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Fermatean fuzzy score function and distance measure based group decision making framework for household waste recycling plant location selection

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The household waste (HW) disposal and recycling have become a significant challenge due to increasing quantities of generated household wastes and increased levels of urbanization. Selecting locations/sites for building new HW recycling plant comprises numerous sustainability dimensions, thus, this work aims to develop new decision-making model for evaluating and prioritizing the HW recycling plant locations. This paper is categorized into three phases. First, we propose new improved score function to compare the Fermatean fuzzy numbers. Moreover, an example is presented to validate the effectiveness of proposed score function over the extant ones. Second, we introduce new distance measure to estimate the discrimination degree between Fermatean fuzzy sets (FFSs) and further discuss its advantages over the prior developed Fermatean fuzzy distance measures. Third, we introduce an integrated methodology by combining the method with the removal effects of criteria (MEREC), the stepwise weight assessment ratio analysis (SWARA) and the measurement alternatives and the ranking according to compromise solution (MARCOS) approaches with Fermatean fuzzy (FF) information, and named as the "FF-MEREC-SWARA-MARCOS" framework. In this method, the FF-distance measure is used to find the weights of involved decision-making experts. Moreover, an integrated criteria weighting method is presented with the combination of MEREC and SWARA models under the context of FFSs, while the combined FF-MEREC-SWARA-MARCOS model is applied to evaluate and prioritize the locations for HW recycling plant development, which illustrates its feasibility of the developed framework. Comparative study and sensitivity assessment are conducted to validate the obtained outcomes. This work provides a hybrid decision analysis approach, which marks a significant impact to the HW recycling plant location selection process with uncertain information.

Keywords Fermatean fuzzy sets, Distance measure, Household waste recycling plant, MARCOS, Score function, Decision-making

With the rapid urbanization and population growth, annual "household waste (HW)" generation is rapidly increasing in all over the world. The classification of HW activities includes the storage, collection, segregation, transport, disposal and recycling of HW according to the rules or standards[1]. For HW disposal, it is required to segregate them into different categories such as organic, non-hazardous, toxic, e-waste etc. Municipal workers collect waste from different dumping centres into vehicles and send them to recycling plants. The strategic plan

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of waste management targets the segregation, collection, transportation, disposal, recycle, reuse and recovery of HW in order to reduce the public health and environmental risks[2,3]. Mismanagement of HW can expand the risk of environmental problems such water scarcity, air pollution, soil contamination[4].

On account of long progression of technological development and significant demand for daily HW disposal, there is a need for constructing new "household waste recycling plants (HWRPs)". In order to construct a new HWRP, selecting a suitable location is a major concern for the government and non-government organizations[5]. Selection of the most suitable location for HWRP would not only reduce the harmful impacts of global warming but also improve the socio-economic growth of a country. Determination of the HWRPL is a strategic decision, which affects various criteria namely transportation costs, construction costs, job creation, sustainability, customer willingness and awareness and others[6–8]. Therefore, location selection for HWRP development is a "multi-criteria decision making (MCDM)" problem. Due to profound implications of MCDM tools, it can be implemented to address the HWRP locations assessment problem.

In general, data in MCDM problems are imprecise and uncertain because of the subjectivity of human mind and vagueness of available information[9–12]. Zadeh[13] initiated the "fuzzy set (FS)" doctrine to tackle the uncertain situation, which has been widely utilized for dealing with realistic MCDM problems. Further, several extensions of FS have been developed from various perspectives and implemented in numerous disciplines of real-life problems[14–19]. As a generalization of FS, an idea of "Fermatean fuzzy set (FFS)"[20] is a novel tool to express the higher levels of uncertainty. In the FFS theory, the cubes sum of belongingness degree (BD) and non-belongingness degree (NBD) of an object is ≤ 1; therefore, the FFS is more flexible than the FS, the intuitionistic FS (IFS) and the Pythagorean FS (PFS). In comparison with the FSs, IFSs and the PFSs, the FFSs can better describe the uncertainty of complex uncertain problems. Yang et al.[21] proposed a TOPSIS method with weighted distance measure and presented its utility in green low-carbon port evaluation problem from sustainability and uncertainty perspectives. Mishra et al.[22,23] firstly analysed the shortcomings of extent works on FFSs. In addition, they combined two different methods for developing a new decision-making framework from Fermatean fuzzy (FF) information viewpoint. With the use of GIS and TOPSIS methods, Hooshangi et al.[24] evaluated and prioritized the sites for solar farm development in the context of FF information. Zhong et al.[25] proposed novel Muirhead mean operators for aggregating the FF numbers (FFNs). Based on these operators, they proposed an improved Failure mode and effects analysis approach to detect the possible faults in the production management. Gao et al.[26] integrated the best worst method (BWM) and VIKOR approach with FFSs to construct a hybrid decision-making methodology for solving healthcare waste treatment method assessment. Golui et al.[27] modified the Gul et al.'s FF-TOPSIS method[28] using correlation coefficient in place of distance measure. In this regard, they proposed a new correlation coefficient and presented its properties. Apart from these studies, many theories and applications have been presented using FFSs[29–31].

In MCDM, criterion weight-finding approaches are characterized as "*Objective and Subjective*" weighting procedures. Numerous authors[32–34] have presented different objective and subjective weighting procedures. A "method with the removal effects of criteria (MEREC)" is an objective weighting model that utilizes the removal effect of each attribute on the assessment of options and determines the weight of criterion[35,36]. In the literature, several studies have been presented regarding the applications of MEREC method. For example, Haq et al.[37] suggested a new model by incorporating the single-valued neutrosophic MEREC and MARCOS methods with an application in solving the sustainable material assessment problem. A collective decision model has been put forward with the integration of MEREC, ranking sum and DNMA methods with IFSs. Further, they used their model for evaluating the alternative fuel vehicles selection within the context of IFSs[38]. Ecer and Aycin[39] recommended an innovative MEREC-based method to measure the innovation performances of G7 countries. Yu et al.[36] designed new MCDM using the MEREC, the BWM and proximity indexed rating model to evaluate and prioritize offshore wind farm sites from interval 2-tuple linguistic perspective. Yet, no one has incorporated the MEREC, SWARA and MARCOS methods with FFSs for assessing the locations of HWRP development.

For subjective weighting model, Keršuliene et al.[40] pioneered the idea of "stepwise weight assessment ratio analysis (SWARA)" that has lower computational complexity compared to some other methods. During the process of criteria weight computation, the SWARA model estimates the decision-making experts' (DMEs') view related to the importance ratings of criteria. Salamai[41] incorporated the SWARA and the VIKOR models for the purpose of ranking risks of green supply chain with neutrosophic information. Ayyildiz[42] developed an innovative Fermatean fuzzy SWARA model for prioritizing the indicators to attain the goals of sustainable development. Further, Stevic[43] applied the fuzzy SWARA for the assessment of mutual significance of criteria. In addition, they drawn some negative conclusions based on an objective criticism using the fuzzy SWARA model. Pandey and Khurana[44] studied a hybrid PFSs-based SWARA-COPRAS model for mitigating the risks of industry 4.0. Since its appearance, several researches have been presented regarding the SWARA method[45,46].

As an innovative and effective approach, the "measurement of alternatives and ranking according to compromise solution (MARCOS)" model measures and prioritizes the options using the compromise solution[47]. In the recent past, several MCDM methodologies have been introduced using MARCOS method. For instance, Torkayesh et al.[48] presented the hybrid BWM-GIS-MARCOS tool to assess and prioritize the landfill sites for the healthcare waste treatment under the context of grey interval set. Ali[49] discussed the MARCOS model on q-ROFS settings to deal with the MCDM problem. In 2022, Badi et al.[50] discussed MARCOS approach to evaluate and prioritize the wind farm site alternatives in relation to different aspects of sustainability. Du et al.[51] presented an integrated MCDM tool by combining the BWM and the MARCOS models and presented its utility in the assessment of regional distribution network outage loss with reference to numerous indicators. With the use of PFSs, Mishra et al.[22,23] proposed a decision-making tool by incorporating the objective-subjective weighting model with the MARCOS method and applied for the assessment of sustainable circular suppliers by means of twenty-five criteria. Further, many scholars have extended the conventional MARCOS method on

diverse fuzzy sets[52–54]. However, there is no work about the assessment of desirable HWRP locations using a hybrid Fermatean fuzzy decision-making method.

Owing to the broader range of fuzzy information and flexibility in dealing with the practical decision-making problems, this work aims to develop a hybrid MCDM model using FF information and implements to assess the sites for HWRP construction. At present, there is no work which considers the integration of the MEREC, the SWARA and the MARCOS models with FF information for solving HWRPLs problem. Consequently, this study introduces an incorporated decision support system by combining the score function, the distance measure-based model, the MEREC model, the SWARA model and the MARCOS approach with FFSs. Further, we implement the developed methodology for assessing the HWRP locations with respect to different criteria. Based on the abovementioned discussions, the main outcomes of this study are given as.

- To rank the FFNs, new score function is developed, which evades the shortcomings of extant FF-score functions[20,55,56].
- To quantify the distance between FFSs, new FF-distance measure is proposed. The developed FF-distance measures can deal with the shortcomings of extant FF-distance measures[57–59].
- An integrated weight-determining model is developed with the combination of MEREC and SWARA methods with FFSs.
- A hybrid FF-MEREC-SWARA-MARCOS methodology is proposed for dealing with the HWRPL selection under sustainability perspective.
- The proposed methodology is applied to a case study of HWRPL assessment problem with FF information, which proves its efficacy and rationality.

Remaining study is summarized as Sect. "Literature review" gives the comprehensive review related to HWRPLs assessment. Section "Proposed FF-score function and distance measure on FFSs" splits into three subsections: (i) some basic definitions are presented related to this study, (ii) new FF-score function is developed to avoid the limitations of some extant FF-score functions and (iii) new FF-distance measure is introduced to evade the shortcomings of extant FF-distance measures. Section "Proposed hybrid FF-MEREC-SWARA-MARCOS method" presents a hybrid MCDM methodology by incorporating the MEREC, the SWARA and the MARCOS approaches with FFSs. Section "Results and discussion" applies the developed FF-MEREC-SWARA-MARCOS model to a case study of HWPRL assessment problem. Moreover, obtained findings are certified by the comparative discussion and sensitivity investigation. Section "Conclusions" discusses the conclusions and the need for further study.

