goal or expectation. Rather, their engagement stemmed from the inherent value of the activities themselves.

The project design was guided by research on promoting teachers' noticing, centering around two main principles: (1) integration of theoretical and practical activities, and (2) individual and collaborative work, wherein the ACSs and ACS report format (elaborated on later) served as instructional tools. The course dealt with various aspects of argumentation with a particular emphasis on the two dimensions of argumentation. Dimension 1 relates to characteristics of the actual argumentation situation (structural and dialogic). Dimension 2 relates to aspects that contribute to shaping the argumentation (enabling or inhibiting), including task characteristics, teaching strategies, student cognitive and affective characteristics, and socio-cultural characteristics.

The course included two cycles each with a similar format (see Fig. 2) that took place at two points in time: analysis of an ACS focused on 'abbreviated multiplication formulas' (ACS2) and analysis of an ACS focused on 'inequalities' (ACS3) (see Figs. 1 and 2 in the Supplementary Material). In addition, the SMTs were asked to analyze the same ACS on 'analytic geometry' (ACS1, see below) at the first and last sessions of the course.

During the course, the instructor used several additional ACSs which the SMTs were asked to partially analyze for practice and internalization of the material taught in the specific session; for example, attending to structural aspects of argumentation (for details of the course sessions see Fig. 3 in the supplementary material).

Research tools

The argumentative classroom situations (ACSs). During the course, the teachers analyzed several ACSs, focusing on different topics in mathematics. The ACSs took place in classes of teacher-colleagues, who recorded them after class and later shared them with us. As noted in the section "Research context and participants", the SMTs were asked to analyze some of the ACSs partially and to fully analyze three of them using the ACS report format. ACS1, ACS2, and ACS3 comprised the main data sources for characterizing the change in SMTs' noticing skills following their participation in the course (RQ1).

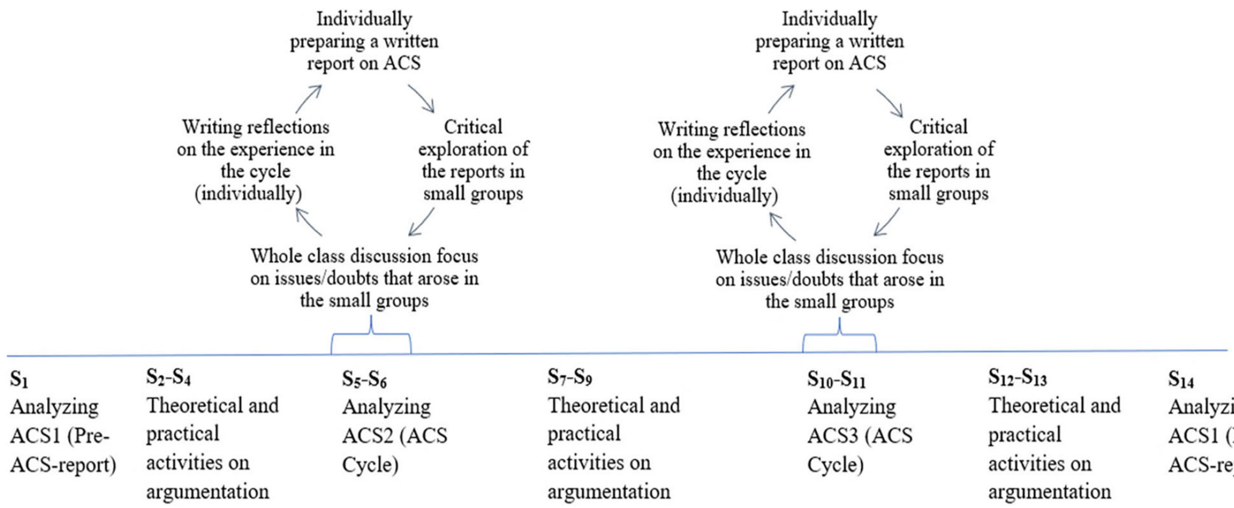


Fig. 2 The course description and schedule. Si stands for the session number.

Table 1 presents the speaker and her/his contribution in the context of ACS1 in chronological order.

To test our choice of ACSs, we asked a group of 11 Ph.D. and M.A. students in mathematics education to analyze the classroom situations individually (by using the ACS-report format; see below) and then share and discuss their analyses. Based on their analysis and discussion, we reached a final consensus that the three situations meet the criteria of an ACS (section "Conceptualization of teachers' noticing of argumentation").

The ACS report format. Table 2 presents the ACS report format, which includes prompts related to two skills of noticing argumentation: attending and interpreting (adapted from Jacobs et al. 2010).

The ACS rubric format. The rubric displayed in Table 3 served as a research tool for analyzing the SMTs' ACS reports. It was designed in accordance with the prompts included in the ACS report format, and reflects the extent to which the SMT attends to structural and dialogic aspects of argumentation and makes sense of the argumentation through the different factors that enabled/inhibited it. The rubric was devised and validated in a course centered around argumentation in mathematics teaching a year prior to the present study (see Nama and Ayalon (2024) for details). In this paper, we used the same rubric as in Nama and Ayalon (2024), except for coding levels of attending to structural aspects. Following review comments we received on a previous paper (Nama et al. 2023), we modified the coding scheme to capture the teachers' proficiency in both identifying the arguments and evaluating their accuracy. For validation, we shared the modified rubric and some data with three Ph.D. students in mathematics education. Through discussion, we reached a consensus on the modified components.

An example of a 'high-level' analysis for one of the three ACSs, the 'Abbreviated Multiplication Formulas' situation, using the rubric, is provided as supplementary material. In addition, some examples of the quality levels of noticing of argumentation in the teachers' reports are presented in the Findings Section. Examples for each quality level of attending to the dialogic aspect and interpreting are given in Tables 1 and 2 in the supplementary material.

Data collection. The data used for this study were collected from the SMTs' reports on ACSs written using the above format (Table 2). For RQ1, SMTs' ACS reports focused on the analysis of three ACSs submitted at four measurement points, capturing their noticing during the course: a report on ACS1 in the first session (pre-ACS) (see Table 1), a report on ACS2 in the fifth session, a report on ACS3 in the tenth session, and a report on ACS1 in the final session (post-ACS), totaling 124 reports. These reports served as the primary data source for characterizing participants' noticing of argumentation skills over time. Initially, we intended to employ four different ACSs throughout the course. However, concerns arose regarding potential influences on the results, as different ACSs may present varying affordances and limitations in stimulating noticing. To mitigate this potential bias, we adopted a methodology commonly used in similar studies, which involves employing the same pre- and post-tests to examine changes in skills during intervention (Santagata et al. 2021, 2007; Schack et al. 2013). Therefore, we opted to use the same ACS in both the first and last sessions (ACS1). We acknowledge that re-encountering the same ACS (ACS1) could influence participants' noticing. To minimize this effect, participants were not exposed to ACS1 or any discussion or references to it between the two assessments. This approach was intended to ensure that changes in noticing reflected the impact of the intervention rather than familiarity with the specific ACS.

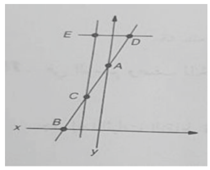
For RQ2, two types of data were collected:

(i) *Written reflections*: At the end of the course, the SMTs submitted written reflections focusing on their experiences throughout the sequence of activities, the differences observed in their analysis of the ACSs, and possible underlying causes. All written reflections from the 31 SMTs were analyzed to identify factors that either promoted or inhibited changes in the SMTs' noticing of argumentation, from the teachers' perspective. The SMTs were prompted with the following questions: What are the differences between your analysis of the ACSs over the course? What factors in your experience contributed (promoted or inhibited) to a change in your work? Please explain in detail.

(ii) *Individual, semi-structured interviews*: The first author (not the course instructor) conducted individual, semi-structured interviews with a random selection of 10 of the 31 SMTs. These interviews aimed to gain additional insight into the factors influencing changes in the SMTs' noticing of argumentation from their perspective. The random selection of 10 participants was guided by the significant changes observed in the responses of the 31 SMTs. This sample size was chosen as a balance between obtaining detailed insights and addressing the practical limitations of the study. It was feasible given the available resources,

Table 1 ACS1: "Analytic geometry".

