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Electrical analogue of one-dimensional and quasi-one-dimensional Aubry–André–Harper lattices

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This work explores the potential for achieving correlated disorder in electrical circuits by utilizing reactive elements. By establishing a direct correspondence between the tight-binding Hamiltonian and the admittance matrix of the circuit, a novel approach is presented. The localization phenomena within the circuit are investigated through the analysis of the two-port impedance. To introduce correlated disorder, the Aubry–André–Harper (AAH) model is employed. Both one-dimensional and quasi-one-dimensional AAH structures are examined and effectively mapped to their tight-binding counterparts. Notably, transitions from a high-conducting phase to a low-conducting phase are observed in these circuits, highlighting the impact of correlated disorder.

The Aubry–André–Harper (AAH) model^{1,2} represents a classic example of a one-dimensional (1D) quasi-crystal, possesses several intriguing features. In the nearest-neighbor tight-binding (TB) framework, the 1D AAH model with an incommensurate potential exhibits a sharp localization-delocalization transition, where all the eigenstates are delocalized below a critical point, while all of them are completely localized beyond that critical point. Due to the incommensurate potential, this 1D model shows a gapped and fractal-like energy spectrum. Beyond the minimal nearest-neighbor TB model, energy-dependent mobility edges have also been predicted analytically³ in the 1D AAH chain. Such localization transition and mobility edges have also been found in coupled AAH chains⁴. Recently, 1D AAH quasi-crystal has been realized experimentally using waveguides⁵ by Kraus et al. The authors in their work showed that the edge states of the fabricated photonic quasi-crystal are topologically nontrivial. Owing to such striking properties and several others, this model has been investigated widely in many contexts over more than three decades^{6–14}.

Recently, it has been shown that *various topological states that are difficult to observe in condensed matter experiments, can be simulated with electric circuits*^{15–17}. Even though electric circuits represent classical systems, with a proper choice of the reactive elements, the corresponding admittance matrix becomes equivalent to the tight-binding Hamiltonian^{15–19}. By means of circuits, the energetics and topological phases of various physical systems have been investigated in recent years, such as Su–Schrieffer–Heeger (SSH) model¹⁶, Weyl semimetal¹⁸, Chern and quantum spin Hall insulators¹⁹, topological Anderson insulators²⁰, breathing kagome and pyrochlore lattices²¹, and many others^{22,23,24–28}. Most of the aforementioned works focused on the study of topological phases based on the close correspondence between such electric circuits and TB models. The motivation of this work is twofold—simulation of a TB AAH system using an electrical circuit and inspection of localization phenomena.

First, we construct electric circuits comprising inductors and capacitors (see Fig. 1) that describe TB AAH systems. The cosine modulation of usual AAH TB site energies is incorporated into the circuit by connecting different values of capacitors at the nodes of the circuit.

Primarily, we focus on designing circuits that are analogous to the TB 1D chains with nearest-neighbor (NN) and next nearest-neighbor (NNN) connections and then a two-stranded ladder network. We detect any localization behavior present in those circuits by computing a two-port impedance (TPI)²⁹.

The notable features of this work are: (i) realization of correlated disordered systems with electrical components, (ii) direct mapping of NN and NNN AAH TB chains and two-stranded AAH ladder with electrical circuits, and (iii) exact correspondence of admittance spectra of electrical circuits with energy spectra of TB AAH systems. Our analysis can be implemented to any other such fascinating correlated systems.