Literature review

This section highlights the previous studies on location selection problem of waste recycling plant. Existing studies made rich developments on the assessment of different types of waste recycling plant using MCDM approaches. For instance, Shi et al.[7] incorporated the "genetic algorithm (GA)"-based framework for assessing suitable location of construction waste recycling plant opening. In that study, the authors firstly used GA for getting the fundamental concepts for determining the optimum result. Kumar et al.[60] used a collective tool by combining the BWM and the VIKOR approach to assess and prioritize the recycling plant sites for "waste electrical and electronic equipment (WEEE)". Their findings reveal that the plant location selection for WEEE recycling assists to maximize the recovery rate of important assets and minimize the harmful effects on the public health and atmosphere. Zhang et al.[61] assessed the sites for a HW processing plant development from Pythagorean fuzzy perspective. Moreover, they proposed an innovative Pythagorean fuzzy aggregation operators-based decision model to deal with the problem of HW processing plant location in Shanghai, China. Sheriff et al.[62] identified the most appropriate site for battery recycling plant construction. In that study, they introduced a hybrid MCDM tool to evaluate the construction plant locations for battery recycling plant from sustainability perspective. Roy et al.[63] gave the GIS-based MCDM tool for assessing municipal solid waste sites. To assess the recycling center sites for plastic waste in the urban region, Torkayesh and Simic[64] presented a hybridized MCDM framework by integrating the stratified BWM, "combined compromise solution (CoCoSo)" and the WASPAS with considered criteria. Their research findings concluded that the Pendik district is the optimal site by considering the sustainability dimensions. Till now, very few studies have worked on HWRPLs assessment using uncertain information[61].

Based on the extant studies, we are unable to find that "which one is the most suitable location for a HWRP construction considering the sustainability aspects?" To conquer the question, the given questions should to be solved:

- What are the prime factors/indicators for selecting the most suitable location under uncertain environment?
- Which is the most significant factor for HW recycling plant location assessment problem?
- Which is the most suitable decision-making method for assessing the sustainable locations for a HWRL?

The main objectives are summarized as.

- To determine the main factors for HWRP locations assessment through literature survey and DMEs' preferences.
- To introduce a weight-determination model for determining the criteria weights.
- To propose a MCDM model for choosing the suitable sites for a HWRP under FFSs environment.

Proposed FF-score function and distance measure on FFSs

The current part of this study firstly presents the fundamental definitions and then, proposes a hybrid FF-MEREC-SWARA-MARCOS model to deal with MCDM problems.

Preliminaries

Definition 3.1 Mathematically, an FFS T on finite universal set $\Omega = \{e_1, e_2, \ldots, e_n\}$ is given as[20]

$$T = \{\langle e_i, (\hbar_T(e_i), \lambda_T(e_i))\rangle | \ e_i \in \Omega\},$$

wherein $\hbar_T, \lambda_T : \Omega \to [0, 1]$ denote the BD and NBD of an object $e_i \in \Omega$ to T, respectively, satisfying $0 \leq (\hbar_T(e_i))^3 + (\lambda_T(e_i))^3 \leq 1$. The degree of indeterminacy is $\pi_T(e_i) = \sqrt[3]{1 - \hbar_T^3(e_i) - \lambda_T^3(e_i)}, \forall e_i \in \Omega$. The term $(\hbar_T(e_i), \lambda_T(e_i))$ is defined as "Fermatean fuzzy number (FFN)", and simply denoted as $\alpha = (\hbar_\alpha, \lambda_\alpha)$ satisfying $\hbar_\alpha, \lambda_\alpha \in [0, 1]$ and $0 \leq \hbar_\alpha^3 + \lambda_\alpha^3 \leq 1$.

Definition 3.2 Consider a FFN $\alpha = (\hbar_\alpha, \lambda_\alpha)$. Then the score and accuracy functions on given FFN are given[20] as

$$\mathbb{S}_{SY}(\alpha) = \left((\hbar_\alpha)^3 - (\lambda_\alpha)^3\right), \tag{1}$$

$$H_{SY}(\alpha) = (\hbar_\alpha)^3 + (\lambda_\alpha)^3, \tag{2}$$

where $\mathbb{S}_{SY}(\alpha) \in [-1, 1]$ and $H_{SY}(\alpha) \in [0, 1]$. Further, Rani and Mishra[55] proposed the normalized score function for a FFN $\alpha = (\hbar_\alpha, \lambda_\alpha)$, given as

$$\mathbb{S}_{RM}(\alpha) = 0.5\left((\hbar_\alpha)^3 - (\lambda_\alpha)^3 + 1\right). \tag{3}$$

Later, Sahoo[56] presented an improved score function for a FFN $\alpha = (\hbar_\alpha, \lambda_\alpha)$, given as

$$\mathbb{S}_S(\alpha) = 0.5\left((\hbar_\alpha)^2 - (\lambda_\alpha)^2 + 1\right)|\hbar_\alpha - \lambda_\alpha|. \tag{4}$$

Definition 3.3 (Senapati and Yager[20]). Let $\alpha = (\hbar_\alpha, \lambda_\alpha), \alpha_1 = (\hbar_{\alpha_1}, \lambda_{\alpha_1})$ and $\alpha_2 = (\hbar_{\alpha_2}, \lambda_{\alpha_2})$ be any three FFNs. Then, some operational laws on given FFNs are presented as

(i) $\alpha^c = (\lambda_\alpha, \hbar_\alpha)$,
(ii) $\alpha_1 \cap \alpha_2 = (\{\hbar_{\alpha_1} \wedge \hbar_{\alpha_2}\}, \{\lambda_{\alpha_1} \vee \lambda_{\alpha_2}\})$,
(iii) $\alpha_1 \cup \alpha_2 = (\{\hbar_{\alpha_1} \vee \hbar_{\alpha_2}\}, \{\lambda_{\alpha_1} \wedge \lambda_{\alpha_2}\})$,
(iv) $\alpha_1 \oplus \alpha_2 = \left(\sqrt[3]{\hbar_{\alpha_1}^3 + \hbar_{\alpha_2}^3 - \hbar_{\alpha_1}^3\hbar_{\alpha_2}^3}, \ \lambda_{\alpha_1}\lambda_{\alpha_2}\right)$,
(v) $\alpha_1 \otimes \alpha_2 = \left(\hbar_{\alpha_1}\hbar_{\alpha_2}, \ \sqrt[3]{\lambda_{\alpha_1}^3 + \lambda_{\alpha_2}^3 - \lambda_{\alpha_1}^3\lambda_{\alpha_2}^3}\right)$,
(vi) $\gamma\alpha = \left(\sqrt[3]{1 - (1 - \hbar_\alpha^3)^\gamma}, \ (\lambda_\alpha)^\gamma\right), \ \gamma > 0$,
(vii) $\alpha^\gamma = \left((\hbar_\alpha)^\gamma, \ \sqrt[3]{1 - (1 - \lambda_\alpha^3)^\gamma}\right), \ \gamma > 0$.

New score function for FFNs

For $p > 1$, a new FF-score function is defined for a FFN $\alpha = (\hbar_\alpha, \lambda_\alpha)$.

$$\mathbb{S}(\alpha) = \left(\frac{(\hbar^3)^p + (1 - \lambda^3)^p}{2}\right)^{1/p}. \tag{5}$$

The developed FF-score function, given by Eq. (5), holds the following properties:

Theorem 3.1 *The developed FF-score function, given by Eq. (5), is monotonically increasing over $\hbar$ and monotonically decreasing over λ.*

Proof The proof of this theorem is given in Section S1 of supplementary file.

Theorem 3.2 The developed FF-score function of a FFN $\alpha = (\hbar_\alpha, \lambda_\alpha)$ fulfils the following results:

(p1) $\mathbb{S}(0, 1) = 0$ and $\mathbb{S}(1, 0) = 1$.
(p2) $0 \leq \mathbb{S}(\alpha) \leq 1$.

Proof The proof of this theorem is given in Section S1 of supplementary file.

Example 3.1 Let us consider $\alpha_1 = (0.4, 0.4)$ and $\alpha_2 = (0.5, 0.5)$ be the given FFNs. It can be noted that the prior developed FF-score functions by Senapati and Yager[20], Rani and Mishra[55] and Sahoo[56] cannot discriminate

the given FFNs because $\mathbb{S}_{SY}(\alpha_1) = \mathbb{S}_{SY}(\alpha_2) = 0$, $\mathbb{S}_{RM}(\alpha_1) = \mathbb{S}_{RM}(\alpha_2) = 0.5$ and $\mathbb{S}_S(\alpha_1) = \mathbb{S}_S(\alpha_2) = 0$. However, the proposed FF-score function computes the results as $\mathbb{S}(\alpha_1) = 0.6634$ and $\mathbb{S}(\alpha_2) = 0.625$. Thus, $\alpha_1 > \alpha_2$. It implies that introduced FF-score function (4) can effectively distinguish the considered FFNs.