The following situation took place in a 12th-grade mathematics classroom in a 3-unit level class, with 20 students. The topic was "analytic geometry". In the first part of the lesson, the student Vered asked the teacher to solve homework question 12, because she had struggled to match the equations to the given lines.

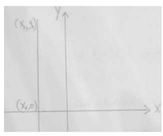


Question 12 Pictured are 3 equations:

x=-3, y= 15, 5x - 3y + 30 = 0

Find the x and y values of the points marked in the picture.

After discussing the solution to the question as a class, the teacher turned to the class and asked: What about the equation x=-3, does it represent a linear function?

1	Teacher	Very nice. I see that you've reached an agreement that the equation $y = 15$ represents a line parallel to the x-axis, and that the equation $x = -3$ represents a line parallel to the y-axis. Yuval, you explained very nicely that $y = 15$ is a constant linear function and that its slope is zero. What about the equation $x = -3$, does it represent a linear function?
2	Adam	I think that it's a constant linear function; we can clearly see that it's a straight line, but the difference between it and $y = 15$ is that it's parallel to the y-axis.
3	Students	Yes, it's a linear function.
4	Zuhair	No, Adam, it's not a function at all.
5	Teacher	Zuhair, I'm really glad you're joining the discussion. Can you please explain your claim?
6	Zuhair	According to the definition of a function: "for each x there is a single unique y," and on the line $x = -3$ that's not true. I drew it in my notebook.
7	Teacher	Can you please draw it on the board?
8	Zuhair	I'd rather not.
9	Teacher	Come on, don't be shy, come up here so that we can all understand what you're saying.
10	Zuhair	[Zuhair draws on the board and explains:]  According to the definition of a function, each x has a single unique y. If we take a random point on the given line [he marks (x_1,0)] and another point on the line (x_1,y_1) [marks this as well], they will have the same x value. So, on this line each x has more than one y, and therefore it isn't a function.
11	Yana	I don't agree with you.
12	Zuhair	Come on, Yana, this time there's nothing to argue; this is a definition we learned in 8th grade.
13	Hala	I agree with Zuhair, can I explain why?
14	Teacher	Just a minute, Hala, let's go with Yana first. Yana, can you explain why you don't agree with Zuhair. [2 minutes of silence] I see that it's quiet. OK, what do you all think?
15	Yana	Wait, teacher, give us another few minutes to think?
16	Teacher	OK, take another few minutes to think. Students, you can try to figure it out together.
17	The teacher walks around among the students [after 5 minutes]	

allowing for an in-depth analysis without being too large to manage effectively or too small to restrict the scope of understanding. The interviews lasted ~90 min each. Prior to each interview, participants received their own reports and assessments, as well as the main findings concerning the changes in the entire group's noticing of argumentation, several days prior to the interview, allowing ample time for reflection. The interviews centered around key inquiries: (1) Do the research findings make sense to you? In what ways? (2) Please share possible explanations for the findings from your viewpoint. (3) Which elements of the course do you believe facilitated or hindered the shift in teachers' noticing of argumentation? (3) What aspects of your experience contributed to your analysis of the ACSs throughout the course? Please elaborate with examples. Teachers were encouraged to provide detailed responses, citing examples where possible, to enrich the interview process.

Data analysis. For RQ1, the data analysis aligned with our objective of examining the change in the SMTs' noticing of argumentation as reflected in their ACS reports over time. The analysis comprised three main stages, outlined below.

18	Teacher	Yana and Ali, I see that you're trying to reach a conclusion! Did you succeed?
19	Yana and Ali	Yes, we're trying.
20	Teacher	What do you think about sharing what you've done so far with the class?
21	Ali	Yes, gladly! Yana thought that if we find the slope of two points, that would help her to decide if it's a function, so I suggested that we use the formula for finding the slope of a line. If we take the two points that Zuhair marked on the line parallel to the y-axis, and try to calculate the slope of the line, we get $m = \frac{(y1-0)}{(x1-x1)} = \frac{y1}{0}$ That's where we stopped. It's not clear what we got, maybe you can help us guys!
22	Teacher	Good for you, you did nice work! You got an expression that is not defined. Who can explain the result in terms of the slope?
23	Maya	Right, teacher, we learned that dividing by zero is not defined, so the slope that they got, $\frac{y1}{0}$, is not defined. That means it is not a function, and definitely not a linear function.
24	Yana	You're right, Zuhair, it is not a function. I was wrong!
25	Khaled	Cool, teacher, my idea was right! I want to share what I did; can I?
26	Teacher	Yes, of course! Please share with us, Khaled.
27	Khaled	I also used the graph, I wanted to find the rate of change of the line by drawing a "step" that's the slope, but I didn't manage to do that. Now I understand from Yana and Ali's use of the formula for finding the slope that this line doesn't have a slope because the expression wasn't defined, so it's not a linear function.
28	Teacher	Can we conclude that the equation $x = -3$ isn't a function?
29	Students	Yes.
30	Teacher	Good for you, students, you did excellent work!

The teacher moves on to the next exercise.

Table 2 The ACS report format.

Attending to prompts
What do you notice in the situation?
Describe in detail those parts of the ACS which you deem important in terms of argumentation, with a specific reference to (1) structural aspects, meaning, the claims, evaluation of the correctness of the claims, the justifications, if such exists, and classifying the types of justifications that emerged in the ACS, and (2) dialogic aspects, i.e., the interactions between the participants, relating to such aspects as co-constructing arguments, critiquing arguments, exhibiting mutual respect, and working toward consensus building. Provide a detailed description of how each aspect is manifested in the ACS, bring examples, and note the specific line numbers.
Interpreting prompts
Provide possible answers to the question: "Why did the ACS occur as it did?" by referring to possible factors that impacted (enabled and/or inhibited) the sequence of argumentation, such as characteristics of the mathematical task, the teaching strategies, student cognitive and affective characteristics, socio-cultural characteristics, or anything else that comes to mind.
Base your interpretation on the ACS itself, in as much detail as possible, including evidence-based details of how the factor impacted the sequence of argumentation; bring quotes, and note specific line numbers; theoretical perspectives and/or teaching experience.

Stage 1: With regard to participants' *attending to argumentation*, for each ACS report, we classified the participant's responses according to the two aspects of argumentation: structural and dialogic. We examined the participant's response to the attending prompts of the structural and dialogic aspects as they appear in the ACS report format (see Table 2). Responses classified as *attention to structural aspects* included attending to the elements of arguments, which include claims, evaluating the correctness of the claims, justifications, and classifying the type of justifications. Responses classified as *attention to dialogic aspects* included attending to aspects of co-constructing of arguments, critique of arguments, mutual respect, and working toward consensus-building. With regard to participants' interpreting of argumentation, for each ACS-report we classified the participant's responses according to the content of the interpretation, i.e., the factors through which the argumentation was interpreted (Task characteristics, Teaching strategies, Student cognitive characteristics, Student affective characteristics and Socio-cultural characteristics).

Stage 2: For each ACS report submitted by each SMT (pre-ACS1 report, ACS2 report, ACS3 report and post-ACS1 report), we employed directed content analysis with a focus on the quality levels of the two noticing skills: *attending to* structural and dialogic aspects (for each of the four dialogic aspects), and *interpreting* of argumentation (for each of the five factors) (section "The ACS rubric format"). It is important to note that we did not evaluate whether a participant's interpretation was correct or incorrect. Instead, our focus was on examining the evidence they provided to support their interpretation, recognizing that different teachers may interpret the same

Table 3 The ACS rubric format.

