past the N-halogeno imides, such as Ziegler's reagents, have often been represented as yielding either halogen cations (*a*), or molecular halogens (*b*) and not free halogen atoms (*c*):

(*a*) $R_2N\text{–}Cl \rightleftharpoons R_2N^- + Cl^+$ (unimolecular ionic reaction).

(*b*) $R_2N\text{–}Cl + HCl \rightleftharpoons R_2N\text{–}H + Cl_2$ (bimolecular reaction)

(*c*) $R_2N\text{–}Cl \rightleftharpoons R_2N + {\cdot}Cl$ (free radical reaction).

The prime reason for my comment in *Nature* of December 16, 1944, was to direct attention to this evidence for reaction by mechanism (*c*), in view of its theoretical significance in relation to the possible modes of covalent bond fission in solution, rather than to claim priority for proposing a mechanism for chlorination of olefines in the allyl position.

In establishing criteria for recognizing free radical reactions in solution, analogies with gas reactions are particularly valuable, though in commenting briefly on a particular reaction it is impossible to do more than cite recent relevant reviews (for example, my references 5 and 8).

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April 29.

Operators in Quantum Theory

IN his article on "Causality or Indeterminism ?" in *Nature* of March 10, Prof. H. T. H. Piaggio refers to the restriction on quantum mechanical operators made by the condition that they be hypermaximal, and suggests that this conceals important requirements. In fact, the literature on the subject does not discuss the physical meaning of this requirement. Dirac, in his "Principles of Quantum Mechanics", substitutes the condition that "only those Hermitian operators that satisfy the expansion theorem represent observables": this is mathematically equivalent to hypermaximality, but also has no clear physical meaning.

The condition that an operator be Hermitian has the simple basis that the corresponding observable must have real values. It seems worth while to inquire what physical requirement on the operators can replace the above conditions. Now it can be shown that those, and only those, Hermitian operators are hypermaximal for which the corresponding Schrödinger equation

$$H\ \psi(t) = \frac{1}{\iota}\,\frac{d\psi}{dt}, \qquad . \quad . \quad . \quad . \quad (1)$$

with the initial condition $\psi(0) = \varphi$ has a solution for all positive and negative t and for every wave-function φ lying in the domain of the operator, H.

If, for example, we take the momentum operator for a line infinite in both directions—the operator $\frac{1}{\iota}\frac{d}{dx}$ operating on wave functions $\varphi(x)$ with

$$\int_{-\infty}^{\infty} |\,\varphi\,(x)\,|^2\,dx = 1,$$

then the equation (1) with the initial condition $\psi(x,0) = \varphi(x)$, has the solution $\psi(x,t) = \varphi(x+t)$ for all values of t.

If, on the other hand, we consider a half-infinite line, from 0 to ∞, the wave-function is $\varphi(x)$ with $0 \leqslant x < \infty$. As is easily seen, the operator $\frac{1}{\iota}\frac{d}{dx}$ operating on wave-functions in this half-infinite line is Hermitian if, and only if, its domain is confined to wave-functions with $\varphi(0) = 0$. The equation (1), with initial conditions $\psi(x,0) = \varphi(x)$, has the solution $\psi(x,t) = \varphi(x + t)$ for all negative t, where we take $\varphi(x) = 0$ *if* $x < 0$; *but for* $t > 0$ *the solution is* formally $\varphi(x+t)$, which does not lie in the domain of the operator if $\varphi(t)$ is not zero. Hence the equation does not have a solution for all positive t. This corresponds to the obvious fact that a particle cannot move in a half-infinite strip with constant momentum for an infinitely long past and future.

The examples illustrate what happens in the general case; since the equation (1) is the equation of a system for which the probability that the observable corresponding to the operator H should have a given value, or should have a value lying in a given interval, is constant, we can restate the condition in the following form:

A Hermitian operator is hypermaximal if, and only if, it is possible for a physical system to maintain a constant probability distribution of the observable corresponding to the operator for a time infinite in both directions.

This condition makes it fairly obvious why only hypermaximal operators obey the expansion theorem. For that theorem to hold, the system must be capable of stationary states, that is, of states in which the probability distribution of the values of the observable is constant from $t = -\infty$ to $t = \infty$.

This approach to operator theory seems to link it more closely with the physical theory than does the theory of von Neumann and Stone. In a paper to appear in the *Quarterly Journal of Mathematics*, I have shown that it leads to a proof of the spectral theorem rather simpler than those of von Neumann and Stone, and also have proved the equivalence of the condition stated with hypermaximality.

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Reform of the Patent Law in Britain

THE article on "Reform of the Patent Law in Britain"[1] criticizes my proposals for patent reform on the grounds that they would reduce the profitability of inventions to financial backers. I would maintain that my proposals would in general have no such effect. I have recommended that the Government should pay patentees rewards in proportion to the economic advantage arising from the application of their patents. Hence the wider these applications (be it by competitors of the patentees or by others) the higher the gains accruing to patentees. It is in fact an essential point of the proposed reform that it would not on the whole take away any of the financial stimulus for the exploitation of patents while leaving everyone free to use them. Even though a closer analysis might reveal cases in which the present form of patents would be somewhat more lucrative to patentees, these cases could be balanced by others in which the reverse would hold.

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[1] *Nature*, 155, 612 (1945).