New FF-distance measure

To determine the dissimilarity degree between FFSs, a new FF-distance measure is proposed, which can successfully handle the shortcomings of extant FF-distance measures[57–59]. Let S and T be two FFSs. Then new FF-distance measure is given by

$$d(S, T) = \frac{1}{4t} \sum_{i=1}^{t} \begin{bmatrix} \left| \hbar_S^3(e_i) - \hbar_T^3(e_i) \right| + \left| \lambda_S^3(e_i) - \lambda_T^3(e_i) \right| + \left| \hbar_S^3(e_i) \lambda_T^3(e_i) - \hbar_T^3(e_i) \lambda_S^3(e_i) \right| \\ + \left(\left| \min\left\{ \hbar_S^3(e_i), \lambda_T^3(e_i) \right\} - \min\left\{ \hbar_T^3(e_i), \lambda_S^3(e_i) \right\} \right| \\ + \left| \max\left\{ \hbar_S^3(e_i), \lambda_T^3(e_i) \right\} - \max\left\{ \hbar_T^3(e_i), \lambda_S^3(e_i) \right\} \right| \right) \end{bmatrix}. \quad (6)$$

Theorem 3.3 For $S, T \in FFSs(\Omega)$, the real-valued function $d(S, T)$ satisfies the given properties:

(a_1). $0 \le d(S, T) \le 1$,
(a_2). $d(S, T) = d(T, S)$,
(a_3). $d(S, T) = 0 \Leftrightarrow S = T$,
(a_4). If $R \subseteq S \subseteq T$, then $d(R, T) \ge d(R, S)$ and $d(R, T) \ge d(S, T)$, $\forall R, S, T \in FFSs(\Omega)$.

Proof The proof is given in Section S1 of supplementary file.

In order to verify the efficiency of introduced distance measure $d(S, T)$, we compare it with various extant measures as[57–59]. The computational results are discussed in Section S2 of supplementary file. For this purpose, we take some FFSs to execute the experimental results of introduced and extant measures. From Table 1, we can see that the previously developed measures $d_{A1}(S, T)$, $d_{A2}(S, T)$, $d_{A3}(S, T)$, $d_{A4}(S, T)$, $d_{A5}(S, T)$, $d_{A6}(S, T)$ (we take $\alpha = 0.4$, $\beta = 0.6$), $d_{A7}(S, T)$, $d_{G1}(S, T)$, $d_{G2}(S, T)$, $d_{G3}(S, T)$, $d_{G4}(S, T)$ and $d_k(S, T)$ present unreasonable results. In the line, we discuss some drawbacks of previously developed measures $d_{A1}(S, T)$, $d_{A2}(S, T)$, $d_{A3}(S, T)$, $d_{A4}(S, T)$, $d_{A5}(S, T)$, $d_{A6}(S, T)$, $d_{A7}(S, T)$, $d_{G1}(S, T)$, $d_{G2}(S, T)$, $d_{G3}(S, T)$, $d_{G4}(S, T)$ and $d_k(S, T)$:
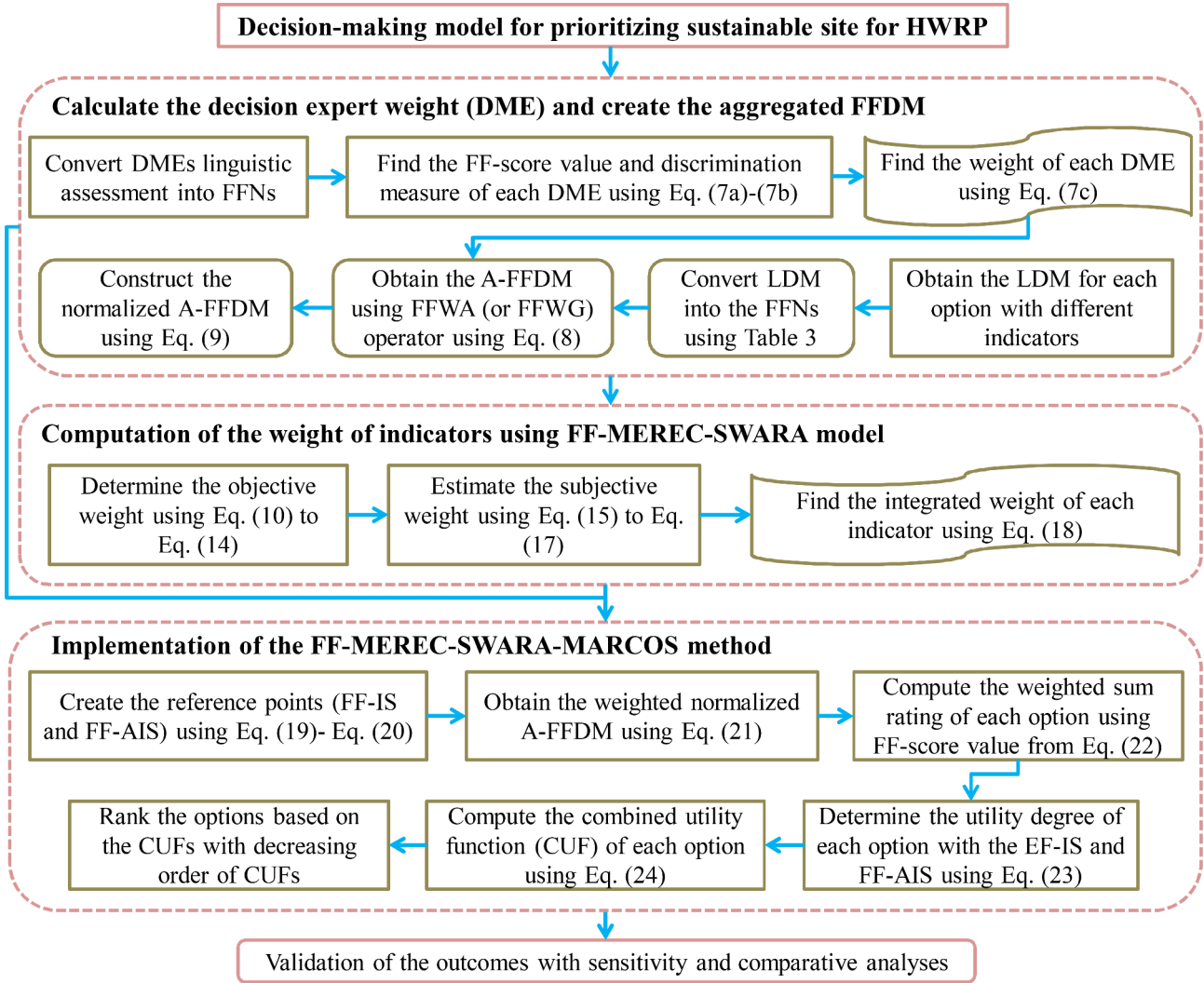
- Ashraf et al.'s measure $d_{A6}(S, T)$ presents the division by zero problem for the sets $S = \{(e_1, 1.0, 0.0)\}$ and $T = \{(e_1, 0.0, 0.0)\}$.
- For the Cases 1 and 5, Ashraf et al.'s measure $d_{A2}(S, T)$ does not hold the property (a_3) of FF-distance measure. Additionally, other measures present the following results: $d_{A1}(S, T) = d_{A4}(S, T) = d_{A7}(S, T) = d_{G1}(S, T) = d_{G2}(S, T) = d_{G3}(S, T) = d_{G4}(S, T) = 0$ (in Case-3) and $d_{A6}(S, T) = d_{A7}(S, T) = 0$ (in Case-4), which are indeed not a crisp number.
- Previously developed measures $d_{A3}(S, T)$, $d_{A4}(S, T)$, $d_{A5}(S, T)$, $d_{A6}(S, T)$, $d_{A7}(S, T)$, $d_{G1}(S, T)$, $d_{G2}(S, T)$, $d_{G3}(S, T)$, $d_{G4}(S, T)$ (in Case-1 and Case-2) have no capabilities to define the positive discrimination from negative discrimination. Similar cases occur for $d_{A4}(S, T)$ and $d_{A5}(S, T)$ (in Case-5 and Case-6).

Thus, it follows that the proposed FF-distance measure $d(S, T)$ is more robust than various extant FF-distance measures.

S T	Case-1 $\{(e_1, 0.3, 0.3)\}$ $\{(e_1, 0.4, 0.4)\}$	Case-2 $\{(e_1, 0.3, 0.4)\}$ $\{(e_1, 0.4, 0.3)\}$	Case-3 $\{(e_1, 1, 0)\}$ $\{(e_1, 0, 0)\}$	Case-4 $\{(e_1, 0.5, 0.5)\}$ $\{(e_1, 0, 0)\}$	Case-5 $\{(e_1, 0.4, 0.2)\}$ $\{(e_1, 0.5, 0.3)\}$	Case-6 $\{(e_1, 0.4, 0.2)\}$ $\{(e_1, 0.5, 0.2)\}$
$d(S, T)$	0.018	0.038	0.5	0.062	0.04	0.031
$dA1(S, T)$	0.074	0.037	**1**	0.25	0.08	0.061
$dA2(S, T)$	**0**	0.037	0.5	**0**	0.021	0.03
$dA3(S, T)$	**0.037**	**0.037**	0.75	0.125	0.05	0.046
$dA4(S, T)$	**0.037**	**0.037**	**1**	0.125	**0.061**	**0.061**
$dA5(S, T)$	**0.929**	**0.929**	**0**	0.778	**0.885**	**0.885**
$dA6(S, T)$	**0.578**	**0.578**	**NaN**	**1**	0.617	0.195
$dA7(S, T)$	**0.578**	**0.578**	**1**	**1**	0.526	0.459
$dG1(S, T)$	**0.071**	**0.071**	**1**	0.222	0.078	0.061
$dG2(S, T)$	**0.073**	**0.073**	**1**	0.234	0.079	0.061
$dG3(S, T)$	**0.074**	**0.074**	**1**	0.25	0.08	0.061
$dG4(S, T)$	**0.074**	**0.074**	**1**	0.246	0.08	0.061
$dk(S, T)$	0.315	0.305	0.854	0.389	0.325	0.316

Table 1. Computational results of the proposed and extant distance measures (Bold and highlighted cell shows unreasonable results and "NaN" represents division by zero problems).