Noticing skill	Levels of noticing
Attending to structural aspects[a]	1. $s \leq \frac{1}{4}X$ 2. $\frac{1}{4}X < s \leq \frac{3}{4}X$ 3. $s > \frac{3}{4}X$
Attending to dialogic aspects[b]	1. Paid no attention to the dialogic aspect. 2. Paid attention to the dialogic aspect, lacking or general description of how the aspect is manifested in a given situation. 3. Paid attention to the dialogic aspect, with a detailed description of how the aspect is manifested in a given situation.
Interpreting[c]	1. Did not address the factor. 2. Addressed the factor with a largely descriptive or evaluative response and little or no use of evidence to support claims. 3. Addressed the factor, with some evidence to support claims (based on the ACS/theoretical perspectives/teaching experience) 4. Addressed the factor, with robust evidence to support claims (based on the ACS/theoretical perspectives/teaching experience)

[a]s represents the score for attending to the structural aspects; X denotes the number of arguments presented in the ACS. For more details, see the section "Data analysis" for Stage 1 of RQ1.
[b]For each of the four dialogic aspects (collaborating on constructing arguments, critiquing arguments, mutual respect, and working toward consensus building).
[c]For each of the five factors (task characteristics, teaching strategies, cognitive and affective student characteristics, socio-cultural characteristics) (Note that we did not find reference to additional factors in the teachers' reports).

argumentative discourse differently. Therefore, the key aspect of our analysis was the justification and evidence supporting each interpretation.

With regard to attending to structural aspects of argumentation, for each of the X arguments presented in the ACS, we applied the following coding scheme:

- In terms of attending to the claim:
  - Participants who identified the claim received a score of 1; otherwise, a score of 0 was assigned.
  - Participants who accurately evaluated the correctness of the claim received a score of 1; otherwise, a score of 0 was assigned.
- In terms of attending to the justification:
  - Participants who identified the justification received a score of 1; otherwise, a score of 0 was assigned.
  - Participants who correctly classified the type of the justification received a score of 1; otherwise, if the participant did not classify the type of justification or incorrectly classified the type of justification, a score of 0 was assigned.

Overall, for each of the X arguments raised in the ACS, a participant could receive a score between 0 and 4. We then calculated the sum of scores for all X arguments, denoted by s. To determine the total score:

- Participants who received a score $s \leq \frac{1}{4} \cdot X$ were assigned a score of 1.
- Participants who received $\frac{1}{4} \cdot X < s \leq \frac{3}{4} \cdot X$ were assigned a score of 2.
- Participants who received $s > \frac{3}{4} \cdot X$ were assigned a score of 3.

With regard to participants' *attending* to dialogic aspects (for each of the four dialogic aspects) and interpreting of argumentation (for each of the five factors), we employed directed content analysis focusing on the quality levels of these two noticing skills, as presented in the rubric format (see the section "The ACS rubric format"). The scoring for each participant was given as follows: scores were given according to the number of quality levels for each skill (for example, a score of 3 for level 3). For *attending to dialogic aspects*, scores ranged from 1 to 3. For *interpreting argumentation*, scores ranged from 1 to 4.

To further validate our analysis, 20% of the data were also coded by additional research team members, comprised of 10 Ph.D. and M.A. students in mathematics education. We used Cronbach's alpha as a criterion to check the score's reliability. Reliability scores for the structural aspects of attending were 0.93, and the reliability scores for all four dialogic aspects ranged from 0.94 to 0.99. Reliability scores for the five factors of interpretation ranged from 0.93 to 0.98, exceeding the recommended minimum level of 0.7, as recommended by Nunnally (1978).

Stage 3: To determine whether a change occurred in SMTs' noticing of argumentation, non-parametric methods were employed due to the ordinal nature of the variables under examination. The following statistical tests were used:

- *Friedman test*: This test was used to determine whether there was a statistically significant change across the SMTs' ACS reports concerning their attention to both structural and dialogic aspects (for each of the four aspects) and interpretation (for each of the five factors).
- *Post-hoc comparisons with the Wilcoxon Signed-Rank Test*: Following the Friedman test, the Wilcoxon Signed-Rank test was applied to identify significant differences between pairs of SMTs' ACS reports. The effect size for the Wilcoxon Signed-Rank tests was also calculated using the formula: $r = z/\sqrt{N}$ (Pallant, 2011). According to Cohen (1988), effect sizes are categorized as follows: "small" ($r = 0.1$), "medium" ($r = 0.3$), and "large" ($r = 0.5$).

For RQ2, we conducted an interpretive and in-depth qualitative analysis of the SMTs' written reflections ($n = 31$) and interview transcripts ($n = 10$) (Creswell, 2007). Employing inductive line-by-line coding, we identified descriptions of the factors that shaped changes in SMTs' noticing of argumentation, based solely on participants' accounts. The themes were generated purely inductively, without reliance on or influence from pre-existing theoretical frameworks during the analysis phase. The process involved multiple iterations of data sorting and continual comparisons both within and across developing categories, ensuring the themes authentically reflected participants' perspectives.

The analysis yielded a coding scheme with four themes grouped into two main categories:

Table 4 The structural aspects of argumentation (scale 1–3), Friedman test and descriptive statistics for each SMT's ACS report (pre-ACS1, ACS2, ACS3, and post-ACS1).

					Friedman test		
		Median	Mean	SD	χ^2	df	Asymp. sig.
Structural aspects	Pre-ACS1 report	3	2.35	0.91	15.917	3	0.001
	ACS2 report	3	2.71	0.46			
	ACS3 report	3	2.94	0.25			
	Post-ACS1 report	3	2.94	0.25			

Table 5 The structural aspects of argumentation (scale 1–3) Wilcoxon test and effect size for all the pairs of ACSs reports.

		Wilcoxon test			
	Time	*Z* statistic	Asymp. sig. (2-tailed)	Adjusted-*P*	Effect size (*r*)
Structural aspects	Pre-ACS1–ACS2	−1.99	0.047	0.28	0.36
	Pre-ACS1–ACS3	−2.97	0.003	0.02	0.53
	Pre-ACS1–post-ACS1	−2.97	0.003	0.02	0.53
	ACS2–ACS3	−2.33	0.020	0.12	0.42
	ACS2–post-ACS1	−2.33	0.020	0.12	0.42
	ACS3–post-ACS1	0.00	1.000	1.000	0.00

1. Factors inherent to the activity: This category included three themes related to (a) the artifacts used in the course, (b) the group and whole-class discussions, and (c) the literature they were exposed to.
2. *Incorporating their teaching experiences into the activity*: This category included one theme related to participants reflecting on their actual teaching practices.

Interpretations were collaboratively discussed by the two authors and refined through cycles of check-coding until consensus was reached (Miles and Huberman, 1994). While the themes emerged inductively, a subsequent phase of analysis involved situating these themes within existing literature on teacher noticing and argumentation. Specifically, we examined how the identified themes aligned with, expanded upon, or challenged prior research. This theoretical triangulation helped refine our interpretations and position our findings within the broader discourse on teachers' noticing of argumentation.

Findings

In this section, we first outline quantitative data on shifts in SMTs' noticing of argumentation, followed by examples from their ASC reports (RQ1). We then examine the factors that SMTs themselves identified as contributing to changes in their noticing of argumentation (RQ2).

Quantitative findings on SMTs' *attending to* and *interpreting of* argumentation over time. Overall, the results provide evidence of improvement in the SMTs' noticing skills of attending to and interpreting argumentation following their participation in the ACS-based course. Specifically, their level of attention to structural aspects improved, as they attended to more claims, and accurately evaluated their correctness, and attended to more justifications and correctly classified their types. Moreover, their attention to dialogic aspects became more versatile, as they attended to more aspects with a detailed description of how each aspect is demonstrated in the ACS. Likewise, their interpretations became more versatile as they interpreted the argumentation through more factors (enabling or inhibiting the argumentation), and became more evidence-based.

Change in SMTs' attending *to structural aspects of argumentation over time*. The results of the Friedman test, presented in Table 4, indicate a statistically significant change in SMTs' skills of attending to *structural aspects* between the SMTs' ACSs reports ($\chi^2 = 15.917$, $\text{df} = 3$, $p = 0.001$). The results of the Wilcoxon Signed-Rank tests for the pairs of the four SMTs' ACSs show a significant change from ACS1 up to ACS3.

As shown in Table 5, significant changes in attending to structural aspects were observed from pre-ACS1 to ASC3, with this trend persisting through post-ACS1. No significant changes were observed between adjacent measures.