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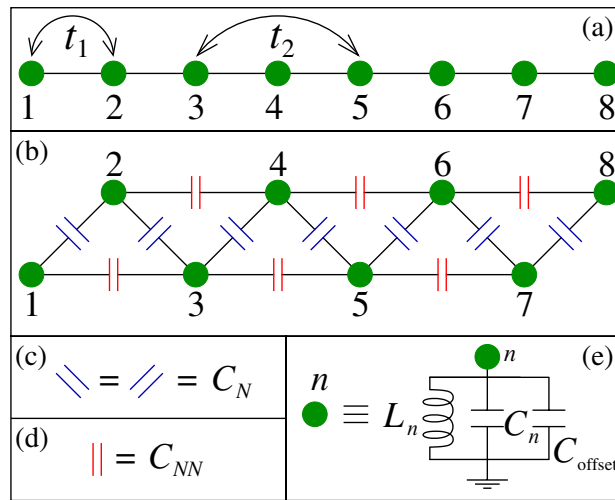


Figure 1. (a) Schematic diagram of a 1D quasiperiodic TB chain consisting of eight sites. t_1 and t_2 are the nearest-neighbor and next nearest-neighbor hopping integrals, respectively. (b) Schematic circuit analogous to 1D quasiperiodic TB chain. (c) Blue capacitors (C_N 's) connect the nearest-neighbor nodes. (d) Red capacitors (C_{NN} 's) connect the next nearest-neighbor nodes. (e) The grounded connection of each node n .

System and theoretical framework

The TB Hamiltonian modeled on a 1D chain (Fig. 1a) within a non-interacting electron picture considering both NN and NNN hoppings can be written as⁴,

$$H = \sum_n \varepsilon_n c_n^\dagger c_n + t_1 \sum_{\langle nm \rangle} c_n^\dagger c_m + t_2 \sum_{\langle\langle nm \rangle\rangle} c_n^\dagger c_m. \quad (1)$$

Here t_1 is the NN hopping integral and $\langle \rangle$ represents the NN sites of the 1D chain, while t_2 is the NNN hopping strength and $\langle\langle \rangle\rangle$ denotes the NNN sites of the 1D chain. ε_n is the on-site potential at site n . The AAH disorder is introduced through the on-site potential and it is²

$$\varepsilon_n = W \cos(2\pi bn + \phi_v) \quad (2)$$

where W is the modulation strength, b is an irrational number and it is chosen as $b = (\sqrt{5} - 1)/2$, n is the site index, and ϕ_v is the AAH phase factor.

To map the TB 1D lattice model (Fig. 1a) that satisfies Eq. (1), we design an electrical circuit which is given in Fig. 1b. Kirchhoff's law at node n in the given LC circuit reads as¹⁹

$$\dot{I}_n = \sum_m C_{nm} (\ddot{V}_n - \ddot{V}_m) + C_n \ddot{V}_n + \frac{1}{L_n} V_n. \quad (3)$$

Here $\dot{V} = \frac{dV}{dt}$. I_n and V_n are the current and voltage at node n , respectively. The sum over m is taken for the first and second nearest-neighbor nodes. C_{nm} is the capacitor connected between nodes n and m . In the second term of Eq. (3), the capacitor C_n and the inductor L_n (in the third term) are connected between the node n and ground.

Following the Fourier transformation of Eq. (3), the relationship between the current and voltage at frequency ω becomes

$$I_n(\omega) = \sum_m J_{nm}(\omega) V_m(\omega). \quad (4)$$

Here J_{nm} is known as the admittance matrix and it becomes

$$J_{nm} = j\omega \left[C_{nm} + \left(C_n + C_{\text{offset}} + \left(\sum_m C_{nm} - \frac{1}{\omega^2 L_n} \right) \right) \delta_{nm} \right]. \quad (5)$$

A one-to-one correspondence can be established between the admittance matrix J_{nm} and the TB Hamiltonian (Eq. 1) with

$$t_{1(2)} = -j\omega C_{nm} \quad (6)$$

$$\varepsilon_n = j\omega \left(C_n + C_{\text{offset}} + \left(\sum_m C_{nm} - \frac{1}{\omega^2 L_n} \right) \right). \quad (7)$$

Apart from the term $j\omega$, the NN hopping integral t_1 can be identified with the capacitor C_N (Fig. 1b) and the NNN hopping integral t_2 with C_{NN} (Fig. 1c). Setting the frequency of the input as $\omega = 1/\sqrt{L_n \sum_m C_{nm}}$, the term $(\sum_m C_{nm} - \frac{1}{\omega^2 L_n})$ in Eq. (7) becomes zero. Then the capacitor C_n (connected between the n -th node and the ground) can be used to incorporate the AAH disorder into the admittance matrix. The inclusion of the additional capacitor C_{offset} (Fig. 1e) serves a specific purpose, as explained in the results section. A comprehensive derivation of the mapping between the admittance matrix and the TB Hamiltonian is provided in the Supplementary material (section 1), with detailed explanations.