Decision-making model for prioritizing sustainable site for HWRP

Calculate the decision expert weight (DME) and create the aggregated FFDM

Convert DMEs linguistic assessment into FFNs → Find the FF-score value and discrimination measure of each DME using Eq. (7a)-(7b) → Find the weight of each DME using Eq. (7c)

Construct the normalized A-FFDM using Eq. (9) ← Obtain the A-FFDM using FFWA (or FFWG) operator using Eq. (8) ← Convert LDM into the FFNs using Table 3 ← Obtain the LDM for each option with different indicators

Computation of the weight of indicators using FF-MEREC-SWARA model

Determine the objective weight using Eq. (10) to Eq. (14) → Estimate the subjective weight using Eq. (15) to Eq. (17) → Find the integrated weight of each indicator using Eq. (18)

Implementation of the FF-MEREC-SWARA-MARCOS method

Create the reference points (FF-IS and FF-AIS) using Eq. (19)- Eq. (20) → Obtain the weighted normalized A-FFDM using Eq. (21) → Compute the weighted sum rating of each option using FF-score value from Eq. (22)

Rank the options based on the CUFs with decreasing order of CUFs ← Compute the combined utility function (CUF) of each option using Eq. (24) ← Determine the utility degree of each option with the EF-IS and FF-AIS using Eq. (23)

Validation of the outcomes with sensitivity and comparative analyses

Fig. 1. Flowchart of proposed decision-making methodology.

QTs	FFNs
Extremely skilled (ES)	(0.9,0.2)
Much skilled (MS)	(0.8,0.4)
Skilled (S)	(0.7,0.5)
Slight skilled (SS)	(0.6,0.7)
Less skilled (LS)	(0.5,0.8)

Table 2. Ratings of attributes and DMEs in QTs.

Proposed hybrid FF-MEREC-SWARA-MARCOS method

The present section develops a hybrid MARCOS method based on the combination of DMEs' weighting model, the FF-weighted averaging operator, and an integrated objective-subjective weighting model with FF information. Figure 1 presents the pictorial representation of the proposed framework. This method involves the following steps:

Step 1: Formation of "qualitative decision matrix (QDM)".

A panel $\{g_1, g_2, ..., g_\ell\}$ of DMEs is formed to assess a set of options $F = \{F_1, F_2, ..., F_p\}$ over the criteria set $V = \{V_1, V_2, ..., V_q\}$. Here, each DME presents his/her views for the performance of each option concerning diverse criteria in the form of "qualitative term (QT)" as Likert scale, which are given in Tables 2, 3 and adopted from Rani and Mishra[55]. Let $\Xi = \left[o_{ij}^{(k)} \right]_{p \times q}$ be the "qualitative decision-matrix (QDM)", in which $o_{ij}^{(k)}$ specifies the rating of each option F_i over criterion V_j by kth DME.

Step 2: Estimation of weight of experts.

QTs	FFNs
Extremely preferred (EP)	(0.95,0.2)
Most preferred (MP)	(0.9,0.3)
Preferred (P)	(0.85,0.4)
Quite preferred (QP)	(0.8,0.5)
Slightly preferred (SP)	(0.75,0.6)
Average (A)	(0.6,0.7)
Slightly unreferred (SU)	(0.5,0.8)
Quite unreferred (QU)	(0.4,0.85)
Unreferred (U)	(0.3,0.9)
Much unreferred (MU)	(0.2,0.95)

Table 3. QTs of the alternatives over diverse criteria for HWRPLs selection.

To find the weight of DME, various techniques have been discussed. Some of them only focus on the score function-based formula or merely pay attention on discrimination information of DMEs. In this study, we present a combined weight model to obtain the weight of DME using the FF-score function-based method and proposed FF-distance measure.

Based on QT and associated FFN $g_k = (\hbar_k, \lambda_k)$, we determine the individual assessment degree of each DME using FF-score value and given as

$$\Phi_k^1 = \frac{\hbar_k^3 \left(2 - \hbar_k^3 - \lambda_k^3\right)}{\sum\limits_{k=1}^{\ell} \left[\hbar_k^3 \left(2 - \hbar_k^3 - \lambda_k^3\right)\right]}, \forall \ k = 1, 2, ..., \ell. \tag{7a}$$

Here, $\Phi_k^1 \geq 0$ and $\sum\limits_{k=1}^{\ell} \Phi_k^1 = 1$.

We compute the discrimination degree of each DME with developed FF-distance measure to estimate the normalized weight of DME as follows:

$$\Phi_k^2 = \frac{\frac{1}{k-1} \sum\limits_{t=1}^{\ell} d\left(g_k, \ g_t\right)}{\sum\limits_{k=1}^{\ell} \left(\frac{1}{l-1} \sum\limits_{t=1}^{\ell} d\left(g_k, \ g_t\right)\right)}, \quad k = 1, 2, ..., \ell. \tag{7b}$$

Here, $\Phi_k^2 \geq 0$ and $\sum\limits_{k=1}^{\ell} \Phi_k^2 = 1$.

Hence, the combined weight Φ_k for each DME can be calculated in the following expression:

$$\Phi_k = \frac{\Phi_k^1 * \Phi_k^2}{\sum_{k=1}^{\ell} \Phi_k^1 * \Phi_k^2}, \tag{7c}$$

Here, $\Phi_k \geq 0$ and $\sum\limits_{k=1}^{\ell} \Phi_k = 1$. Thus, the higher the rating of Eq. (7c) is, the larger weight should be given to the DME $g_k, k = 1, 2, ..., \ell$.

Step 3: Aggregate the individual opinions.

To aggregate the QDM $\Xi = \left[o_{ij}^{(k)}\right]_{p \times q}$, FF-weighted averaging operator[20] is implemented on QDM and obtained an aggregated FF-decision matrix (A-FF-DM) $A = (\upsilon_{ij})_{p \times q}$, where

$$\upsilon_{ij} = (\hbar_{ij}, \lambda_{ij}) = FFWA_{\Phi_k} \left(o_{ij}^{(1)}, o_{ij}^{(2)}, ..., o_{ij}^{(\ell)}\right) = \left(\sqrt[3]{1 - \prod_{k=1}^{\ell} \left(1 - \left(\hbar_{ij}^{(k)}\right)^3\right)^{\Phi_k}}, \ \prod_{k=1}^{\ell} \left(\lambda_{ij}^{(k)}\right)^{\Phi_k}\right). \tag{8}$$

Step 4: Determination of weight of criteria.

Let $W = (\omega_1, \omega_2, ..., \omega_q)^T$ be a collection of criteria weights satisfying $\sum\limits_{j=1}^{q} \omega_j = 1$ and $\omega_j \in [0, \ 1]$. Then, a scheme for finding weight of criteria is discussed as follows:

Case 1: Obtain the objective weight of attributes.

Step 4.1: Normalization of the A-FF-DM.

If the A-FF-DM contains cost and benefit types of criteria, then it is needed to create the normalized A-FF-DM (NA-FF-DM) $\mathbb{N} = (\varsigma_{ij})_{p \times q}$, where ς_{ij} is a normalized FFN, defined by

$$\varsigma_{ij} = \left(\overline{\hbar}_{ij}, \overline{\lambda}_{ij}\right) = \begin{cases} \upsilon_{ij} = \left(\hbar_{ij}, \lambda_{ij}\right), & j \in V_b, \\ \left(\upsilon_{ij}\right)^c = \left(\lambda_{ij}, \hbar_{ij}\right), & j \in V_n, \end{cases} \tag{9}$$

where V_b is benefit type criterion and V_n is cost type criterion.

Step 4.2: Computation of FF-score matrix.

Applying Eq. (5) to develop the FF-score matrix $\Theta = (\delta_{ij})_{p \times q}$ of each FFN ς_{ij} using Eq. (10) as follows:

$$\delta_{ij} = \left(\frac{(\overline{\hbar}_{ij}^3)^p + (1 - \overline{\lambda}_{ij}^3)^p}{2}\right)^{1/p}. \tag{10}$$

Step 4.3: Define performance of option.

By means of Step 4.2, the performance of each option over considered attributes is revealed in Eq. (11) as

$$\Psi_i = \ln\left(1 + \left(\frac{1}{q}\sum_j |ln\left(\delta_{ij}\right)|\right)\right). \tag{11}$$

Step 4.4: Estimation of performance of option removing of each criterion.

The assessment degree of ith option by eliminating jth attribute, discussed as

$$\Psi'_{ij} = \ln\left(1 + \left(\frac{1}{q}\sum_{k,k \neq j} |ln\left(\delta_{ik}\right)|\right)\right). \tag{12}$$

Step 4.5: Computation of absolute deviation of each attribute.

We compute the deviations of jth criterion with the Step 4.3 and Step 4.4. Let $\mho_j$ denotes the deviation of jth criterion, then

$$\mho_j = \sum_i \left|\Psi'_{ij} - \Psi_i\right|. \tag{13}$$

Step 4.6: Find the weight value of each criterion.

Considering the absolute deviation, we estimate the weight of each attribute as

$$\omega_j^o = \frac{\mho_j}{\sum\limits_{j=1}^{q} \mho_j}. \tag{14}$$

Case 2: Determination of subjective weight with the FF-SWARA model.

The SWARA method firstly finds the score value of each criterion using the DMEs' opinions. On the basis of obtained FF-score values, arrange the criteria from higher to lower FF-score values. This process comprises following phases:

Step 4.7: Each DME assesses the criteria set and presents their opinions regarding each criterion in terms of LVs. With the help of FFWA operator, find the aggregated value $G = (\alpha_j)_{1 \times q}$, of criteria performances given by each DME as follows:

$$G = (\alpha_j)_{1 \times q} = FFWA_{\Phi_k}\left(\alpha_j^{(1)}, \alpha_j^{(2)}, ..., \alpha_j^{(\ell)}\right), j = 1, 2, ..., q.$$

Also, we compute the FF-score rating using Eq. (5) for each criterion.

Step 4.8: Determine the relative importance (s_j) of attribute using FF-score rating.

Step 4.9: Find the comparative coefficient using Eq. (15), and given by

$$k_j = \begin{cases} 1, & j = 1, \\ s_j + 1, & j > 1. \end{cases} \tag{15}$$

Step 4.10: Calculate the weight of attribute using the expression (16) as

$$r_j = \begin{cases} 1, & j = 1 \\ \dfrac{r_{j-1}}{k_j}, & j > 1 \end{cases} \tag{16}$$

Step 4.11: Computation of overall weight of attribute with Eq. (17), and given by

$$\omega_j^s = \frac{r_j}{\sum_{j=1}^q r_j}, \; j = 1, 2, ..., q. \tag{17}$$

Case 3: Based on Case 1 and Case 2, we compute the aggregated or combined weight given by

$$\omega_j = \vartheta \, \omega_j^s + (1 - \vartheta) \, \omega_j^o, \tag{18}$$

where ϑ is the aggregating coefficient of decision precision parameter within the range of 0 to 1.

