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The Ising model celebrates a century of interdisciplinary contributions

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The centennial of the Ising model marks a century of interdisciplinary contributions that extend well beyond ferromagnets, including the evolution of language, volatility in financial markets, mood swings, scientific collaboration, the persistence of unintended neighborhood segregation, and asymmetric hysteresis in political polarization. The puzzle is how anything could be learned about social life from a toy model of second order ferromagnetic phase transitions on a periodic network. Our answer points to Ising's deeper contribution: a bottom-up modeling approach that explores phase transitions in population behavior that emerge spontaneously through the interplay of individual choices at the micro-level of interactions among network neighbors.

The centennial of the Ising model offers an invitation to celebrate the historical contributions of interdisciplinary scholarship at the intersection of the social and physical sciences. Social physics – the study of statistical regularities in human behavior and social interaction – dates back at least to Quetelet's *Physique Sociale*[1]. Although individual actions are highly idiosyncratic, Quetelet introduced the idea, later codified by Durkheim[2] as "social facts," that population level aggregates – like crime rates, suicide rates, and marriage rates – vary systematically with other aggregate measures across cultures, regions, and time periods. This macroscopic quantitative approach to the study of social life was transformative and provides the backdrop against which the interdisciplinary contributions of the Ising model can be appreciated[3].

The Ising model was invented in 1924 by the physicist Ernst Ising[4], using a concept suggested by his doctoral supervisor Wilhelm Lenz. The original model assumed an undirected chain-like one-dimensional lattice in which a single thermal excitation can break the long-range order, hence no spontaneous magnetization or phase transition can occur at a non-zero temperature. Twenty years later, Lars Onsager discovered that a phase transition is possible with a two-dimensional periodic lattice, such that spontaneous order can arise at low temperatures and the decay of magnetization with rising temperature is continuous, as empirically observed[5]. In general, phase transitions in physical systems occur when a small change in an external parameter (such as temperature) causes a system-wide qualitative change, and it can be also discontinuous as when liquid water turns to ice.

Lenz and Ising assumed that atoms possess a quantized dipole magnetic moment (a magnetic spin that can be up or down), citing calculations of Otto Stern from 1920 who had shown that magnetic moments of molecules in paramagnetic crystals cannot oscillate in all directions since that would contradict the Curie law for magnetic susceptibility[6]. The model was limited to a dipole moment directed along one axis, which Renfrey Potts[7] later generalized to any number of mutually orthogonal states. The level of magnetization is the difference between the number of up and down spins. Lenz and Ising anticipated that spins can interact between nearest neighbors in a magnetic crystal, in which interaction is limited to sites that are directly adjacent. Although the nature of these interactions was unknown to Lenz and Ising, they postulated that the energy of a parallel configuration (in which adjacent neighbors spin in the same direction) was lower than the energy of an antiparallel configuration. Energy minimization tends to align the spins S_i in a parallel configuration, while thermal noise introduces random perturbations that disrupt order, with the potential for spontaneous transition between ferromagnetic and paramagnetic phases. Energy is given by

$$H(S) = -\sum J_{ij} S_i S_j \qquad (1)$$

where J_{ij} quantifies the strength of the interaction between nearest neighbors i and j and $S_i \in \{-1, 1\}$ corresponds to top/down directions of the dimensionless spin S_i located at the site i.

The physics of social life

Applications of the Ising model are not limited to ferromagnets. Physicists have used lattice networks with nearest-neighbor influence to model non-magnetic critical behaviors in a broad variety of materials, from glasses to gases. Advances in statistical physics attracted the attention of chemists, biologists, and computer scientists as well, inviting speculation about the

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universality of Ising systems as a lawful regularity in the self-organization of phase transitions.

Inevitably, that speculation extended to the dynamics of opinion bifurcation in social systems[8]. Nevertheless, crossing the disciplinary chasm between the natural and social sciences confronted two ostensibly crippling limitations. First, unlike the orderly behavior of identical individual atoms with binary states, the highly nuanced behavior of heterogenous human individuals arises out of complex and often poorly understood cognitive, emotional, and physiological processes. Like temperature in ferromagnetic systems, these unknowns can produce highly unpredictable and fluctuating individual behavior but the idiosyncrasies cannot be assumed to be random. Second, complex social networks hardly resemble the static, undirected, regular lattice of the Ising model or the random or complete graphs needed to simplify the mathematics of pairwise human interaction.