Change in SMTs' attending *to dialogic aspects of argumentation over time*. The results of the Friedman test, presented in Table 6, indicate a statistically significant change in SMTs' skills of attending to each of the four dialogic aspects between the SMTs' ACS reports.

As shown in Table 6, the majority of SMTs entered the course with a medium level of attending to the dialogic aspects of argumentation: co-construction of arguments (mean = 1.97), critique of arguments (mean = 2.13), mutual respect (mean = 2.23), and working toward consensus building (mean = 2.10). The results, as shown in Table 6, indicate that statistically significant changes occurred in the SMTs' attending to dialogic aspects of argumentation throughout the course.

Table 7 presents the results of Wilcoxon signed-rank tests for pairs of the ACS reports.

As shown in Table 7, significant changes in co-constructing of arguments were observed in each stage of ACS up to ACS3, with no additional changes between ACS3 and post-ACS1. Regarding the three dialogic aspects—critique arguments, mutual respect, and working toward consensus building—significant changes were noted every two or three steps of ASC, specifically between ASC1 and ASC3 or post-ASC1, and between ASC2 and post-ASC1. No significant changes were observed between adjacent measures.

Change in SMTs' interpreting *of argumentation over time*. The Friedman tests presented in Table 8 indicate a statistically significant change in SMTs' skills of *interpreting* argumentation

Table 6 Four dialogic aspects of argumentation (scale 1-3) Friedman test and descriptive statistics for each ACS report (pre-ACS1, ACS2, ACS3, and post-ACS1).

		Median	Mean	SD	Friedman test		
					χ^2	df	Asymp. sig.
Co-constructing of arguments	Pre-ACS1 report	2	1.97	0.84	45.105	3	<0.001
	ACS2 report	3	2.55	0.620			
	ACS3 report	3	2.94	0.25			
	Post-ACS1 report	3	3.00	0.00			
Critique arguments	Pre-ACS1 report	2	2.13	0.76	28.429	3	<0.001
	ACS2 report	2	2.45	0.50			
	ACS3 report	3	2.74	0.51			
	Post-ACS1 report	3	2.94	0.25			
Mutual respect	Pre-ACS1 report	2	2.23	0.67	37.850	3	<0.001
	ACS2 report	3	2.58	0.62			
	ACS3 report	3	2.90	0.40			
	Post-ACS1 report	3	3.00	0.00			
Working toward consensus building	Pre-ACS1 report	2	2.10	0.75	32.707	3	<0.001
	ACS2 report	3	2.55	0.72			
	ACS3 report	3	2.87	0.34			
	Post-ACS1 report	3	3.00	0.00			

Table 7 Four dialogic aspects of argumentation (scale 1-3) Wilcoxon test and effect size for all the pairs of ACS reports.

	Time	*Z* statistic	Wilcoxon test		
			Asymp. sig. (2-tailed)	Adjusted-*P*	Effect size (*r*)
Co-constructing of arguments	Pre-ACS1-ACS2	−2.75	0.006	0.036	0.49
	Pre-ACS1-ACS3	−4.14	<0.001	<0.001	0.74
	Pre-ACS1-post-ACS1	−4.14	<0.001	<0.001	0.74
	ACS2-ACS3	−2.68	0.007	0.042	0.48
	ACS2-post-ACS1	−3.28	0.001	0.006	0.59
	ACS3-post-ACS1	−1.41	0.157	0.942	0.25
Critique arguments	Pre-ACS1-ACS2	−1.98	0.048	0.288	0.36
	Pre-ACS1-ACS3	−3.11	0.002	0.012	0.56
	Pre-ACS1-post-ACS1	−3.99	<0.001	<0.001	0.72
	ACS2-ACS3	−2.07	0.039	0.234	0.37
	ACS2-post-ACS1	−3.64	<0.001	<0.001	0.65
	ACS3-post-ACS1	−1.90	0.058	0.348	0.34
Mutual respect	Pre-ACS1-ACS2	−2.08	0.037	0.222	0.37
	Pre-ACS1-ACS3	−3.72	<0.001	<0.001	0.67
	Pre-ACS1-post-ACS1	−4.18	<0.001	<0.001	0.75
	ACS2-ACS3	−2.07	0.039	0.234	0.37
	ACS2-post-ACS1	−3.13	0.002	0.012	0.56
	ACS3-post-ACS1	−1.34	0.180	1.000	0.24
Working toward consensus building	Pre-ACS1-ACS2	−2.01	0.045	0.27	0.36
	Pre-ACS1-ACS3	−3.69	<0.001	<0.001	0.66
	Pre-ACS1-post-ACS1	−4.18	<0.001	<0.001	0.75
	ACS2-ACS3	−2.00	0.045	0.27	0.36
	ACS2-post-ACS1	−2.89	0.004	0.024	0.52
	ACS3-post-ACS1	−2.00	0.046	0.276	0.36

through the different lenses, representing factors that contribute to shaping argumentation (enabling or inhibiting), between the SMTs' ACSs reports. These factors include task characteristics, teaching strategies, student cognitive and affective characteristics, and socio-cultural characteristics.

As depicted in Table 8, the SMTs initially exhibited low levels of interpreting argumentation through the four lenses: Task characteristics (mean = 1.39), Student cognitive characteristics (mean = 2.19), Student affective characteristics (mean = 1.90), and Socio-cultural characteristics (mean = 1.58), while demonstrating moderate levels of interpreting argumentation through the Teaching strategies lens (mean = 2.64). The results displayed in Table 8 indicate statistically significant changes in the SMTs' skills of interpreting argumentation through different lenses over the duration of the course.

Table 9 presents the results of Wilcoxon signed-rank tests for the pairs of the ACSs reports.

As shown in Table 9, there were significant changes in SMTs' interpretation of argumentation through the lenses of task characteristics occurring at each stage of the ACS up to ACS3. No further change was observed between ACS3 and post-ACS1. Significant changes in SMTs' interpretation of argumentation through the lenses of teaching strategies occurred at each stage of ACS up to post-ACS1. Moreover, significant changes in SMTs'

Table 8 Skills of interpreting the argumentation (scale 1–4) Friedman test and descriptive statistics for each SMT's ACS report (pre-ACS1, ACS2, ACS3, and post-ACS1).

		Median	Mean	SD	Friedman test		
					χ^2	df	Asymp. sig.
Task characteristics	Pre-ACS1 report	1	1.39	0.92	51.681	3	<0.001
	ACS2 report	3	2.32	1.19			
	ACS3 report	4	3.36	1.05			
	Post-ACS1 report	4	3.81	0.65			
Teaching strategies	Pre-ACS1 report	3	2.64	1.08	42.234	3	<0.001
	ACS2 report	3	3.32	0.790			
	ACS3 report	4	3.68	0.480			
	Post-ACS1 report	4	3.97	0.18			
Student cognitive characteristics	Pre-ACS1 report	2	2.19	1.07	52.028	3	<0.001
	ACS2 report	3	2.74	0.930			
	ACS3 report	4	3.45	0.810			
	Post-ACS1 report	4	3.97	0.180			
Student affective characteristics	Pre-ACS1 report	1	1.90	1.10	49.216	3	<0.001
	ACS2 report	2	1.94	1.06			
	ACS3 report	3	3.13	1.06			
	Post-ACS1 report	4	3.84	0.37			
Socio-cultural characteristics	Pre-ACS1 report	1	1.58	0.89	56.635	3	<0.001
	ACS2 report	1	1.81	0.91			
	ACS3 report	4	2.65	1.25			
	Post-ACS1 report	4	3.87	0.34			

Table 9 Skills of *interpreting the argumentation* (scale 1–4) Wilcoxon test and effect size for the pairs of ACSs-reports.