Step 5: Express the positive-ideal rating (PIR) and negative-ideal rating (NIR) on FFNs using Eq. (19) and Eq. (20), respectively, as

$$\alpha_j^+ = \left(\hbar_j^+, \lambda_j^+ \right) = \begin{cases} \left(\max_i \hbar_{ij}, \min_i \lambda_{ij} \right), & j \in V_b, \\ \left(\min_i \hbar_{ij} \max_i \lambda_{ij} \right), & j \in V_n. \end{cases} \tag{19}$$

$$\alpha_j^- = \left(\hbar_j^-, \lambda_j^- \right) = \begin{cases} \left(\min_i \hbar_{ij}, \max_i \lambda_{ij} \right), & j \in V_b, \\ \left(\max_i \hbar_{ij}, \min_i \lambda_{ij} \right), & j \in V_n. \end{cases} \tag{20}$$

Step 6: Compute the weighted normalized A-FF-DM (WNA-FF-DM).

We obtain the WNA-FF-DM $\mathbb{N}_w = \left(\widehat{\varsigma}_{ij} \right)_{p \times q}$ as

$$\widehat{\varsigma}_{ij} = \left(\widehat{\hbar}_{ij}, \widehat{\lambda}_{ij} \right) = \omega_j \varsigma_{ij} = \left(\sqrt[3]{1 - \left(1 - \left(\overline{\hbar}_{ij} \right)^3 \right)^{\omega_j}}, \; \left(\overline{\lambda}_{ij} \right)^{\omega_j} \right). \tag{21}$$

Step 7: Estimation of the FF-score rating of the WNA-FF-DM using Eq. (5) as

$$\Delta_i = \sum_{j=1}^q \mathbb{S} \left(\widehat{\varsigma}_{ij} \right), \; i = 1, 2, ..., p, \tag{22}$$

wherein $\mathbb{S} \left(\widehat{\varsigma}_{ij} \right)$ means the proposed FF-score rating using Eq. (5).

Step 8: Finding utility degree (UD) of options with the PIR and NIR as

$$m_i^- = \frac{\Delta_i}{\Delta_{ais}} \text{ and } m_i^+ = \frac{\Delta_i}{\Delta_{is}}, \tag{23}$$

where Δ_{is} and Δ_{ais} denote summation of FF-score ratings of weighted PIR α_{jw}^+ and NIR α_{jw}^-, respectively.

Step 9: Assessing combined utility function (CUF).

the CUF of each alternative with weighted PIR and NIR is obtained as.

$$g(m_i) = \frac{m_i^+ + m_i^-}{1 + \frac{1 - g(m_i^+)}{g(m_i^+)} + \frac{1 - g(m_i^-)}{g(m_i^-)}}, \text{ where } g\left(m_i^+\right) = \frac{m_i^-}{m_i^- + m_i^+} \text{ and } g\left(m_i^-\right) = \frac{m_i^+}{m_i^- + m_i^+}, \; i = 1, 2, ..., p. \tag{24}$$

Step 10: Prioritizing options with the CUFs and choosing the suitable one with highest CUF rating.

Results and discussion
This section first shows a case study of HWRPLs problem and further implements the proposed FF-MEREC-SWARA-MARCOS approach for choosing an appropriate location for HWRP. Next, it presents the comparison and sensitivity analysis to confirm the obtained findings. Lastly, we discuss the implications of the proposed work.

Case study: household waste recycling plant location (HWRPL) selection
Increasing quantity and complexity of HW concerns about recycling of waste materials. India faces many environmental challenges due to HW mismanagement. In the case study, we have chosen Indore region, cleanest and leading city in the Madhya Pradesh (India) to locate the HWRP. To this aim, we consider a case study of HWRPL selection of an Indian company, located in Indore. This enterprise has been working for last 15 years and is recognized as a market frontrunner in its waste recycling facilities. Due to increasing amount of HW, this enterprise needs to create a new HWRP but it does not have any appropriate system for constructing location. In this work, we concentrate on the application of robust methodology for assessing the HWRPLs which will help the DMEs to assess an appropriate location for HWRP.

To collect the data for the assessment and investigation, we planned in-person meetings with the DMEs. Though we invited nine DMEs and out of which, four DMEs are approved to collaborate with us during the preparation of questionnaires. In the committee of four DMEs, each DME has more than 12 years' expertise in the discipline of MSW management, sustainability and ecological planning and gave their views in taking an appropriate decision. Out of which two of them are from the MSW management, one DME is from sustainability and the other one is from ecological planning. The DMEs supported with scholars during the complete study. They planned some strategies that can be executed in other HWRPL selection problem in India. Then, we have studied the related literature to choose the HWRPLs as alternatives in India. Lastly, we have determined 10 criteria, which are denoted as $V_1, V_2, \ldots, V_{10}$. In this study, economic, social, environmental and risk aspects are considered for assessing the HWRPLs. After preliminary analysis, the panel has selected four locations in Indore as possible alternatives, which are location-1 (F_1), location-2 (F_2), location-3 (F_3) and location-4 (F_4). Table S1 presents the sample questionnaire to evaluate the HWRPLs with respect to multiple criteria (see Section S3 in supplementary file). Table 4 and Fig. 2 present the list of criteria obtained from online questionnaire and experts' opinions. In order to mitigate the subjective randomness, the qualitative data is transformed to FFNs using the tabular values from Tables 2–3. This case study is presented for demonstration purpose of choosing the best HWRPL, which proves the applicability of the developed approach. Readers may diminish or add some attributes as per their requirements.

Implementation process

Here, we apply the proposed FF-MEREC-SWARA-MARCOS model on the aforesaid case study in order to choose the best HWRPL over multiple criteria. In the following, we provide the computational steps of the developed framework:

Steps 1–3: Table 2 presents the QRs and their associated FFNs to state the rating of DMEs and the defined attributes. Tables 3 reports the linguistic values and their corresponding FFNs for evaluating the possible recycling plant locations. Using Table 2 and Eqs. (7a)–(7c), the DMEs' weights are determined in Table 5. Table 6 presents QDM obtained based on four DMEs (g_1, g_2, g_3, g_4), wherein each QR presents the assessment value of each location F_i against the given attributes. From Eq. (8) and Table 6, an A-FF-DM $M = (v_{ij})_{p \times q}$ is established, given in Table 7.

Step 4: Since some sustainability indicators are of benefit-type and remaining are cost-type, so that we create NA-FF-DM in Table 8 using Eq. (9). Further, we constructed the score matrix based on Eq. (10). To obtain the objective weight with the FF-MEREC model, we find the complete performance of each location using Eq. (11) and given as $\Psi_1 = 0.407$, $\Psi_2 = 0.366$, $\Psi_3 = 0.353$ and $\Psi_4 = 0.392$. Then, the overall performance of each location is determined by considering the removal of each criterion using Eq. (12) and presented in Table 9. From Eq. (13), we derive the addition of absolute derivations and finally calculate weight of each criterion through Eq. (14) (see Fig. 3).

Initially, we calculate the assessment ratings and FF-score values of each criterion provided DMEs from Eq. (5), and given in Table 10. From FF-SWARA model given by Eqs. (15)–(17), we computed the subjective weight with the FF-SWARA model and shown in Table 11 (see Fig. 4). In this context, ω_j^s denotes subjective weight of criteria with FF-SWARA and given as $\omega_j^s = (0.1051, 0.1031, 0.1033, 0.1001, 0.1128, 0.1141, 0.0800, 0.0871, 0.0972, 0.0972)$. Next, we have combined the weights obtained by FF-MEREC and FF-SWARA through Eq. (18). Thus, the combined weight set for $\vartheta = 0.5$ is graphically shown in Fig. 5 and presented as $\omega_j = (0.0768, 0.0730, 0.0703, 0.1298, 0.1417, 0.1105, 0.1247, 0.0728, 0.0713)$.

Here, Fig. 5 exhibits weight of diverse attributes for HWRPLs assessment. The factor distance to the sustainability (V_5) (0.1417) has been obtained the most essential attributes for HWRPLs assessment. Amount of household wastes (V_6) (0.1298) is the second essential factor for HWRPLs assessment. Job creation (V_4) with weight 0.1292 is the third significant attribute for HWRPLs assessment and remaining attributes are taken as essential attribute for HWRPLs assessment.

Step 6: Apply Eqs. (19)–(20) and Table 6, the PIR and NIR on FFNs are presented as follows:
$\alpha_j^+ = \{(0.34, 0.899), (0.333, 0.892), (0.274, 0.917), (0.689, 0.623), (0.65, 0.61), (0.64, 0.544), (0.664, 0.529), (0.666, 0.644), (0.237, 0.919), (0.361, 0.91)\}$.