The unavoidable puzzle is how anything could be learned about social life from a toy model of second order ferromagnetic phase transitions on a periodic network. The puzzle is compounded by what Stauffer and Schulze call the "scientific segregation" of the social and physical sciences that is evident in the parallel development of the Ising model and Schelling's classic model of residential segregation: "[W]e see here not only residential segregation, but also scholarly segregation, with physicists ignoring the Schelling model until recently, and sociologists ignoring the similarity of the Schelling to the Ising model until now."[9:2]

Nevertheless, analogs of atomic spins have yielded important insights into the qualitative dynamics of cascades and spontaneous phase transitions in human populations: the critical mass in influence maximization (i.e., how concentrated do influencers need to be in order to trigger an adoption cascade)[10,11], the detection of bot-assisted influence campaigns[12], the institutional procedures for collective decision making[13], the temporal dynamics of co-authorship networks[14], the evolution of language[15], the spread of tax evasion[16,17], volatility[18] and phase transitions[19] in financial markets, mood swings[20], the theory of social impact[21–23], the dynamics of group membership[24], the persistence of unintended neighborhood segregation[25,26], and the dynamics of political polarization[27]. Our review is focused on these last two closely related applications, polarization and segregation, each with immediate relevance to both basic and applied research.

Neighborhood segregation and the micromotives of macrobehavior

Working without the benefit of personal computers, Schelling[25] used red and blue poker chips on a checkerboard to explain the self-organization of residential racial segregation in U.S. cities, even in the absence of exogenous pressures like housing costs, racial redlining, and vitriolic intolerance. Although Schelling was unaware of the Ising model, Stauffer[28:473] argues that the differences were superficial "and his model was therefore more complicated than needed." Nevertheless, his checkerboard model captures Ising's essential principles: *local attraction to and repulsion from nearest neighbors on a spatial network, relative to an external field operating on each individual.*

Schelling posited neighbors on a square lattice with a discrete binary choice to stay or move that is triggered by a critical out-group proportion among nearest neighbors, which Schelling assumed reflected only a very modest out-group intolerance. Dissatisfied neighbors move out of the neighborhood to the nearest empty cell that has an acceptable chromatic distribution. The departure leaves the previous in-group neighbors more likely to follow suit, corresponding to the "white flight" observed in real-world neighborhoods, which Schelling also modeled analytically as a cascade[29]. Follow-ups to Schelling showed that segregation could be obtained even in a multiculturalist population that strictly preferred diversity, due to the weak stability of multicultural configurations when there is a small asymmetry in the aversion to having all out-group neighbors compared to all in-group[30,31].

The importance of Schelling's demonstration goes well beyond the explanation of residential segregation and highlights the deeper contribution of the Ising model to the science of social behavior—a contribution that is captured in the title of Schelling's 1978 book *Micromotives and Macrobehavior* in which his earlier 1971 paper on "the interactive dynamics" of self-organizing segregation is revisited. His segregation model illustrates the book's title by showing how a macro-behavioral pattern of segregation can emerge spontaneously among individuals responding to racially tolerant micromotives.

More broadly, the title encapsulates an approach to the study of social life that complements Durkheimian "social facts" about the macro-behavioral properties of populations that can be studied using cross-cultural comparisons. The Ising model explores the microfoundations of population behavior by showing 1) how spontaneous order (i.e., a systematic, observable, macro behavioral pattern) can emerge internally through what Schelling[25] described as "the interplay of individual choices" at the micro-level of interactions among network neighbors, and 2) how the emergence of spontaneous order through the dynamics of interaction can be highly non-linear, involving phase transitions characterized by qualitative changes in the macro-behavioral patterns.