To study the localization behavior of the circuit, the simplest experimentally measurable quantity is the two-port impedance Z_{nm} between nodes n and m . Z_{nm} is defined as²⁹

$$Z_{nm} = \frac{V_n - V_m}{I} \quad (8)$$

where $V_n - V_m$ is the voltage difference between the nodes n and m . I is the magnitude of current $I = I_n = -I_m$, that is the current I flows into node- n and leaves node- m .

In order to determine Z_{nm} , we need to express the potentials in terms of the input current I and for that, the admittance matrix (Eq. 5) has to be inverted. To do so, first, we write the spectral form of J_{nm} as

$$J_{nm} = \sum_p j_p \psi_{p,n}^* \psi_{p,m} \quad (9)$$

where j_p is the p -th eigenvalue of the admittance matrix and $\psi_{p,n}$ is the p -th eigenfunction at node n . With this, the regularized inverse of the admittance matrix, known as the circuit Green's function, can be written as

$$G_{nm} = \sum_{p: j_p \neq 0} j_p^{-1} \psi_{p,n}^* \psi_{p,m}. \quad (10)$$

The two-port impedance then simplifies to

$$\begin{aligned} Z_{nm} &= G_{nn} + G_{mm} - G_{nm} - G_{mn} \\ &= \sum_p j_p^{-1} |\psi_{p,n} - \psi_{p,m}|^2. \end{aligned} \quad (11)$$

Results and discussion

First, we consider the circuit which is analogous to a 1D TB NN AAH chain. For this case, we do not consider any connection with the capacitors C_{NN} 's. Before we discuss the results, let us specify the values of the capacitors and inductors. We consider the number of nodes in the circuit as $N = 100$ and set $C_N = 1 \mu\text{F}$. The inductor L_n that is connected between n -th node and ground is set at $L_n = L = 1\text{mH}$ except at the extreme two nodes, namely nodes 1 and 100. The inductors between the ground and these two nodes are fixed at $L_1 = L_{100} = 2L$. The frequency of the input is fixed at $\omega = 1/\sqrt{2LC_N}$. The capacitor C_n (Fig. 1e) is considered in the μF range and the magnitude is chosen according to Eq. (2). For instance, the grounded capacitors for a disorder strength $W = 1$ (with $b = (\sqrt{5} - 1)/2$ and $\phi_v = 0$) assume the values, $C_1 = -0.7374 \mu\text{F}$, $C_2 = 0.0874 \mu\text{F}$, $C_3 = 0.6084 \mu\text{F}$, and so on. We attach another capacitor C_{offset} (shown in Fig. 1e) between each node and the ground, in parallel to C_n . This is crucial to avoid potential issues with the eigenvalues of the admittance matrix. When C_{offset} is omitted, there is a possibility that the eigenvalues of the admittance matrix can approach zero or become very close to zero. In such cases, if we examine the expression as given in Eq. (11), the TPI will diverge, regardless of the nature of the p th eigenmode. To mitigate this problem, it becomes necessary to incorporate C_{offset} in the circuit design. By doing so, we ensure that all eigenvalues of the admittance matrix are significantly greater than zero. This inclusion of C_{offset} guarantees that the nature of the eigenmodes can be accurately captured, preventing any numerical instabilities or unrealistic outcomes in the TPI calculation.

We set $C_{\text{offset}} = 10\text{mF}$. The effect of such a connection can readily be observed in the admittance spectra as shown in Fig. 2.