Dimensions	Criteria	Meanings
Economic (L_1)	Transportation cost (V_1)	It involves the total costs of transportation used for household waste processing plant location selection
	Recycling cost (V_2)	It contains operation and installation costs of the HWRP
	Construction cost (V_3)	It involves the total amount of costs used in the assessment of HW processing plant location
Social (L_2)	Job creation (V_4)	It considers the formation of new jobs as it produces new service capacity
	Sustainability (V_5)	It considers the recovery and recycling processes conducting in the HWRP
Environmental (L_3)	Amount of household wastes (V_6)	It discusses to household wastes volume to be treated at the processing plant
	Capacity of household waste processing plant (V_7)	It involves the utmost quantity of household wastes that can be treated at the HWRP
	Consumers' environmental awareness and willingness (V_8)	It considers that whether consumers are willing to release and classify their household wastes to waste collection site
Risk (L_4)	Operational risk (V_9)	It considers that shortage in the quantity of waste transported to processing plant
	Social risk (V_{10})	It considers the risk of explosion and fire because of the production of methane gas on processing sites

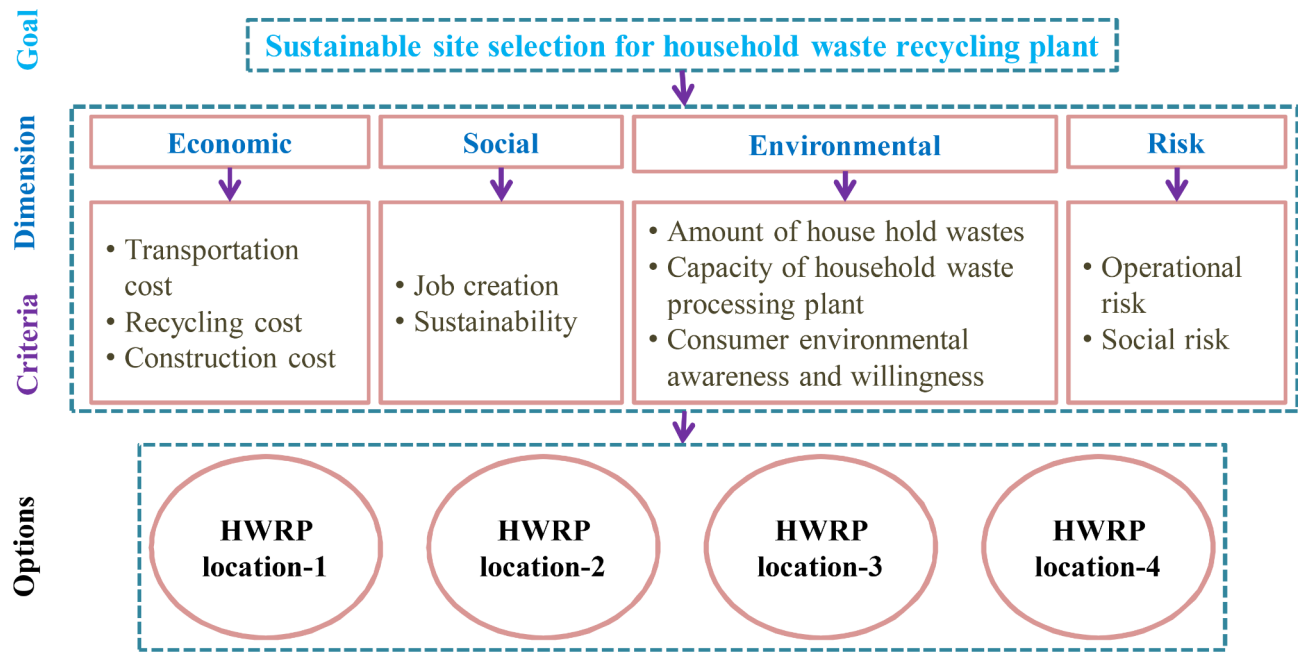**Table 4.** Details of criteria for HWRPL assessment[7,61,65].

Fig. 2. Hierarchical structure of the HWRPL assessment based on 10 indicators/factors.

DEs	QRs	FFNs	Weights
g_1	SS	(0.6,0.7)	0.1252
g_2	ES	(0.9,0.2)	0.3703
g_3	S	(0.7,0.5)	0.2113
g_4	MS	(0.8,0.4)	0.2932

Table 5. Weight of DME for HWRPLs selection.

	F_1	F_2	F_3	F_4
V_1	(QU, SU, SU, U)	(SU, U, A, SU)	(U, SU, A, MU)	(QU, U, SU, SU)
V_2	(A, SU, MU, SU)	(SU, QU, SU, A)	(U, SU, QU, QU)	(MU, QU, SU, QU)
V_3	(U, U, SU, A)	(U, MU, A, QU)	(SU, U, U, SU)	(U, A, U, U)
V_4	(A, SP, QP, P)	(SP, A, MP, P)	(QP, A, MP, SP)	(QP, SP, A, MP)
V_5	(MP, A, A, QP)	(QP, A, QP, EP)	(A, QP, P, A)	(QP, SP, A, A)
V_6	(QP, SP, QP, A)	(MP, SP, SP, A)	(QP, MP, A, SP)	(QP, QP, SP, EP)
V_7	(A, SP, QP, MP)	(SP, QP, P, SP)	(A, MP, QP, EP)	(QP, QP, QP, SP)
V_8	(SP, A, QP, P)	(P, A, P, A)	(QP, SP, QP, SP)	(QP, QP, A, P)
V_9	(QU, A, SU, U)	(MP, SU, A, QU)	(U, SU, SU, SU)	(U, QU, MU, A)
V_{10}	(SU, MU, SU, SU)	(A, U, U, MU)	(SU, A, QU, QU)	(QU, SU, SU, QU)

Table 6. Qualitative decision matrix constructed by four DMEs.

$\alpha_j^- = \{(0.41, 0.843), (0.363, 0.871), (0.339, 0.846), (0.574, 0.704), (0.574, 0.704), (0.644, 0.671), (0.605, 0.684), (0.605, 0.71), (0.402, 0.819), (0.39, 0.823)\}$.

Step 7: From Eq. (21) and Table 8, the WNA-FF-DM for HWRPLs assessment is created and is mentioned in Table 12.

Step 8: Using Eq. (22) and Table 12, the FF-score ratings of options, weighted PIR and NIR are obtained and given in Table 13.

Steps 9–11: Applying Eqs. (23)–(24), we obtain UDs of options as $m_1^+ = 0.819$, $m_2^+ = 0.851$, $m_3^+ = 0.913$, $m_4^+ = 0.875$, $m_1^- = 1.082$, $m_2^- = 1.124$, $m_3^- = 1.206$, $m_4^- = 1.156$ and the CUFs of options as $g(m_1) = 0.617$, $g(m_2) = 0.642$, $g(m_3) = 0.688$ and $g(m_4) = 0.66$. Thus, prioritization of HWRPL options is $F_3 \succ F_4 \succ F_2 \succ F_1$ and the HWRPL-3 (F_3) is best choice with the maximum CUF.

	F_1	F_2	F_3	F_4
V_1	(0.373,0.861)	(0.410,0.867)	(0.410,0.843)	(0.340,0.899)
V_2	(0.363,0.871)	(0.373,0.873)	(0.323,0.878)	(0.333,0.892)
V_3	(0.274,0.917)	(0.379,0.898)	(0.295,0.915)	(0.339,0.846)
V_4	(0.605,0.684)	(0.679,0.637)	(0.689,0.623)	(0.574,0.704)
V_5	(0.601,0.699)	(0.620,0.694)	(0.650,0.610)	(0.574,0.704)
V_6	(0.644,0.671)	(0.665,0.639)	(0.640,0.544)	(0.635,0.637)
V_7	(0.605,0.684)	(0.671,0.598)	(0.664,0.529)	(0.656,0.613)
V_8	(0.605,0.710)	(0.666,0.644)	(0.644,0.655)	(0.591,0.658)
V_9	(0.402,0.819)	(0.406,0.848)	(0.361,0.867)	(0.237,0.919)
V_{10}	(0.361,0.910)	(0.339,0.899)	(0.390,0.823)	(0.373,0.861)

Table 7. The A-FF-DM for HWRPLs evaluation.

	F_1	F_2	F_3	F_4
V_1	(0.861,0.373)	(0.867,0.410)	(0.843,0.410)	(0.899,0.340)
V_2	(0.871,0.363)	(0.873,0.373)	(0.878,0.323)	(0.892,0.333)
V_3	(0.917,0.274)	(0.898,0.379)	(0.915,0.295)	(0.846,0.339)
V_4	(0.605,0.684)	(0.679,0.637)	(0.689,0.623)	(0.574,0.704)
V_5	(0.601,0.699)	(0.620,0.694)	(0.650,0.610)	(0.574,0.704)
V_6	(0.644,0.671)	(0.665,0.639)	(0.640,0.544)	(0.635,0.637)
V_7	(0.605,0.684)	(0.671,0.598)	(0.664,0.529)	(0.656,0.613)
V_8	(0.605,0.710)	(0.666,0.644)	(0.644,0.655)	(0.591,0.658)
V_9	(0.819,0.402)	(0.848,0.406)	(0.867,0.361)	(0.919,0.237)
V_{10}	(0.910,0.361)	(0.899,0.339)	(0.823,0.390)	(0.861,0.373)

Table 8. Normalized A-FF-DM for HWRPLs evaluation.

Parameters	$\left(\Psi'_{ij}\right)$ values				$\mho_j$	ω_j^o
	Ψ'_1	Ψ'_2	Ψ'_3	Ψ'_4		
V_1	0.391	0.350	0.334	0.380	0.062	0.0484
V_2	0.393	0.351	0.339	0.380	0.055	0.0429
V_3	0.398	0.354	0.343	0.375	0.048	0.0372
V_4	0.353	0.321	0.309	0.331	0.204	0.1582
V_5	0.350	0.310	0.307	0.331	0.220	0.1705
V_6	0.357	0.319	0.310	0.344	0.188	0.1454
V_7	0.353	0.323	0.313	0.347	0.182	0.1411
V_8	0.349	0.319	0.302	0.338	0.209	0.1624
V_9	0.387	0.348	0.337	0.383	0.062	0.0483
V_{10}	0.396	0.355	0.333	0.376	0.059	0.0455

Table 9. Objective weight by FF-MEREC for HWRPLs assessment.

Comparative study

In the following, we present comparison of developed and extant MCDM approaches under the context of FFSs. In this regard, some MCDM methods are chosen, which are Mishra & Rani's WASPAS model[66], Gül's ARAS model[67], Simić et al.'s CoCoSo model[68] and Senapati & Yager's TOPSIS model[20] approaches are employed to deal aforesaid problem.