Abelson's puzzle

The Ising model of local influence on a spatial network also led to novel insights into the dynamics of partisan polarization, a problem closely related to neighborhood segregation. Polarization refers to a process that divides a population into nearly disjoint clusters (or "echo chambers") where communication and agreement are far more likely within clusters than between, disagreement is far more likely between than within, opinions and beliefs are relatively extreme, the alignment of opinion spreads from political to cultural differences (including lifestyle choices and consumer preferences), and partisans identify with their in-group and despise those in the out-group.

Several years before Schelling modeled the emergence of segregation on a spatial network, Robert Abelson showed mathematically that models in which agents are influenced by the opinions of others move inexorably toward convergence around a single opinion. "Since universal ultimate agreement is an ubiquitous outcome of a very broad class of mathematical models," Abelson[32:153] pondered, "we are naturally led to inquire what on earth one must assume in order to generate the bimodal outcome of community cleavage studies."

Three decades later, Axelrod[33] rehearsed Abelson's puzzle: "If people tend to become more alike in their beliefs, attitudes, and behavior when they interact, why do not all such differences eventually disappear?" Axelrod's model of "local convergence and global polarization" retained the Ising model's regular lattice, but with pairwise adoption of a single neighbor's dissimilar trait, an updating rule used in the Voter model[34], another equally prominent application of Ising dynamics. Axelrod also replaced static isotropic nearest-neighbor influence with the widely observed tendency for homophily in dynamic networks, in which "birds of a feather flock together." The greater the pairwise similarity between agents, the stronger their social tie, and the stronger the tie, the higher the probability to copy one of the neighbor's dissimilar traits. At each time-step, an agent and one of its neighbors are randomly selected for updating. With a probability equal to the proportion of shared cultural features, the agent adopts one of its neighbor's traits on which they differ. The process continues until all pairs of neighbors are either identical or completely different across all cultural dimensions.

Flache and Macy[35:970] highlighted Axelrod's counter-intuitive insight: "Homophily generates a self-reinforcing dynamic in which similarity strengthens influence and influence leads to greater similarity. This might appear to merely strengthen the tendency towards global convergence." Instead, Axelrod found that convergence can remain local, leading to global diversity, where "the number of stable homogeneous regions decreases with the number of features, increases with the number of alternative traits per feature, decreases with the range of interaction, and (most surprisingly) decreases when the geographic territory grows beyond a certain size."

Axelrod's model can be interpreted as a process of "cultural speciation," in which influence is not possible between individuals that have absolutely nothing in common. However, when Klemm et al.[36] introduced epsilon temperature into Axelrod's model, the results were dramatic: cultural drift leading to the collapse of diversity, a result also reported by Castellano[37] using an Ising model with Glauber dynamics and by Flache and Macy[35]. Temperature in the Ising model corresponds here to behavioral randomness, caused by decision-making that includes an idiosyncratic input along with social influence from network neighbors. The idiosyncratic input can be partially non-deterministic since it depends on the complexity of human cognition. These perturbations can create bridges across the divides that demarcate cultural regions when an agent randomly adopts a cultural trait that is shared with a neighbor with whom they previously had nothing in common. The shared trait restores a positive probability of future influence, thereby destabilizing a previously metastable configuration characterized by multiple cultural islands, leading to globally stable monoculture. This result shows the importance of the temperature parameter in the original Ising model, which has generally been neglected in social science applications that assume stochastic decision making (as in Axelrod's model of behavioral propensity) but do not bound choice propensities away from the limits of probability.

Following Klemm et al.[36], Centola et al.[38] revised Axelrod's model by extending the principle of homophily beyond the preference to interact with similar neighbors, to include the Schelling-like possibility to interact outside the confines of the agent's current location on a spatial network. While Axelrod's homophily limits interaction to nearest neighbors based on threshold values for both cultural and spatial proximity, Centola et al. relax spatial proximity as a second requirement for interaction. Unlike the Schelling model, these agents cannot change neighborhoods; nevertheless, neighbors with nothing in common can break their tie and search randomly for a replacement from anywhere on the lattice, including a member of a cultural region that would otherwise be inaccessible due to spatial instead of cultural distance.