	Time	Wilcoxon test			
		Z statistic	Asymp. sig. (2-tailed)	Adjusted-*P*	Effect size (*r*)
Task characteristics	Pre-ACS1–ACS2	−2.98	0.003	0.018	0.53
	Pre-ACS1–ACS3	−4.22	<0.001	<0.001	0.76
	Pre-ACS1–post-ACS1	−4.86	<0.001	<0.001	0.87
	ACS2–ACS3	−2.95	0.003	0.018	0.53
	ACS2–post-ACS1	−3.90	<0.001	<0.001	0.70
	ACS3–post-ACS1	−2.12	0.034	0.204	0.38
Teaching strategies	Pre-ACS1–ACS2	−2.96	0.003	0.018	0.53
	Pre-ACS1–ACS3	−3.78	<0.001	<0.001	0.68
	Pre-ACS1–post-ACS1	−4.27	<0.001	<0.001	0.77
	ACS2–ACS3	−2.52	0.012	0.072	0.45
	ACS2–post-ACS1	−3.47	<0.001	<0.001	0.62
	ACS3–post-ACS1	−3.00	0.003	0.018	0.54
Student cognitive characteristics	Pre-ACS1–ACS2	−2.24	0.025	0.15	0.40
	Pre-ACS1–ACS3	−3.48	<0.001	<0.001	0.63
	Pre-ACS1–post-ACS1	−4.56	<0.001	<0.001	0.82
	ACS2–ACS3	−2.71	0.007	0.042	0.49
	ACS2–post-ACS1	−4.50	<0.001	<0.001	0.80
	ACS3–post-ACS1	−3.12	0.002	0.012	0.56
Student affective characteristics	Pre-ACS1–ACS2	−0.12	0.901	1.000	0.02
	Pre-ACS1–ACS3	−3.37	<0.001	<0.001	0.60
	Pre-ACS1–post-ACS1	−4.58	<0.001	<0.001	0.82
	ACS2–ACS3	−3.52	<0.001	<0.001	0.64
	ACS2–post-ACS1	−4.51	<0.001	<0.001	0.81
	ACS3–post-ACS1	−3.37	<0.001	<0.001	0.60
Socio-cultural characteristics	Pre-ACS1–ACS2	−1.10	0.273	1.000	0.20
	Pre-ACS1–ACS3	−3.40	<0.001	<0.001	0.61
	Pre-ACS1–post-ACS1	−4.88	<0.001	<0.001	0.88
	ACS2–ACS3	−3.12	0.002	0.012	0.56
	ACS2–post-ACS1	−4.79	<0.001	<0.001	0.86
	ACS3–post- ACS1	−3.89	<0.001	<0.001	0.70

interpretation of argumentation through the lenses of student cognitive characteristics, student affective characteristics, and socio-cultural characteristics were evident at each stage of the ACS from ACS2 up to post-ACS1.

The statistical results presented above suggest that the significant changes observed in the SMTs' perception of argumentation—both attending and interpreting—were not confined to the pairs of pre-ACS1 and post-ACS1 (the identical

ACS scenarios). This indicates that the changes observed throughout the course cannot solely be attributed to the repetition of the same ACS, underscoring a broader change in teachers' noticing capabilities.

Examples of changes in SMTs' noticing of argumentation. Below, we provide examples of changes in SMTs' levels of attending to structural aspects, attending to dialogic aspects, and interpreting argumentation, as evidenced in their pre-ACS1 and post-ACS1 reports. These examples were selected based on two key criteria: (1) they illustrate the most frequently observed types of changes in SMTs' noticing and (2) they align with the broader patterns identified in the quantitative analysis (see Tables 4–9), ensuring that these examples are not isolated cases but rather indicative of patterns found across multiple participants. While these examples do not capture all possible variations, they serve as representative cases of the shifts we observed in teachers' noticing.

Change in SMTs' attending to structural aspects of argumentation. Below, we present an example of a change in SMT14's level of *attending to structural aspects* of argumentation from level 1 to level 3, as evidenced in her pre-ACS1 and post-ACS1 reports.

This example illustrates a commonly observed pattern in our dataset, where teachers progressed from recognizing isolated claims and justifications to a more systematic and comprehensive identification of argumentation structures. Similar shifts were documented across multiple participants, contributing to the overall trend of improved attention to structural aspects of argumentation (see Table 4).

In the pre-ACS1 report, SMT 21 wrote:

> Argument 1: Claim: "I think that it's a constant linear function" incorrect claim (line 2)".
> Argument 2: Claim: "The equation $x = -3$ is not a function. Justification: According to the definition of a function: "for each x there is a single unique y," and on the line $x = -3$ that's not true. I drew it in my notebook."
> Argument 3: Claim: "The equation $x = -3$ is not a function. Justification: the slope that they got, $\frac{y1}{0}$, is not defined."

In the post-ACS1 report, SMT14 wrote:

> Argument #1: incorrect claim (line 2): "I think that it's a constant linear. The equation $x = -3$ is a linear function!"
> Justification: line 2; "Based on visual perception. Unaccepted justification in the math class".
> Argument #3: Correct claim (line 23,24): "The equation $x = -3$ is not a function and definitely not a linear function."
> Justification: line 6: "Deductive, based on definition supported by a visual (graphic) representation (line 10). An accepted justification in the math class".
> Justification: "Lines 21, 23; algebraic, based on the formula for finding the slope of a line. An accepted justification in the math class".
> Argument #4: Correct claim (line 27): "The equation $x = -3$ not is a linear function"."
> Justification: "Line 27, based on the formula for finding the slope of a line (algebraic) and the rate of change of the line by drawing a "step" that's the slope (Graphic). An accepted justification in the math class".

For the pre-ACS1 report, SMT21 achieved a score 6 for *attending to structural aspects*:

1. She identified three claims (of the three arguments). A score of 1 received for each claim.
2. She accurately evaluated the correctness of the one claim. Therefore, a score 1 was received.
3. She identified two justifications. Therefore, a score of 1 was received for each justification.

Consequently, her work was rated at level 2. For the post-ACS1 report, SMT21 achieved a score of 16 for attending to structural aspects, correctly identifying and evaluating all claims and justifications presented in the ACS. Consequently, her work was rated at level 3.

Change in SMTs' attending to dialogic aspects of argumentation. Below, we present an example of a change in SMT24's level of *attending to critique of arguments* aspect from level 2 to level 3, as evidenced in her pre-ACS1 and post-ACS1 reports. While this example captures one particular instance of increased attention to *critique of arguments* aspect, similar developments were found in other cases. Many participants showed enhanced ability to describe and evaluate critique in argumentation, aligning with the general trends observed in our analysis (see Table 6).

In the pre-ACS1 report, SMT24 wrote:

> Zuhair did not agree with Adam's argument.

In the post-ACS1 report SMT 24 wrote (abbreviated):

> I identify several cases of critique arguments in the ACS: (1) Zuhair disagreed with Adam's solution "No, Adam, it's not a function at all" (line 4), refuted his argument by using the definition of a function and a graphic justification: … (line 6), "… (line 10). (2) Yana objected to Zuhair's argument "I don't agree with you" (line 11). Zuhair defends his argument based on definition of a function "Come on, Yana, this time there's nothing to argue, this is a definition we learned in 8th grade". Yana and Ali search together for an additional justification to evaluate Zohair's argument. Ali said: Yana thought that if we find the slope of two points, … so I suggested that we use the formula for finding the slope of a line…" (line 21). Maya helped Yana and Ali to explain the result they got: "… we learned that dividing by zero is not defined…" (line 23). Yana conceded her claim following the justification they raised and was convinced by Zohir's argument; she expressed agreement with Zuhair "You're right, Zuhair, it isn't a function. I was wrong". (3) Hala (line 13) also critiqued Zuhair's argument, expressing agreement with Zuhair's argument: "I agree with Zuhair", but without justifying her claim because the teacher did not allow her to. (SMT24)

For the pre-ACS1 report example, SMT24 attained a score of 2 for attending to the Critiquing arguments aspect, as she merely noted that Zuhair disagreed with Adam's argument without providing a detailed description of how the critique was manifested in the situation. Conversely, for the post-ACS1 report example, SMT24 attained a score of 3 for her detailed description of how the Critiquing arguments aspect was manifested.