In Fig. 2 we plot the eigenvalues j_n of the admittance matrix as a function of AAH phase ϕ_v . As mentioned earlier, the structures of the admittance and the TB Hamiltonian matrices are identical, apart from the term $j\omega$ (Eq. 5). The eigenvalues j_n of the admittance matrix are expressed in units of Ω^{-1} and ϕ_v in units of π . The modulation strength is fixed at $W = 0.5$. As mentioned above, C_{offset} shifts all the eigenvalues towards a positive value and none of the eigenvalues is close to zero. The spectrum is divided into three branches and they are almost constant with ϕ_v . A few modes are also seen to cross from one branch to another through the gaps with ϕ_v . Overall, the behavior of the admittance spectrum as a function of ϕ_v is identical to the energy spectrum as a function of ϕ_v , computed for 1D tight-binding Aubry chain by Kraus et al.⁵, and thus, we can claim that our circuit setup is correct.

Now, let us look at the localization behavior of the circuit. Here we consider two particular modes in the gap, namely the modes at $\phi_v = 0.48\pi$ (the red hollow circle in Fig. 2) and $\phi_v = 0.68\pi$ (the orange hollow circle in Fig. 2), having the same eigenvalues. We compute the two-port impedance for these two modes as a function of node index as shown in Fig. 3a. We fixed one port at node 50 and the other one is taken through all the nodes in order to compute the two-port impedance. The results for $\phi_v = 0.48\pi$ and $\phi_v = 0.68\pi$ are displayed in red and orange colors, respectively. In Fig. 3a, we see that the two-port impedance $|Z|$ is maximum at the extreme left node, namely node 1 for $\phi_v = 0.48\pi$ and abruptly decreases to zero from the left side to the right of the

1D electric circuit. On the other hand, for $\phi_v = 0.68\pi$, we observe a complete mirror-symmetric feature of the previous case. The maximum impedance is now observed at node 100, which is at the extreme right of the 1D circuit and then gradually decreases from the right side to the left. We also choose a mode from the bulk band as shown by the black circle in Fig. 2 and computed the two-port impedance. The corresponding result is shown in Fig. 3b. In the given case, $|Z|$ is about an order of magnitude less than the previous two cases, indicating that all the nodes are well extended for the chosen bulk mode. Overall, the behavior of two-port impedance with node index is very much consistent with the established localization behavior of 1D TB AAH chain⁵ as a function of ϕ_v .

Next, we vary the disorder strength via the grounded capacitors C_n s and study the behavior of two-port impedance. Here we set $\phi_v = 0$ and all other circuit parameters are taken same as mentioned earlier. In the density plot of Fig. 4, we show the behavior of the natural log of two-port impedance as functions of disorder strength and eigenvalues of admittance matrix (measured in Ω^{-1}). Here the two-port impedance

at each of the eigenmodes is computed by keeping one port at node-1 and varying the other port at all the other nodes and then taking the maximum of $|Z|$. The values in the colorbar denote the natural log of two-port impedance, where lower values are denoted with dark color and higher values with bright ones. The eigenspectrum of the admittance matrix is divided into three branches as expected. Below $W = 2C_N$, the computed two-port impedance for all the modes is vanishingly small, as is clearly seen in Fig. 4. Beyond $W = 2C_N$, the eigenvalue spectrum becomes brighter, indicating that the TPI for all the modes is much higher than that in $W < 2C_N$ region. Therefore, a sharp transition occurs at $W = 2C_N$ (critical W is $W_c = 2C_N$) from a highly conducting zone (vanishingly small $|Z|$) to a low conducting one (relatively large $|Z|$). Such a sharp transition in the context of localization has already been studied in 1D TB AAH chain⁷.