An even simpler solution to Abelson's puzzle is the Ising model's assumption that local reactions can be negative as well as positive. When asked to define "social influence," a naive response will point to processes like persuasion and conformity. However, influence can also be aversive, in which the interaction between neighbors who disagree on most issues leads them to move farther apart on an issue on which they previously happened to agree.

A signed graph is a key assumption in Hopfield's Ising-derived recurrent neural network model of associative memory[39]. Like the atoms with up and down spins in the Ising model, Hopfield's neurons have dipolar states of 1 (when the neuron fires) or −1 (when it does not), and the state of a neuron depends on the states of its network neighbors, with the neural analog of both ferromagnetic and antiferromagnetic properties, depending on a signed pairwise edge parameter, usually denoted J_{ij} in Ising models and w_{ij} in Hopfield applications. For $J_{ij} > 0$ in the Ising specification, energy minimizes in the parallel configuration, but if $J_{ij} < 0$, neighboring spins prefer to point in the opposite direction of their neighbors. In both models, neighbors i and j will either tend to converge (i.e., point in parallel) or to diverge (in an antiferromagnetic configuration), depending on the sign of the edge-weight parameter.

Following Hopfield, psychologists have applied the Ising model to psychopathology, with mental disorders conceptualized as a stable configuration of interacting symptoms that are susceptible to phase transitions[40]. The Ising approach has also been applied to attitude dynamics, modeled as an entropy-based dynamic network of feelings, cognitive representations, beliefs, and behaviors[41]. Belief networks can be modeled within-individual, where the edges correspond to an undirected elective affinity, such that longitudinal change in one node will be associated with change in an adjacent belief[42,43]. Belief networks have also been modeled between-individual as the cultural projection of a bipartite social network, where the edges correspond to the co-occurance of attitudes and beliefs, as might be measured using survey data[44].

Xenophobia in dynamic networks

A key difference between Hopfield and Ising is that w_{ij} is updated while J_{ij} is not. Unlike the Ising model's fixed edges between nearest neighbors on a regular lattice, the Hopfield model assumes a complete undirected graph with signed dynamic edge weights. Any network node in the Hopfield model has the potential to be wired to any other, but the synaptic sign and strength of their connection depends on the states of the two nodes. As with homophily in social networks, "cells that fire together, wire together" (or more precisely, "cells that are repeatedly active at the same time will tend to become 'associated' so that activity in one facilitates activity in the other," an updating rule introduced by Hebb[45:70] and incorporated (along with other learning algorithms) in models of artificial neural networks.

The adage that "birds of a feather fly together" turns out to be incomplete. Equally important is the complementary rule that "birds not of a feather cannot be tethered," as in the xenophobic reaction to encounters with those perceived to be different. For theoretical physicist Lawrence Kraus[46], "The question is, what do you do to the people who are wrong because they're not part of your group? Well, in many cases you kill them or you ostracize them or you send them to hell…the same kind of xenophobia." Xenophobia is the mirror image of homophily, in which differences repel. Just as homophily reinforces social influence by strengthening positive ties among those who are similar, xenophobia reinforces social differentiation by strengthening negative ties to those who are different.

The Hopfield model combines the Ising model's antiparallel differentiation with Axelrod's dynamic network. The result is a model with both attraction and repulsion that offers a robust solution to Abelson's puzzle, with an important implication for theories of partisan polarization. From a random start, a connected graph of dipolar nodes tends to gravitate into a small number of densely connected clusters within which the edges are mainly positive and anyone who ventures too close will be assimilated, while edges between the clusters are mainly negative, thereby reinforcing cultural differentiation.

Although bifurcation is the most likely outcome, the number of stable clusters is not always two. Consider a matrix of binary (e.g. "agree" or "disagree") opinions across d dimensions, where each dimension corresponds to a discreet issue, such as whether to legalize marijuana. In the case $d = 2$ there are four possible ideological profiles, where a profile consists of one of the four possible combinations of agreement or disagreement on two issues. For example, during much of the 20th century, politics in the US was characterized by partisan pluralism in a two-dimensional ideological configuration in which social and economic liberalism were largely orthogonal, yielding four equidistant groups. Republicans were internally divided into roughly equal east coast and west coast ideological wings, and Democrats were divided between New Deal northerners and Dixiecrat segregationists.