Change in SMTs' interpreting argumentation. Below, we present an example of a change in an SMT20's level of interpreting argumentation through the lens of 'teaching strategies', improving from level 3 in the pre-ACS1 report to level 4 in the post-ACS1 report. This example illustrates how SMTs began to integrate theoretical insights and teaching experience into their interpretations. While the depth and focus of these interpretations varied across participants, this example reflects a trend toward more evidence-based and theoretically grounded interpretations, as supported by the quantitative findings (see Table 8).

In the pre-ACS1report, SMT20 referred to the factor of teaching strategies in her interpretation:

> The teacher in the scenario helped to promote argumentation. She asked students to give justifications, encouraged them to participate, and allowed them to come to the board and present their solutions (for example, lines 5, 7, 9, 14). Furthermore, the teacher did not assess the students' arguments nor tell the students if their ideas were correct or not. That helped the students to delve deeper and discuss the ideas that arose. On the other hand, the teacher hindered argumentation at times. For instance, when Hala asked to explain why she agreed with Zuhair's argument, the teacher didn't give her the opportunity to speak and instead asked Yana to continue (line 14). Even after Yana gave her justification, the teacher did not return to Hala (SMT20).

In the post-ACS1 report, SMT20 wrote (abbreviated):

> According to Conner et al. (2014), there are several kinds of questions that the teacher can use to support collective argumentation (requesting evaluation, requesting an idea, etc.) and other supportive actions (directing, promoting, …). Such kinds of questions and supportive actions are clearly expressed in this ACS: In the pre of the situation, the teacher *challenged* the students with a question that was both interesting and confusing (line 1). The student Adam raised an incorrect argument, "we can clearly see that it's a straight line …" (line 2), and the student Zuhair disagreed with him, "No, Adam, it's not a function at all." (line 4) … The teacher asked him to explain his claim (*requesting an explanation*) "Can you please explain your argument?" (line 5), Zuhair acquiesced to the teacher's request and supplied a verbal explanation "According to the definition of a function: …" (line 6), but the teacher asked him to expand on it by drawing on the board (*requesting a method*) "Can you please draw it on the board?" (line 7). Zuhair's initial response was to decline the teacher's request, *but the teacher encouraged him* (*promoting*), using words that increased his motivation, "Come on, don't be shy…" (line 9). Zuhair acquiesced to the teacher's request and expanded on his justification by drawing an explanation on the board [Zuhair draws on the board and explains], "According to the definition of a function …" (line 10) …
>
> In addition, after Zuhair presented his solution and drew on the board, *the teacher did not assess Zuhair's solution*, and that gave the students the opportunity to assess his solution and develop an argumentative dialog around it (from my experience as a teacher, if the teacher had been satisfied with Zuhair's correct verbal argument, and not paid attention to the assessment of the rest of the students, it could be that the students would have immediately agreed with Zuhair because the teacher showed that she agreed with him, and the exercise would have ended there)…

In the pre-ACS1 example, SMT20 addressed the factor of teaching strategies, noting how certain teacher actions either facilitated or hindered argumentation within the ACS, but provided minimal evidence to back her claims, merely referencing lines from the text. By contrast, in the post-ACS1 example, SMT20 provided robust evidence to support her claims, using Conner's framework (2014) to analyze the teacher's actions depicted in the ACS and providing evidence from the ACS itself to explain how these actions contributed to the students' engagement in argumentation. Additionally, she incorporated insights from her own teaching experience to enrich her interpretation.

This example represents a broader trend observed in our data, in which SMTs' interpretations of argumentation became more grounded in theoretical perspectives, teaching experience, and textual evidence from the ACS. While not all participants reached this level of interpretation, many demonstrated a shift towards more structured and theory-informed analyses, as reflected in the general patterns identified in Table 8.

Thematic analysis of SMTs' written reflections and interviews. This section examines the factors contributing to changes in SMTs' noticing of argumentation, from their perspective (RQ2). The thematic analysis of written reflections and interview transcripts revealed four themes, each reflecting a different way in which SMTs engaged with course activities to develop their noticing. These themes highlight both shared learning experiences and variations in how SMTs interpreted and applied what they learned.

Although all SMTs participated in similar course activities, their responses indicate that the way these activities influenced their noticing varied. For instance, some SMTs found that working with ACSs helped them better recognize structural aspects of argumentation, such as identifying claims and justifications, whereas others focused more on dialogic interactions, such as critique and collaborative reasoning. Similarly, group discussions enabled some SMTs to refine their interpretations of teacher actions, while others used them to explore how students' emotions and classroom norms shape argumentation.

To structure these findings, we grouped the themes into two overarching categories: (1) Factors inherent to the activity – Three themes related to the artifacts used in the course, group and whole-class discussions, and the literature on argumentation; (2) Incorporating teaching experiences into the activity—One theme capturing how SMTs connected their own classroom experiences to their engagement with ACSs. Each theme is illustrated with a representative example, demonstrating how SMTs engaged with these influences. While a single example is provided for each theme, we acknowledge that SMTs' responses varied. Throughout our analysis, we highlight these variations to provide a nuanced understanding of the different ways in which SMTs' noticing developed.

Themes related to factors inherent to the activity. SMTs identified three key factors inherent to the course activities that contributed to their development in noticing argumentation: engaging with a diverse set of ACSs, participating in group and whole-class discussions, and exposure to literature on argumentation. Table 10 presents these themes, including the number of SMTs who addressed each theme and a description. Representative examples of teachers' responses (abbreviated), along with short analyses, are provided in Appendix 1 in the Supplementary Material. These examples illustrate both commonalities and variations in how these factors supported changes in SMTs' noticing of argumentation. While the examples do not represent all possible responses, they were selected to illustrate common patterns in how SMTs engaged with these aspects of the course. At the same time, SMTs' responses indicate variation in how these factors influenced their noticing, depending on their prior experiences, perspectives, and areas of focus. For instance, in the theme addressing the role of group discussions, SMTs reported different ways in which these interactions supported their noticing. Some

Table 10 Themes related to factors inherent to the activity.

Theme	# of SMTs (N = 31)	Description of the theme
Theme #1: Developing knowledge and analytical skills for noticing argumentation through engagement with ACSs	31	From SMTs' perspective, engaging with ACSs refined their understanding of argumentation and supported their noticing by providing concrete classroom situations for analysis. SMTs reported that ACSs helped them identify and differentiate claims and justifications, while others noted that ACSs enhanced their awareness of dialogic aspects, such as critique and co-construction of arguments. Additionally, some SMTs reported that ACSs helped them recognize and understand the factors influencing argumentation. This engagement deepened their ability to analyze argumentation more precisely.
Theme #2: Enhancing analytical skills for noticing argumentation through the application of theoretical frameworks	30	From SMTs' perspective, engaging with theoretical frameworks supported the refinement of their noticing skills by providing structured tools for analyzing argumentation. SMTs reported that applying these frameworks helped them interpret classroom interactions more systematically, identify patterns in students' reasoning and teacher facilitation, and evaluate the effectiveness of different argumentative moves. While some SMTs focused on using theoretical constructs to assess teacher actions, others applied them to better understand students' justifications and dialogic interactions.
Theme #3: Enhancing analytical skills for noticing argumentation through small-group and whole-class discussions	30	From SMTs' perspective, participating in small-group and whole-class discussions helped refine their noticing of argumentation by exposing them to multiple interpretations and perspectives. Through peer interactions, SMTs reported gaining new insights into teacher and student actions, recognizing alternative ways to interpret argumentation processes, and refining their analytical reasoning. While some SMTs emphasized that discussions helped them better understand teacher facilitation strategies, others highlighted how these interactions increased their awareness of students' reasoning, emotions, and classroom norms that shape argumentation.

emphasized that peer discussions refined their interpretations of teacher actions, helping them better understand instructional strategies that facilitate argumentation. Others highlighted how discussions enhanced their awareness of students' emotions and classroom norms, recognizing how these social and affective dimensions impact argumentation processes. Similarly, SMTs who engaged with literature on argumentation reported that it shaped their noticing in different ways: some used theoretical frameworks to analyze teacher moves, while others found the readings most valuable for understanding the structure of students' justifications. These variations suggest that while the identified factors were consistently meaningful for SMTs' learning, their influence was not uniform. Instead, the way each SMT engaged with these elements was shaped by their individual backgrounds and focus areas.