Now, we bring in the capacitor C_{NN} to make the circuit analogous to a 1D NNN TB AAH chain. We fix the capacitor $C_{NN} = 0.25 \mu\text{F}$. The inductor L_n that is connected between n -th node and ground is set at $L_n = L = 1\text{mH}$ except at the extreme four nodes, namely nodes 1, 2, 99, and 100. We fix $L_1 = L_{100} = 2L$ and

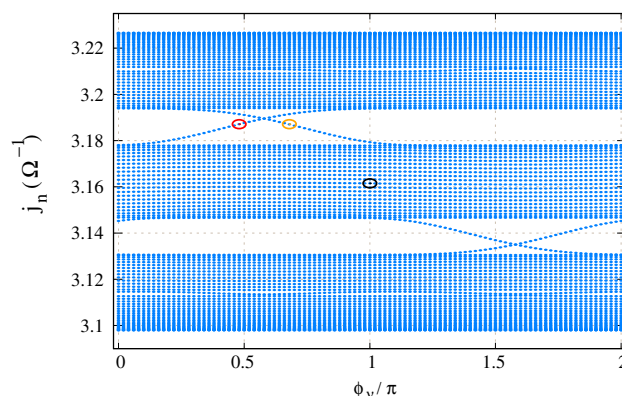


Figure 2. Admittance spectrum as a function of Aubry phase ϕ_v considering NN connection. Number of nodes $N = 100$. The AAH disorder strength is fixed at $W = 0.5$. Other circuit parameters are described in the text. The nodes within the red, orange, and black circles are picked up to compute the two-port impedance in Fig. 3.

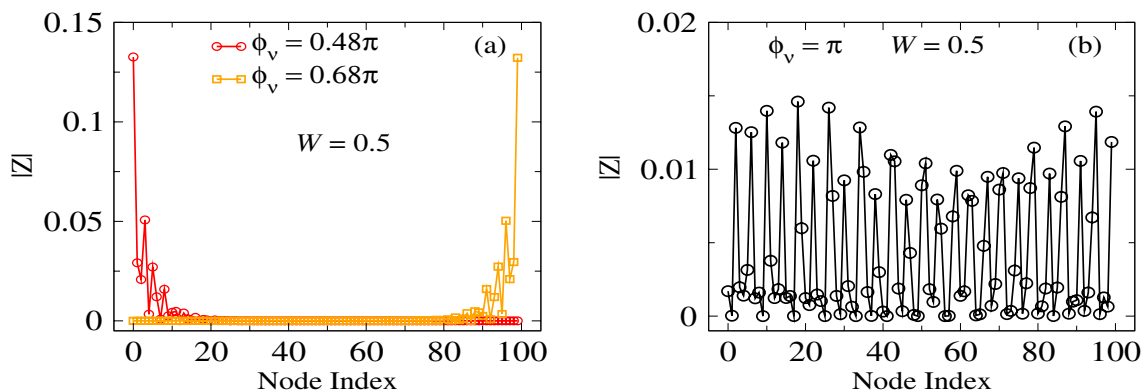


Figure 3. Visualizing edge states. Two port impedance $|Z|$ as a function of node index. One port is fixed at node 50. (a) The red curve is for $\phi_v = 0.48\pi$ and the considered eigenmode is chosen from Fig. 2, marked with red circle. The orange curve is for $\phi_v = 0.68\pi$ and the considered eigenmode is chosen from Fig. 2, marked with orange circle. Both modes have the same eigenvalue. (b) The black curve is for $\phi_v = 0.48\pi$ and the considered eigenmode is chosen from Fig. 2, marked with black circle.

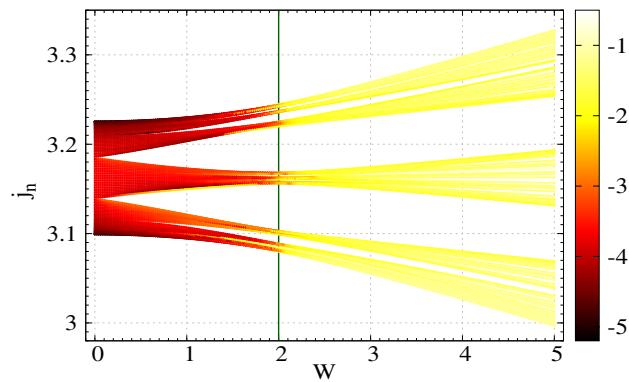


Figure 4. Density plot of the natural log of two-port impedance as functions of modulation strength W and eigenvalues of admittance matrix j_n , considering NN connection. Number of nodes $N = 100$. The colorbar denotes the values of $\ln |Z|$. The green vertical line at $W = 2C_N$ denotes the sharp transition from the highly conducting region to the low conducting one. Thus the critical W is $W_c = 2C_N$.