However, there can also be more than two salient controversies, involving opinions with more than two discreet states. This in turn implies a much larger opinion matrix, with many sparsely populated profiles that are therefore susceptible to being pulled into another location in the matrix. Simulations with an Ising-like model reveal the consequent destabilizing of a pluralist equilibrium, allowing pluralism to collapse into bifurcation, as has happened in the US in recent decades[47,48]. As pluralism collapses, idiosyncratic correlations can arise between substantively unrelated (and even logically contradictory) preferences, such as pro-life advocacy of capital punishment, free-marketer support for government regulation of bedroom behavior, support for presidential criminal immunity among conservative critics of government overreach, and endorsement of woke intellectual intolerance among progressive intellectuals[47,49].

Energy minimization in the Ising model can leave the network in an imbalanced state, with energy trapped in a stable configuration. Triadic imbalance among positive and negative edge weights incorporates "geometrical frustration" in the nomenclature of statistical physics, "cognitive dissonance" in psychological models, and "structural imbalance" in social network analysis. A node's triadic relations are structurally balanced when the product of the three signed relations is positive.

The concept of balanced relationships was introduced by Heider in 1946[50] and later generalized for network structures by Cartwright and Harary[51] as a formalization of the adage, "the friends of my enemies are my enemies." The cognitive dissonance experienced by agents in an imbalanced triad can lead to changes in node attributes (as when two friends disagree on a political issue), and it can also lead to changes in signed relations (as when two enemies have a friend in common). The psychological aversion to cognitive dissonance is an alternative to the assumption of homophily-weighted local influence in social science applications of the Ising model. However, these are not contradictory explanations for opinion dynamics. Rather, Górski et al.[52:1] (see also Pham et al.[53]) show that structural balance and homophily "are complementary mechanisms thought to shape social groups leading to, for instance, consensus or polarization."

An Ising-like model with dynamic edge weights has also been applied to the investigation of tipping points and hysteresis in the dynamics of polarization among political elites[54]. In the past, common threats – like Spanish Flu, the subprime mortgage crisis, and the attacks on Pearl Harbor and the World Trade Center – resulted in bipartisan responses by the representatives of both parties in the U.S. Congress. In contrast, in early 2020, a deadly COVID-19 pandemic threatened massive loss of life and the collapse of the medical system and the economy, yet common sense public health measures became a source of bitter partisan division in the U.S., with protective masks transformed into a political battle crest. Rather than bridging the divisions, the pandemic became a divisive partisan controversy. The contentious response to a common threat poses a troublesome question: is there a critical point above which polarization becomes difficult or perhaps impossible to reverse, even in response to a communal threat?

To find out, the authors used a computational model of self-reinforcing opinion dynamics driven by the interplay of attraction and repulsion. Members of two equal-size parties traverse a multi-dimensional issue space of positions on ten binary issues. From a random start, each agent updates their weights and states by moving closer to neighbors with whom they mostly agree and farther from those with whom they disagree, where the magnitude of the move depends not only on the level of agreement or disagreement but also two control parameters: the agent's tolerance of disagreement and the strength of party identification relative to their ideological commitment to the issues. The results revealed critical values for phase transitions into polarization and recovery, with asymmetric hysteresis trajectories in which the tipping point for recovery can be far below the critical value for polarization.

Even more disconcerting was the response of the system to exogenous shocks, modeled as the sudden appearance of a new issue on which all agents were in initial agreement. At low polarization, a shared interest promoted bipartisan compromise on all ten issues, but above this point the new issue itself became polarized.