Theme related to incorporating their teaching experiences into the activity. Beyond engaging with course activities, SMTs reported that connecting their noticing of argumentation to their own teaching experiences played a key role in refining their analytical skills. By reflecting on classroom situations they had previously encountered, SMTs were able to analyze ACSs through the lens of their professional experience, recognize patterns in student reasoning and teacher facilitation, and interpret classroom interactions in a more nuanced way.

While almost all SMTs drew connections between ACSs and their teaching, they did so in different ways. Some SMTs used their prior instructional knowledge to compare ACSs to real classroom discussions they had observed, while others engaged in critical reflection on past teaching situations, reinterpreting events through the perspective of argumentation.

Table 11 presents this theme, along with the number of SMTs who addressed it and a description. An abbreviated example of a teacher's response, illustrating how SMTs connected their noticing to their teaching experiences, along with a short analysis, is provided in Appendix 2 in the Supplementary Material. The example reflects a common approach; responses varied in focus, with some teachers emphasizing how their experience helped them recognize teacher moves, while others focused on how it shaped their understanding of student engagement in argumentation.

Discussion and conclusion

This study examined the changes in SMTs' noticing of argumentation (RQ1) and the factors that contributed to these changes from their perspective (RQ2) within the context of their participation in an ACS-based course. The findings reveal that over time, SMTs demonstrated significant improvements in both attending to and interpreting argumentation. Specifically, they became more adept at identifying mathematical claims, justifications, and dialogic interactions, as well as interpreting argumentation through multiple lenses (e.g., teaching strategies, student characteristics, and classroom norms). Thematic analysis of teachers' reflections and interviews further indicated that SMTs attributed these changes to four key factors: (1) engaging with ACSs, (2) applying theoretical frameworks, (3) participating in collaborative discussions, and (4) connecting their learning to their teaching experiences. These findings provide insights into how professional development activities can foster teachers' noticing of argumentation and contribute to their instructional awareness.

Changes in SMTs' noticing of argumentation. Previous studies that did not focus on argumentation noted developments in teachers' noticing of various classroom features after their participation in teacher education programs utilizing classroom

Table 11 A theme related to incorporating teaching experiences into the activity.

heme	# of SMTs ($N = 31$)	Description of the theme
Theme #4: Enhancing analytical skills for noticing argumentation through connections to personal teaching experiences	28	From SMTs' perspective, connecting their noticing of argumentation to their own teaching experiences helped refine their ability to analyze classroom interactions more critically and meaningfully. Reflecting on familiar instructional situations enabled them to identify patterns in argumentation, interpret teacher and student moves, and recognize factors influencing the development of argumentation in ways that felt directly relevant to their practice. While some SMTs used their experiences to compare ACSs to real classroom discussions they had observed, others revisited past instructional situations, interpreting them differently in light of their engagement with argumentation-focused analysis. The table provides an example illustrating one way in which SMTs used their teaching experiences to enhance their noticing, though responses varied in focus.

scenarios (e.g., Star et al. 2011; van Es and Sherin, 2008). Our findings align with these studies, demonstrating significant improvements in SMTs' noticing of argumentation following their involvement in a professional development course. A particularly notable improvement was observed in teachers' ability to attend to the structural elements of argumentation. By the course's conclusion, especially in ACS3 and post-ACS1, SMTs were able to identify all or nearly all of the arguments presented in the classroom transcripts, including mathematical claims and justifications. Similarly, SMTs exhibited a more refined ability to attend to dialogic aspects of argumentation, such as identifying student critiques and co-construction of arguments. These findings suggest that teachers attained a higher level of proficiency in noticing core components of mathematical argumentation, an essential skill for fostering productive mathematical discourse (Weber et al. 2008).

Since structural and dialogic aspects of argumentation are both critical for supporting student learning and deeper mathematical understanding, teachers' increased attention to these elements is particularly meaningful. However, prior literature highlights a persistent tension in teachers' instructional decision-making between promoting student participation and adhering to mathematical standards for valid arguments (Ayalon et al. 2022; McClain and Cobb, 2001; Nathan and Knuth, 2003; Staples, 2007). While our study documents changes in SMTs' noticing of argumentation, we did not find evidence that teachers explicitly grappled with or modified how they manage this tension in their instructional reasoning. Further research is needed to explore whether and how teachers learn to balance student engagement and mathematical rigor in their facilitation of argumentation.

Beyond attending to argumentation, SMTs also demonstrated growth in their ability to interpret argumentation through multiple lenses. Prior research by Ayalon and Nama (2024) explored what SMTs referred to when asked to envision argumentation in their classroom. Their responses predominantly referenced two factors: task characteristics and teaching strategies, with only minimal or almost no mention of student characteristics and socio-cultural aspects. A similar pattern emerged at the start of our course, where most participants primarily focused on teaching strategies when interpreting argumentation. However, by the end of the course, SMTs demonstrated more nuanced interpretations, considering multiple interrelated factors, including student reasoning, emotions, and classroom norms, when analyzing argumentation within ACSs. The factors used by teachers in their interpretations are directly linked to teaching practices that foster opportunities for students to engage in argumentation (e.g., Mueller et al. 2012; Staples, 2014). Thus, the teachers' discussions of these factors and their influence on shaping the argumentation situation are of significant importance.

This focus on noticing diverse aspects of argumentation—both structural and dialogical—along with the factors influencing it, supports teachers in recognizing their critical role in facilitating argumentation. As Kosko et al. (2014) noted, teachers' perceptions of their role in supporting argumentation often conflict with reform expectations, with some assuming that argumentation requires minimal scaffolding. The ACS-based course directly addresses this misconception by emphasizing the deliberate and active role of the teacher in fostering effective argumentation.

Factors contributing to the change in noticing. Our research design does not allow us to definitively state the causes behind the observed changes in participants' noticing of argumentation. However, thematic analysis of teachers' reflections and interviews provides insights into the factors they identified as contributing to these changes. These factors fall into two main categories: (1) Three factors inherent to the course activities, including (a) engaging with ACSs, (b) applying theoretical frameworks, and (c) participating in collaborative discussions, and (2) One factor related to teachers' own classroom experiences, namely, connecting their noticing of argumentation to their prior teaching experiences. Notably, the SMTs did not report any factors that they felt hindered the change in their noticing.

One of the strongest themes in teachers' reflections was the role of ACSs in enhancing their noticing of argumentation. Many SMTs emphasized that engaging with multiple ASCs allowed them to explore various aspects of argumentation and continually reassess its diverse elements. Their feedback supports our intention of using various transcripts from different classrooms covering multiple mathematics topics, as this approach was intended to train teachers to notice argumentation in diverse instructional settings. Prior research confirms that using structured representations of classroom interactions—whether in video or text-based formats—can effectively support teacher noticing by directing attention to key pedagogical moments (Santagata et al. 2021; Scherrer and Stein, 2013). In our study, we employed various transcripts from different classrooms covering multiple mathematics topics for two main purposes. Firstly, the transcripts allow us to 'observe' entire sequences of moves while enabling the analysis of each individual move within the context of its sequence (Scherrer and Stein, 2013). This approach is beneficial for highlighting the discussion features we aim to promote. Secondly, various transcripts from different topics in mathematics and different classrooms afford teachers the opportunity to apply noticing skills of argumentation across diverse educational settings. The teachers' reflections confirm that these experiences deepened their ability to differentiate claims and justifications, identify critical moments of critique, and recognize the role of teacher facilitation in shaping argumentative discussions.

Another key factor identified by teachers was the role of theoretical frameworks in refining their noticing. Teachers reported that exposure to research on argumentation improved their ability to analyze ACSs more systematically. Specifically, they noted that reading and discussing literature on teacher moves (e.g., Conner et al. 2014) helped them recognize instructional strategies that promote or hinder argumentation. This aligns with prior research showing that theoretical knowledge enhances teacher noticing, as it provides structured tools for making sense of classroom interactions (Bruckmaier et al. 2016; Dreher and Kuntze, 2015; König et al. 2014).