$L_2 = L_{99} = 2(C_N + C_{NN})L/(2C_N + C_{NN})$. The frequency of the input is fixed at $\omega = 1/\sqrt{2L(C_N + C_{NN})}$. The rest of the circuit parameters are the same as mentioned earlier. The density plot of the TPI, shown in Fig. 5, is computed following the prescription described above in Fig. 4. The spectral nature of the eigenvalues of the admittance matrix with disorder strength is identical to the behavior of the eigenvalue spectrum as a

function of disorder strength for the 1D NNN TB AAH chain. Here also we observe a transition from a conducting region to an insulating one. However, there is no such sharp transition as in the case of NN case (Fig. 4). Rather, *the transition is admittance dependent*. It is important to note that in the 1D AAH tight-binding chain with higher order hopping terms, there exists energy dependent mobility edge, which separates the localized wave functions from the delocalized ones. We also have a similar situation in the present case – *admittance dependent mobility edge*, which separates the highly conducting region from the low conducting one.

Finally, we design a two-stranded ladder network electrically. The schematic diagram for the circuit is shown in Fig. 6a. The upper and lower strands are coupled vertically through the capacitor C_V (Fig. 6b).

In both the strands, the neighboring nodes (marked with green circles) are connected through the capacitor C_H , denoted with red color (Fig. 6c). The crossed nodes (viz, nodes 1 and 5, nodes 2 and 4, etc.) are connected through the capacitor C_D (Fig. 6d). The n -th node is connected to the ground via a parallel LC circuit, as shown in Fig. 6e.

The chosen circuit parameters are as follows. The number of nodes fixed at each strand is $N = 100$. The frequency of the input is fixed at $\omega = 1/\sqrt{(2C_H + 2C_D + C_V)L}$. The inductor L_n that is connected between n -th node and ground is set at $L_n = L = 1\text{mH}$ ($n \neq 1, 100, 101, 200$). The inductors between the ground and these four nodes are fixed at $L_n = (2C_H + 2C_D + C_V)L/(C_H + C_D + C_V)$ ($n = 1, 100, 101, 200$). The capacitor $C_n = C_{n+100}$ (Fig. 6e) is considered in the μF range and the magnitude is chosen according to Eq. (2) as before. In addition to that, we attach another capacitor $C_{\text{offset}} = 10\text{mF}$ (shown in Fig. 6e) between each node and ground, in parallel to C_n to introduce a shift in the admittance spectrum well above the zero line due to the fact mentioned earlier.

With all the said circuit parameters, we show the behavior of the natural log of two-port

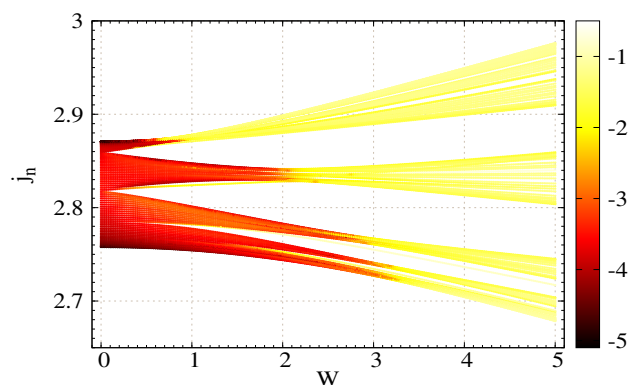


Figure 5. Density plot of the natural log of two-port impedance as functions of modulation strength W and eigenvalues of admittance matrix j_n , considering NNN connection. Number of nodes $N = 100$. The colorbar denotes the values of $\ln |Z|$.