Complex networks

Social scientists have long resisted three simplifications of network structure that are widely assumed in socio-physics: a regular lattice (as in the Ising model), a complete network (as in the Hopfield model), or an Erdős–Rényi random graph (with equal probability of an edge between any two nodes). These network structures simplify the math but have never been observed in large empirical populations. On the contrary, social networks are characterized by complex topologies, with bridge ties that span vast network distances, uneven modularity, non-uniform degree distributions, and a mix of directed, bi-directed, and undirected edges with dynamically variable weight and valence. What happens when Ising-like models are tested instead on complex networks? Will order (e.g., bifurcation into spins that differ in direction) still emerge spontaneously where the translational symmetry of a regular lattice is absent, hence interactions are not restricted to nearest neighbors and instead decay in strength with the distance between neighbors?

The answer, it turns out, is a qualified yes. The first indication of the possibility of spontaneous ordering came from simulations conducted for a scale-free network. The physicists Albert and Barabási[55] showed how scale-free degree distributions could arise in social networks that grow in size through a process of preferential attachment. "Degree" refers to the number of a node's network neighbors, and a scale-free network is characterized by a degree distribution that follows a power-law (at least for large values of the degree). For example, a newcomer to a social media site is more likely to follow a well-known celebrity than a relative unknown. Simply put, preferential attachment exhibits a "Matthew effect"[56] in which "the rich get richer." The outcome, Barabási and Albert discovered, was a scale-free degree distribution described by a power law with the characteristic exponent equal to three. Twitter approximates a scale-free network, as do collaboration networks, although these extreme degree distributions are relatively infrequent in social systems[57].

When the original Ising model is simulated on a scale-free network, a bifurcating phase transition occurs spontaneously, qualitatively similar to what was originally observed on a regular lattice, but with one interesting difference. On a two-dimensional lattice the critical temperature for phase transitions at the thermodynamic limit does not depend on network size (i.e., the number of nodes). In contrast, on scale-free networks, the critical temperature for bifurcation increases logarithmically with network size, as shown by Aleksiejuk, Hołyst and Stauffer[58]. This phenomenon can be explained with a detailed examination of hubs (the highly connected nodes) in scale-free networks. Compared to nodes with average degree, hubs stabilize spontaneous ordering in the scale-free network and are much more resistant to thermal fluctuations. The larger the network, the greater the expected degree of the largest hub. More generally, Leone et al.[59] proved that for the larger class of scale-free networks (with a generic exponent, of which the Barabási-Albert network with exponent three is a prominent but specific example), the critical temperature is proportional to the ratio of the second moment of the degree distribution and the first moment (the mean degree). As a consequence, large populations organized in heterogeneous networks, in which the second moment of the degree distribution diverges, will always fall below the critical temperature and thus show partial order.

The analysis of the Ising model on scale-free networks has generated an important insight into real-world opinion dynamics. The existence of hubs in complex social networks allows for the possibility of changing the opinion of an entire group by applying the societal equivalent of an external field to the most highly connected nodes. The external impetus could take the form of disinformation on social media (or even old-fashioned bribery) from a foreign adversary, or a concerted public health campaign in response to a global pandemic. Given budget constraints on the intervention, which nodes should be targeted? Simulations of the Ising model on scale-free networks reveal the answer: the largest of the hubs[58]. These nodes play the role of opinion leaders, and convincing these leaders is more likely to trigger a cascading phase transition in the opinion of the entire group compared to targeting their followers directly. The intuition is that the largest hubs are the nodes best equipped to convert the critical mass of followers required to sustain the cascade.

An obvious problem is that social influence can be bi-directed, hence hubs may also have high in-degree, in which case the external field targeting the largest hub may be unable to overcome the combined influence of a large proportion of the population. There is a second problem as well: the scale-free network assumes away the variation in edge weights. Suppose somehow the external field is able to flip the largest hub. The largest hub may have the most followers, but are they paying attention? Contrary to the intuitive idea that hubs are the most important "influencers," Quax, Appoloni, and Sloot[60] discovered that this is not generally the case.

Katz and Lazarsfeld's classic empirical study of pre-internet voter behavior in the 1940 U.S. presidential election found that social influence is much stronger in face-to-face personal relationships, compared to the impersonal influence of hub-like mass media[61]. They proposed an alternative model of the "two-step flow of communication" in which information from mass media first reaches local "opinion leaders" who then directly influence their network neighbors through interpersonal communication. The two-step model has been tested empirically many times over the ensuing decades, with mixed results. For example, a 2011 study of "Twitter lists" found considerable support for the two-step model[62], while a 2017 study[63] found that @user mentions in Twitter posts about protest events mainly targeted intermediate opinion leaders whose follower counts were greater than those of the average user but far less than those of media outlets.