The primary goal of exposing teachers to theoretical perspectives on argumentation was to deepen their knowledge in this area, thereby enhancing their ability to notice argumentation. Moreover, engaging teachers in analyzing ACSs with the help of established analytical frameworks (e.g., Conner et al. 2014) aimed to enhance, extend, and refine the teachers' analysis. Teachers reported that reading and discussing scholarly articles on argumentation significantly boosted their understanding, which in turn improved their ability to notice argumentation. This finding is consistent with existing research, which suggests that teachers' knowledge positively influences their noticing skills (e.g., Bruckmaier et al. 2016; Dreher and Kuntze, 2015; König et al. 2014; Meschede et al. 2017; Schoenfeld, 2011). In our study, teachers described using theoretical insights in two key ways: Refining their interpretation of teacher actions, such as identifying when teacher prompts encouraged student reasoning versus when they constrained discussion and applying research-based criteria to evaluate the quality and structure of student arguments. These findings highlight the importance of integrating theoretical perspectives into professional development to help teachers notice and interpret argumentation in more nuanced ways.

Engaging in small-group and whole-class discussions, involving consolidating analyses, critiquing evidence from the ACSs, and incorporating theoretical insights and personal teaching experiences, was another critical factor that teachers identified as supporting their development in noticing. This approach is supported by research emphasizing the benefits of sharing and critiquing educational experiences to improve noticing, particularly in mathematical contexts (van Es, 2012; van Es and Sherin, 2008). Moreover, engaging teachers in critiquing peers' analyses exposes them to diverse ideas and perspectives, which enhances their ability to discern quality differences in argumentation (Ayalon and Wilkie, 2020; Nama and Ayalon, 2024). Collectively, these activities contributed to the development of teachers' noticing skills during the course, as reported by the participants.

Finally, many teachers described how reflecting on their own classroom experiences helped them refine their noticing of argumentation. Some teachers explicitly compared the ACSs to real classroom interactions they had observed or facilitated, while others used their past experiences to rethink how they approached student participation in argumentation. While previous studies have examined teachers' reflections on argumentation (e.g., Kosko et al. 2014), our findings indicate that teachers actively used their past experiences to refine their noticing of argumentation in classroom interactions. Specifically, SMTs reported that connecting ACSs to real instructional challenges—such as student participation barriers or difficulties in structuring arguments—allowed them to develop a more nuanced understanding of how argumentation unfolds in classroom settings.

Overall, the themes identified in this study reflect well-documented elements of effective professional development identified in the literature. These include the use of structured representations of classroom instruction (e.g., Santagata et al. 2021), engaging teachers with theoretical perspectives on argumentation (e.g., Bruckmaier et al. 2016; Dreher and Kuntze, 2015), encouraging collaborative discussions to refine noticing (e.g., van Es, 2012; van Es and Sherin, 2008), and integrating teachers' personal classroom experiences into their analysis (e.g., Bastian et al. 2022; Gold and Holodynski, 2017).

A unique contribution of this study is its evidence for the role of ACSs in enhancing teacher noticing, as reflected in the participants' reflections. As one teacher noted:

> The argumentation situations offered valuable insights into the various aspects of argumentation and the factors that influence it. Through clear and concise dialog, they enabled me to focus on these key elements.

Another teacher highlighted how the variety of ACSs expanded their perspective:

> The written event provided me with the opportunity to read it several times, each time trying to identify additional aspects related to argumentation. Furthermore, each event presented a different situation focused on a different topic in mathematics, which contributed to broadening my noticing skills.

These reflections reinforce the idea that exposure to diverse, well-structured argumentation scenarios is a powerful tool for developing teachers' noticing, an insight that holds promise for future teacher education initiatives.

Contribution, future directions, and limitations. This study did not investigate teachers' subsequent implementation of argumentation in their classroom practice. Further research is needed to clarify and strengthen the links between teachers' noticing of argumentation and their practical teaching methods. Although classroom implementation was beyond the scope of this study, we are encouraged by reports from some participants who experimented with argumentative activities in their classrooms during or after the course. These teachers described being more attuned to students' reasoning and argumentation structures, suggesting that their enhanced noticing had an impact on their instructional awareness. However, a systematic examination of this transition—from noticing to practice—is needed.

This research contributes to the literature on professional learning by demonstrating the potential of an ACS-based course to enhance teachers' noticing of key argumentation practices in mathematics. While prior studies have examined general teacher noticing, this study provides an in-depth exploration of how teachers develop noticing specifically in relation to argumentation—an aspect of mathematics instruction that remains challenging to implement effectively. Our findings highlight how multiple dimensions of teacher learning (e.g., engagement with structured instructional representations, theoretical frameworks, collaborative discussions, and reflective teaching connections) contribute to developing noticing skills in argumentation.

In addition to these conceptual contributions, this study also offers practical tools for mathematics teacher education. By developing and refining tools such as ACSs, ACS rubrics, and structured report formats, we provide resources that can be used in professional development and teacher education programs to support teachers' ability to analyze, assess, and ultimately implement argumentation in their classrooms. These tools allow teachers to systematically explore and assess a variety of ACSs across different mathematical topics and instructional settings, thereby strengthening peer-assessment, critical analysis, and collaborative professional learning (Nama and Ayalon, 2024). Mathematics teacher educators can use these tools to structure discussions around how argumentation unfolds in classrooms, helping teachers move beyond general pedagogical insights toward specific instructional strategies that support student reasoning.

One limitation of this study is the uncertainty of the duration of improvement in noticing. Specifically, it is unclear whether, or for how long, teachers will maintain a high level of noticing when analyzing different ACSs after the course. Future research should investigate whether, and under what conditions, teachers maintain a high level of noticing when analyzing different ACSs beyond the course setting. Another potential limitation of this study is the use of the same ACS (ACS1) in both the initial and final phases of the course. As a result, the shift in noticing may have been influenced by the repeated exposure to the same ACS. Future studies should explore alternative methods for assessing long-term growth in noticing, such as introducing entirely new ACSs in post-course assessments. Finally, this study did not examine an additional skill of teacher noticing, namely, responding. Participants were asked to describe what they would do in the teacher's place in each ACS to promote argumentation. We are currently analyzing the teachers' responses to understand the characteristics of the teaching alternatives they proposed, how these alternatives relate to their attentiveness and interpretations of the argumentation in the ACSs, and whether and how their proposed responses evolve alongside the development of their other noticing components.

By addressing these questions, future research can deepen our understanding of how teacher noticing of argumentation develops, how it connects to classroom practice, and how professional development can be designed to support sustained learning and instructional change.

Data availability

The datasets generated and/or analyzed during this study are not publicly available due to compliance with general data protection regulations (GDPR). As a result, we are unable to share the data publicly, as it contains sensitive information that could potentially compromise the privacy of individuals involved in the study. However, we are open to sharing relevant parts of the data upon reasonable request, provided that proper confidentiality agreements and ethical clearances are established. Please let us know if you need any further clarification on this matter.

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Author contributions

Michal Ayalon and Samaher Nama: Conceptualization. Samaher Nama and Michal Ayalon: Methodology, writing—original draft, writing—review and editing. Samaher Nama: Data collection and data analysis.

Competing interests

The authors declare no competing interests.

Ethical approval

The methodology for this study was approved by the Human Research Ethics Committee, Faculty of Education, University of Haifa (approval number 485/20). The research was performed in accordance with relevant guidelines/regulations applicable when human participants are involved—Department of Health and Human Services, USA. Approval was issued on 15 November 2020 and is valid for four years (15 November 2020–14 November 2024). https://research.hevra.haifa.ac.il/ethics.

Informed consent

Written informed consent was obtained from all participants in this study before data collection, via signed agreements collected by the first author between 1 and 10 March 2022. Data collection occurred between 11 March and 10 June 2022. The consent covered participation, data use, and consent to publish. Participation was voluntary. All participants were fully informed that their anonymity was assured, why the research was being conducted, how their data would be utilized, and that there were no risks to them in participating. They were also informed that they could withdraw from the study at any time without consequences.

Additional information

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