Mishra & Rani's WASPAS model
Steps 1–4: Follow the steps of developed framework.
   Step 5: Finding the values of weighted sum rating and product rating using Eq. (25) and Eq. (26), respectively.

$$\wp_i^{(1)} = \overset{q}{\underset{j=1}{\oplus}} \omega_j \varsigma_{ij}, \tag{25}$$

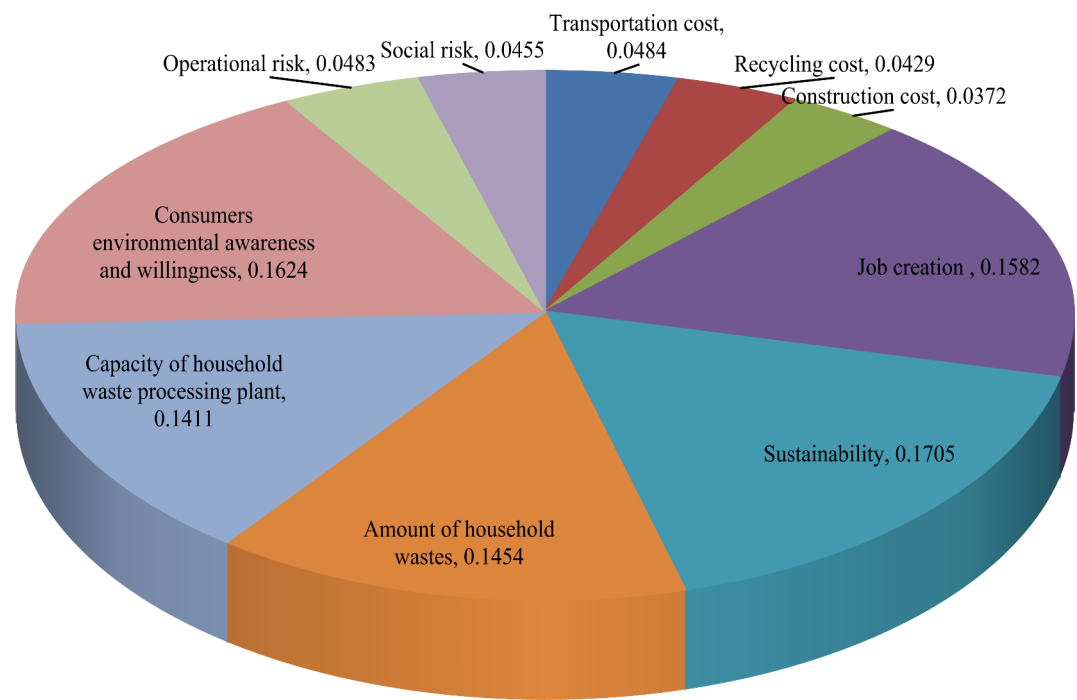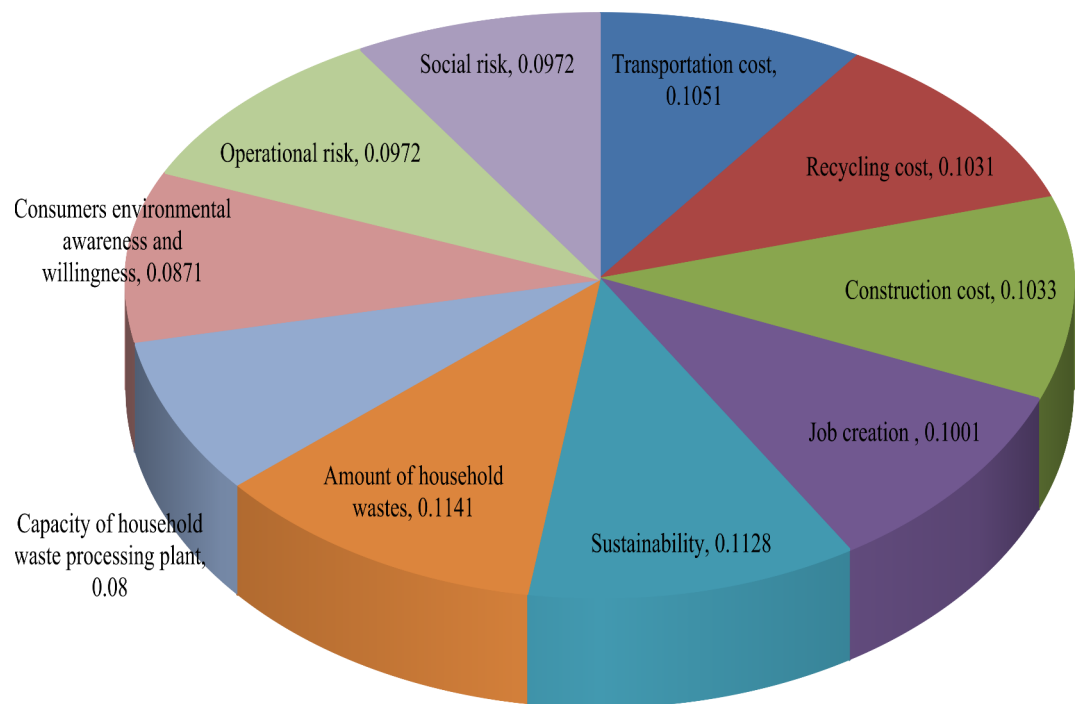**Fig. 3**. Presentation of objective weight by FF-MEREC for HWRPLs assessment.

Criteria	g_1	g_2	g_3	g_4	A-FFNs	Score rating
V_1	A	SP	P	QP	(0.638,0.652)	0.509
V_2	SP	QP	QP	P	(0.644,0.627)	0.490
V_3	SQP	SP	QP	MP	(0.644,0.655)	0.492
V_4	MP	QU	QP	A	(0.651,0.700)	0.467
V_5	A	SP	SP	P	(0.578,0.711)	0.583
V_6	A	A	QP	SP	(0.576,0.724)	0.594
V_7	A	SU	SU	P	(0.417,0.840)	0.240
V_8	QP	A	QU	SU	(0.501,0.776)	0.329
V_9	SP	SU	P	A	(0.629,0.712)	0.444
V_{10}	P	A	SP	P	(0.615,0.701)	0.444

Table 10. The FF-score values of criteria for locations assessment by DMEs.

Criteria	Crisp degrees	s_j	k_j	r_j	ω_j^s
V_6	0.594	-	1.0	1.0	0.1141
V_5	0.583	0.011	1.011	0.9891	0.1128
V_1	0.509	0.074	1.074	0.9209	0.1051
V_3	0.492	0.017	1.017	0.9055	0.1033
V_2	0.490	0.002	1.002	0.9037	0.1031
V_4	0.467	0.023	1.023	0.8774	0.1001
V_9	0.444	0.023	1.023	0.8518	0.0972
V_{10}	0.444	0.000	1.000	0.8518	0.0972
V_8	0.329	0.115	1.115	0.7639	0.0871
V_7	0.240	0.089	1.089	0.7015	0.0800

Table 11. Subjective weight using the FF-SWARA approach.

Fig. 4. Presentation of subjective weight by the FF-SWARA for locations assessment.
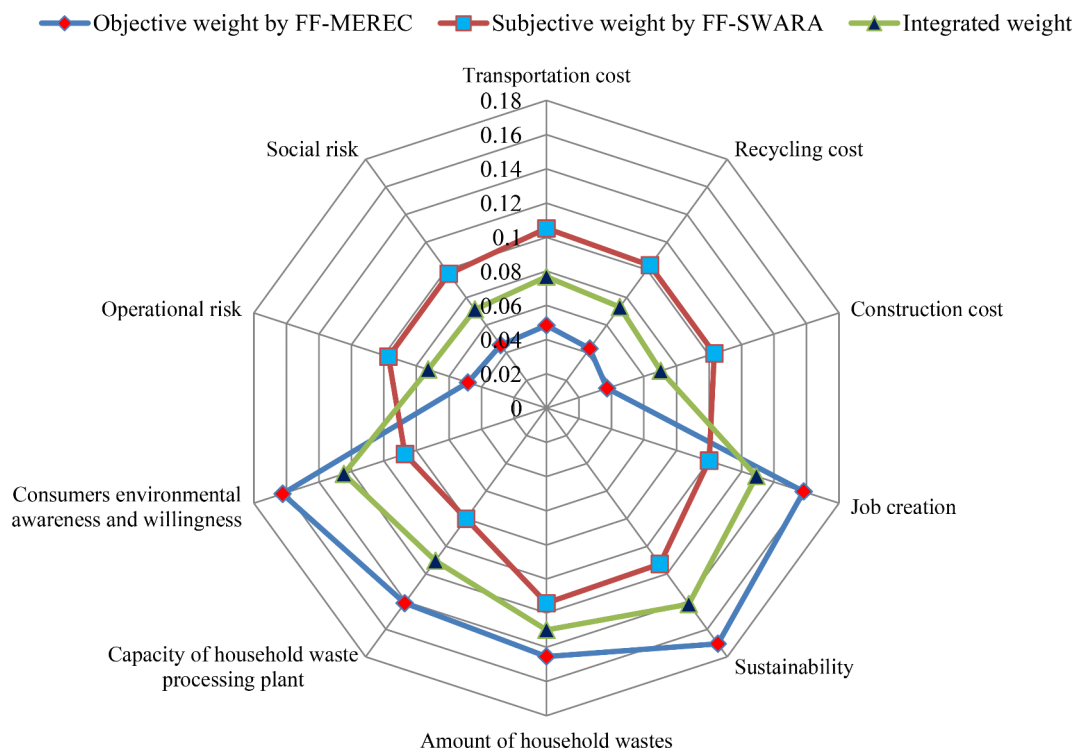
Fig. 5. Weight of indicator for locations assessment with the FF-MEREC-SWARA model.

$$\wp_i^{(2)} = \underset{j=1}{\overset{q}{\otimes}} \varsigma_{ij}^{\omega_j}, \; i = 1, 2, ..., p. \tag{26}$$