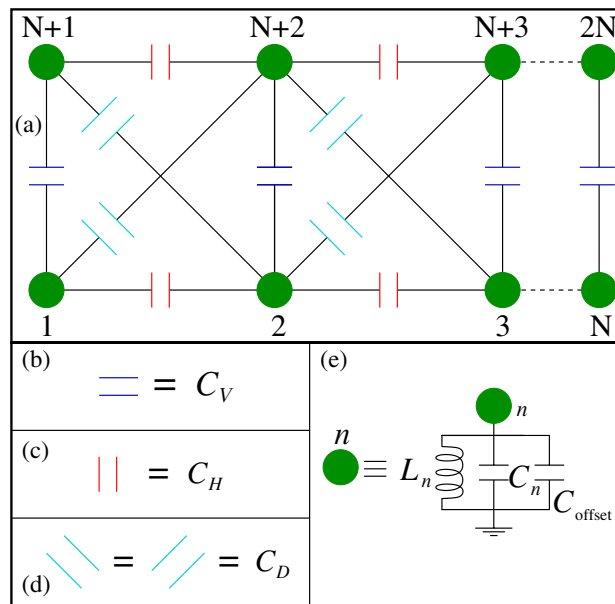


Figure 6. (a) Schematic diagram of two-stranded ladder network having N nodes in each strand. (b) Blue capacitors C_V 's connect the two strands vertically. (c) Red capacitors C_H 's connect the nearest-neighbor nodes horizontally in each strand. (d) Cyan capacitors C_D 's connect the nodes diagonally. (e) The grounded connection of each node n .

impedance as functions of disorder strength and eigenvalues of the admittance matrix in Fig. 7. The computed two-port impedance for a particular eigenmode is chosen by considering all the possible two ports in the present circuit setup and then we take the maximum impedance among them. Here, the natural log of the maximum impedance is plotted for the density plot. The color convention for the colorbar is same as before. In the present case, we identify two critical points. The first one is $W_{c1} = 2(C_H - C_D) = 1.6 \mu\text{F}$ (shown by the magenta vertical line), below which all the modes are highly conducting. The second one is $W_{c2} = 2(C_H + C_D) = 2.4 \mu\text{F}$, beyond which all the modes are poorly conducting in nature (shown by the green vertical line). Within the range $W_{c1} < W < W_{c2}$, there is a mixed phase zone, where the highly conducting modes and poorly conducting modes coexist. Such a feature is also in good agreement with the localization behavior of the two-stranded ladder network in the TB framework⁴.

It is important to note here that our system is a 'classical circuit' composed of traditional components such as capacitors and inductors. Consequently, we do not anticipate the presence of any interference effects in this particular case. However, we can draw an analogy between the admittance matrix of the circuit and the TB Hamiltonian, as demonstrated in our present work (see Supplementary material, section 1 for a detail discussion). In this analogy, the voltage nodes that are connected to the ground through the capacitors and inductors serve as the TB lattice points, while the capacitors between the nodes act as the TB hopping integrals. The eigenfunctions

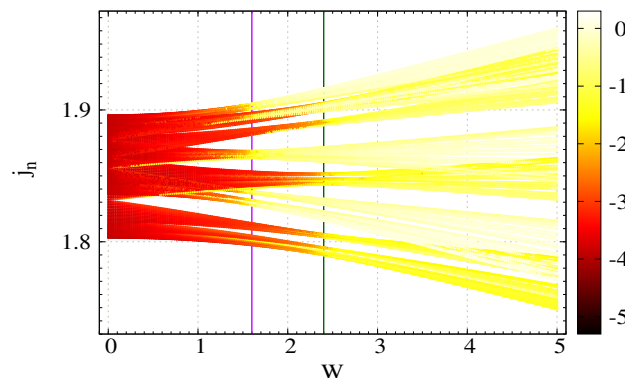


Figure 7. Density plot of the natural log of two-port impedance as functions of modulation strength W and eigenvalues of admittance matrix j_n for two-stranded ladder network. Number of nodes at each strand $N = 100$. The colorbar denotes the values of $\ln|Z|$. The magenta and green vertical lines denote are drawn at two critical points as described in the texts.