Nearly all observed social networks share three common features, namely transitivity, homophily, and clustering[64]. Thus, network neighbors typically have one or more common neighbors and common attributes, and the density of ties within a community greatly exceeds the density between communities. The Ising model has also been simulated on complex networks that exhibit these features[65]. In these systems, it is possible to observe a specific ordering (or distribution of node states) with Ising interactions limited to nearest neighbors. The peculiarity of this ordering lies in the emergence of different local orders for different communities, such that the local distribution of dipolar states within a given community differs more than chance from the global distribution over the entire network.

The emergence of local order in clustered (or modular) networks invites inquiry into the dynamics by which local order might collapse. Recall Abelson's puzzle about the inevitability of opinion convergence. Abelson recognized the dependence of global convergence on network structure, but only for the extreme case of disconnected subgraphs (or network components). What happens to local order as we increase the number of bridges between local communities? Is there a critical point in globalization at which the emergence of spontaneous local order suddenly collapses? If there is a convergence of views among different communities, which opinion will become dominant?

Suchecki and Hołyst[66,67] found that the process of opinion homogenization can exhibit a discontinuous phase transition. Exceeding a critical density of connections between communities leads to a sudden subordination of the opinion of one community to the opinion of another. Surprisingly, the group whose opinion wins out is not necessarily the largest community. On the contrary, holding constant the number of ties within a community, the larger the community the lower the density, and low in-group density can increase vulnerability to out-group influence. A group's bridge nodes can be the most vulnerable to being converted through out-group contact, so when a bridge is added to the graph, the flow of influence will depend on the number of in-group ties at one end of the bridge compared to those of the out-group node at the other end.

The outcome of inter-group competition also depends on the magnitude of fluctuations in average opinions within groups before introducing bridge ties that enable between-group interactions. These fluctuations might be caused, for example, by the introduction of behavioral noise, analogous to temperature in the Ising model. Linear Response Theory[68] suggests that a group with high fluctuations is more susceptible to changes, thus succumbing to the influence of the group with the fewest fluctuations.

Six degrees of separation

In 1967, the psychologist Stanley Milgram[69] posed "the small world problem" using a novel letter-forwarding experiment. Milgram found that, on average, a letter mailed to a friend could reach its final destination in about six repostings. The startling implication is that any two people on the planet have "six degrees of separation," that is, they are separated by five intermediate friends of friends. How is this possible, particularly if people tend to be clustered in a "small circle of friends"?

The answer was revealed by two applied mathematicians, Duncan Watts and Steve Strogatz[70]. They showed that a very small proportion of random ties spanning large network distances can give a highly clustered network the mean geodesic of a random graph, a finding that resonated with the Ising model in physics. "The Ising system," Gitterman[71:8373] noted, "with a small fraction of random long-range interactions, is the simplest example of small-world phenomena in physics." However, the greatest interest has been in applications of "random rewiring" to human interaction. For example, Centola and Macy[72] used simulations of diffusion over randomized lattice networks to expose a hitherto hidden assumption in nearly all models of social influence, going back to Granovetter's classic paper on the "strength of weak ties"[73] – that social contagions mirror the spread of disease and information. Centola and Macy classified disease and information as "simple contagions" for which a single exposure is sufficient: If one learns sports scores from a friend or is infected with a respiratory virus, there is usually no need to have the scores confirmed or the viral load enhanced via an additional independent exposure. In contrast, the adoption of risky innovations, deviant behaviors, or products with positive network externalities may require social reinforcement through exposure to multiple prior adopters. The spread of these "complex contagions" requires dense local structure, with bridges that are "wide" (i.e., composed of multiple short paths between the communities) rather than "long" (i.e., spanning otherwise distant regions of the network). The long bridges found in small-world networks provide shortcuts that facilitate the spread of simple contagions, but wide bridges are needed so that nodes adjacent to one activated neighbor are likely to have other neighbors that are also activated.

Complex contagion was independently incorporated in the Sznajd model, based on the principle "The power of social validation is undeniably very strong."[74:9] The model required two neighboring spins to be aligned in order to influence the spins of their respectively nearest and next-nearest neighbors. A closed community then evolves toward either a dictatorship or a collective stalemate, both of which are problematic for democratic institutions and increasingly evident in contemporary political practice.

Looking forward

This survey of social applications of the Ising model reveals three topics where more attention is warranted. First, research on homophily and xenophobia has analyzed the continuous coevolution of weights and states, but phase transitions have generally been limited to the latter, to the neglect of research on phase transitions in network topology. Consider for example the breakup of the Zachary karate club[75], or the bifurcation of social media into disjoint and mutually reinforcing "echo chambers." A preference to interact with like-minded neighbors and avoid those who differ can create a positive feedback loop where shared in-group beliefs promote insulation from contradictory information or argument, and pressure to differentiate from the out-group strengthens in-group beliefs. This self-reinforcing dynamic suggests the possibility that the topological division may not evolve gradually but might instead be characterized by a critical tipping point[24].

Second, social applications of the Ising model have typically assumed zero temperature, even when decision-making is assumed to be stochastic, and despite the familiar assumption of randomness in opinion sampling, statistical inference, small-world networks, controlled experiments with random assignment, and regression models with unexplained variance. Although the strength of social influence is often varied relative to non-random intrinsic drivers like self-interest, it is rarely manipulated relative to the effects of uncertain information, trembling hands, or the unpredictability of idiosyncratic human behavior. Curiously, noise in Ising-like social science models is generally assumed to be constant but social relationships vary in strength, while in physics it is often the opposite: interaction strengths are held constant as noise is changing. Both parameters are important. The manipulation of temperature is vital not only to ensure the robustness of analytical results (as demonstrated by Klemm et al.[36]) but also because there may be temperature-induced phase transitions that have theoretical importance for the understanding of social dynamics.

Third, the value of Ising-like models goes beyond the contributions to theoretical research. The models also yield testable predictions that need to be confirmed with empirical research, including controlled experiments and longitudinal observational studies. For example, Flamino et al.[24]. model group formation and evolution using an Ising-derived model of the utility of interactions in a cluster as the difference between the squares of the numbers of members with positive and negative spins. They then test the agreement with longitudinal observations of student social groups. Galesic and Stein[76:275] also show that "Simple Ising and Potts models can be parameterized to resemble actual societies," and these models "can reproduce belief dynamics on individual and group levels." There is also a growing literature in statistics that addresses empirical estimation of Ising model parameters in cases with an unknown adjacency matrix, signed continuous edge weights, and node-specific threshold parameters[77–82].

The centennial anniversary of the Ising model is an opportunity to celebrate a long and growing list of contributions to knowledge that extends well beyond ferromagnets, including applications to social dynamics involving positive and negative reactions to network neighbors. That core principle has been retained and extended as the models have evolved to include homophily on dynamic and complex network topologies, with important qualitative insights into problems like residential segregation and political polarization. The shared framework is a bottom-up approach to the study of population behavior that emerges out of the micro-level interactions among constituent individuals.

Celebration, however, is not our intent. Instead, we invite readers to engage in a very different exercise – to ask how anything can be learned about human behavior from a toy model of atomic spins. Humans, after all, are not atoms with binary states that can be influenced by nearest neighbors on an undirected regular lattice.

The answer was put succinctly by Fortunato et al.[83:3] in an assessment of social science applications in statistical mechanics: "In many phase transitions, the large-scale behavior of a many-particle system is independent of particulate details and their microscopic interactions; only a few basic features are relevant. Thus, systems that cannot be fully characterized at the individual level might still display recognizable patterns in the aggregate, if the number of constituents is sufficiently large." We hope our review has helped to bridge the disciplinary divide between the behavioral and physical sciences by offering quantitative social scientists a deeper appreciation of the modeling approach of Ernst Ising.

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