of the admittance matrix characterize the voltage profile across the nodes in the circuit, akin to the spatial variation of the wavefunction in a TB system. Furthermore, the eigenvalues of the admittance matrix correspond to the eigenenergies of the TB Hamiltonian. By introducing distinct capacitors between each node and the ground, we intentionally introduce correlated disorder into the circuit. Through these analogies, we propose that the localization observed in the circuit, which is governed by the analysis of TPI, can be attributed to the following reasons. As previously mentioned, the eigenmodes of the admittance matrix describe the voltage distribution across the nodes in the circuit. According to the definition, the TPI between two nodes becomes significant when a finite density of nontrivial eigenmodes exists. This density is relatively low when the disorder strength is below the critical region (which separates the low to high conducting zones), leading to small TPI values. However, as we surpass the critical region, the density of eigenmodes increases substantially, becoming localized at specific nodes or groups of nodes. This localization phenomenon results in higher TPI values.

In simpler terms, the localization observed in the circuit is not caused by interference effects, as the system is classical. Instead, it arises from the voltage distribution across the nodes, which alters their characteristic properties based on the capacitors connected to the ground. These capacitors introduce disorder into the circuit and play a crucial role in determining the localization behavior.

To gain insights into the localization effect present in our selected circuits, we conducted a thorough analysis of the TPI for the p th eigenmode at varying levels of disorder strength. The detailed analysis and results are presented in Supplementary Material (section 2).

Note 1: The selection of capacitor and inductor values in this study was influenced by the available sources of the input signal. Specifically, we carefully considered the parameter values to determine the resonant frequency in the 1D NN circuit. For instance, in Fig. 1b, with a chosen inductance value of $L = 1\text{mH}$ and a capacitance value of $C_N = 1\text{ }\mu\text{F}$, we can calculate ω as $\omega = 1/\sqrt{2C_N L} = 22.36\text{kHz}$. This is a realistic value for the input signal.

Note 2: In electrical circuits, achieving higher order coupling is relatively straightforward. For example, in Fig. 1, a NNN coupling is achieved by connecting the NNN nodes through the red capacitors. However, this concept can also be extended to mechanical systems. For instance, an LC circuit follows a homogeneous second-order differential equation, similar to a mechanical system composed of springs and dampers¹⁶. Furthermore, the Ref.³⁰ discusses such a possibility in elastic systems, where a third neighbor coupling is established, highlighting the versatility of such systems.

Conclusion

To conclude, we have proposed a way to realize AAH disorder in electrical circuits. One-dimensional and two-stranded ladder networks have been considered for the purpose. We have shown that the AAH disorder strength and the phase can be controlled by tuning the reactive elements of the circuits. Like the inverse participation ratio (IPR), which is one of the measures of the localization phenomena in TB systems, the two-port impedance of the electrical circuits considered in the present work can serve the same purpose. Specifically, we have shown that for 1D NN circuit, the behavior of two-port impedance exhibits a sharp transition from a highly conducting region to a poorly conducting region. We have also observed admittance dependent mobility edge in 1D NNN circuit, which separates the high-conducting region from the low-conducting one. Finally, for the two-stranded ladder network, we have found two critical points, below one of the critical points, all the modes are highly conducting, and beyond the other critical point, all of them are poorly conducting. In between these two critical points, both the low and high conducting modes coexist. All the observations have been carried out based on two-port impedance. We strongly believe that the present analysis provides a direct mapping of AAH lattices with electrical circuits.

Data availability

Derived data supporting the findings of this study are available from the corresponding author on request.

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Author contributions

SG and SKM conceived the project. SG performed the numerical calculation. SG and SKM analyzed the data and co-wrote the paper.

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Additional information

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