Step 6: Estimation of UD of options as

	F_1	F_2	F_3	F_4	α_{jw}^+	α_{jw}^-
V_1	(0.422,0.927)	(0.427,0.934)	(0.407,0.934)	(0.456,0.920)	(0.456,0.921)	(0.408,0.934)
V_2	(0.423,0.929)	(0.425,0.930)	(0.429,0.921)	(0.442,0.923)	(0.442,0.923)	(0.423,0.929)
V_3	(0.462,0.913)	(0.442,0.934)	(0.459,0.918)	(0.398,0.927)	(0.462,0.913)	(0.398,0.927)
V_4	(0.317,0.952)	(0.362,0.943)	(0.368,0.941)	(0.299,0.956)	(0.368,0.941)	(0.299,0.956)
V_5	(0.324,0.951)	(0.336,0.950)	(0.354,0.932)	(0.308,0.951)	(0.354,0.932)	(0.308,0.951)
V_6	(0.340,0.949)	(0.353,0.944)	(0.338,0.924)	(0.335,0.943)	(0.338,0.924)	(0.341,0.950)
V_7	(0.301,0.959)	(0.339,0.945)	(0.335,0.932)	(0.330,0.947)	(0.335,0.932)	(0.301,0.959)
V_8	(0.313,0.958)	(0.350,0.947)	(0.336,0.949)	(0.305,0.949)	(0.350,0.947)	(0.313,0.958)
V_9	(0.383,0.936)	(0.405,0.936)	(0.419,0.929)	(0.469,0.901)	(0.469,0.901)	(0.383,0.936)
V_{10}	(0.457,0.930)	(0.446,0.926)	(0.384,0.935)	(0.412,0.932)	(0.456,0.930)	(0.384,0.935)

Table 12. A WNA-FF-DM for HWRPLs assessment.

	F_1	F_2	F_3	F_4	α_{jw}^+	α_{jw}^-
V_1	0.139	0.132	0.127	0.157	0.157	0.127
V_2	0.138	0.136	0.149	0.150	0.150	0.136
V_3	0.169	0.136	0.162	0.134	0.169	0.134
V_4	0.084	0.104	0.109	0.077	0.109	0.077
V_5	0.088	0.091	0.117	0.084	0.117	0.084
V_6	0.092	0.102	0.125	0.099	0.125	0.092
V_7	0.073	0.098	0.114	0.093	0.114	0.073
V_8	0.076	0.097	0.092	0.087	0.097	0.076
V_9	0.118	0.123	0.136	0.186	0.186	0.118
V_{10}	0.145	0.148	0.120	0.130	0.148	0.120
Δ_i	1.121	1.165	1.25	1.198	1.37	1.036

Table 13. FF-score ratings of each option for HWRPLs evaluation.

Options	$\wp_i^{(1)}$	$\wp_i^{(2)}$	$\mathbb{S}\left(\wp_i^{(1)}\right)$	$\mathbb{S}\left(\wp_i^{(2)}\right)$	Q_i	Ranks
F_1	(0.762,0.54)	(0.697,0.619)	0.643	0.551	0.6427	4
F_2	(0.776,0.532)	(0.731,0.584)	0.658	0.596	0.6582	3
F_3	(0.768,0.491,0.754)	(0.726,0.540,0.772)	0.667	0.612	0.6670	1
F_4	(0.766,0.509,0.748)	(0.693,0.597,0.769)	0.659	0.560	0.6588	2

Table 14. UDs of options for locations assessment.

$$Q_i = \iota\,\wp_i^{(1)} + (1-\iota)\,\wp_i^{(2)},\, i = 1,2,...,p. \tag{27}$$

where ι is the utility coefficient within the range of 0 to 1.

Step 7: Choosing the best option with highest FF-score rating of UDs.
From Eqs. (25) to (27), the UD of option for HWRPLs are demonstrated in Table 14.
Therefore, the ranking of option for locations assessment is $F_3 \succ F_4 \succ F_2 \succ F_1$ and the option location-3 (F_3) is an ideal location with maximum degree.

Gül's ARAS model
Steps 1–4: Similar to the developed framework.
Step 5: Defining an "optimal alternative rating (OAR)".

$$h_0 = \left(\hbar_j^0,\,\lambda_j^0\right) = \begin{cases} \left(\max_i \hbar_{ij}, \min_i \lambda_{ij}\right), & j \in V_b, \\ \left(\min_i \hbar_{ij} \max_i \lambda_{ij}\right), & j \in V_n. \end{cases} \tag{28}$$

Step 6: Find the "relative assessment rating (RAR)" and the UD of each option.

Using the WNA-FF-DM $\mathbb{N}_w = \left(\widehat{\varsigma}_{ij} \right)_{p \times q}$, given in Eq. (21), the RAR of option is obtained as

$$\Delta_i = \sum_{j=1}^{q} \mathbb{S} \left(\widehat{\varsigma}_{ij} \right), \, i = 1, 2, ..., p. \tag{29}$$

The UD is computed using the RAR (Δ_i) and OAR (h_0). The UD (Q_i) of option F_i, $i = 1,2,\ldots,p$ is given as

$$Q_i = \frac{\Delta_i}{h_0}, i = 1, 2, ..., p. \tag{30}$$

The prioritize the option in ascending UD (Q_i), $i = 1,2,\ldots,p$.

From Eq. (28) and the A-FF-DM, we define the OAR to the HWRPLs assessment as follows:

$h_0 = \{(0.34, 0.899), (0.333, 0.892), (0.274, 0.917), (0.689, 0.623), (0.65, 0.61), (0.64, 0.544), (0.664, 0.529), (0.666, 0.644), (0.237, 0.919), (0.361, 0.91)\}$. Using Eq. (5) and Eq. (29), we obtain the ROR of HWRPLs assessment. Applying Eq. (30), the UD Q_i is computed as $Q_1 = 0.8187$, $Q_2 = 0.8506$, $Q_3 = 0.9128$ and $Q_4 = 0.8746$. Based on the UD (Q_i), the prioritization of sites to establish new HWRP is $F_3 \succ F_4 \succ F_2 \succ F_1$ and thus, the location-3 (F_3) is the ideal location over various criteria.

Simic et al.'s FF-CoCoSo model
Steps 1–5: Follow to FF-WASPAS framework.

Step 6: Estimating the "relative degree (RD)" of each option as

$$t_i^{(1)} = \frac{\mathbb{S} \left(\wp_i^{(1)} \right) + \mathbb{S} \left(\wp_i^{(2)} \right)}{\sum_{i=1}^{p} \left(\mathbb{S} \left(\wp_i^{(1)} \right) + \mathbb{S} \left(\wp_i^{(2)} \right) \right)}, \tag{31}$$

$$t_i^{(2)} = \frac{\mathbb{S} \left(\wp_i^{(1)} \right)}{\min_i \mathbb{S} \left(\wp_i^{(1)} \right)} + \frac{\mathbb{S} \left(\wp_i^{(2)} \right)}{\min_i \mathbb{S} \left(\wp_i^{(2)} \right)}, \tag{32}$$

$$t_i^{(3)} = \frac{\vartheta \, \mathbb{S} \left(\wp_i^{(1)} \right) + (1 - \vartheta) \, \mathbb{S} \left(\wp_i^{(2)} \right)}{\vartheta \, \max_i \mathbb{S} \left(\wp_i^{(1)} \right) + (1 - \vartheta) \max_i \mathbb{S} \left(\wp_i^{(2)} \right)}, i = 1, 2, ..., p, \tag{33}$$

where ϑ is the RCR coefficient within the range of 0 to 1.

Step 7: Assessment of the compromise rating (CR) of options.

The CR (t_i) of each option is given by

$$t_i = \left(t_i^{(1)} t_i^{(2)} t_i^{(3)} \right)^{1/3} + \frac{1}{3} \left(t_i^{(1)} + t_i^{(2)} + t_i^{(3)} \right), i = 1, 2, ..., p. \tag{34}$$

Hence, choose the best option with the highest CD (t_i).

Apply Eqs. (31)–(33), the RDs of HWRPLs assessment is presented in Table 15. From Eq. (34), the CR of each HWRPL is calculated and is mentioned in Table 15. As per CRs, the preference of HWRPLs is $F_3 \succ F_4 \succ F_2 \succ F_1$, and thus, the HWRPL-3 (F_3) is the best site over different criteria.

FF-TOPSIS method
The procedure for FF-TOPSIS is.

Steps 1–5: Similar to the proposed approach.

Step 6: Computing the weighted FF-distances $E_i^+ = dis \left(v_{ij}, \alpha^+ \right)$ and $E_i^- = dis \left(v_{ij}, \alpha^- \right)$ of options based on the FF-Euclidean distance measure[20].

Step 7: The relative closeness rating (RCR) of options from FF-PIR is estimated in the given expression:

$$R \left(F_i \right) = \frac{E_i^-}{E_i^+ + E_i^-}, \, \forall i. \tag{35}$$

Rank the options according to the RCRs.

We have implemented the TOPSIS method on the aforesaid case study. We have computed FF-distance measures as follows: $E_1^+ = 0.096$, $E_1^- = 0.035$, $E_2^+ = 0.062$, $E_2^- = 0.076$, $E_3^+ = 0.037$, $E_3^- = 0.094$, $E_4^+ = 0.082$ and $E_4^- = 0.058$. Based on Eq. (35), the relative closeness coefficient to the FF-PIR is presented as follows: $R \left(F_1 \right) = 0.267$, $R \left(F_2 \right) = 0.549$, $R \left(F_3 \right) = 0.718$, and $R \left(F_4 \right) = 0.416$. The ranking of plant locations is $F_3 \succ F_2 \succ F_4 \succ F_1$, thus, the location-3 (F_3) is the best location for establishing the HWRP. The key benefits of the proposed hybrid framework are presented as follows (see Fig. 6):

	$t_i^{(1)}$	$t_i^{(2)}$	$t_i^{(3)}$	t_i	Ranking
F_1	0.2413	2.0	0.9635	1.843	4
F_2	0.2535	2.106	0.9867	1.923	2
F_3	0.2587	2.15	1.0	1.959	1
F_4	0.2465	2.042	0.9877	1.8845	3

Table 15. The RDs and CRs findings for HWRPLs assessment.
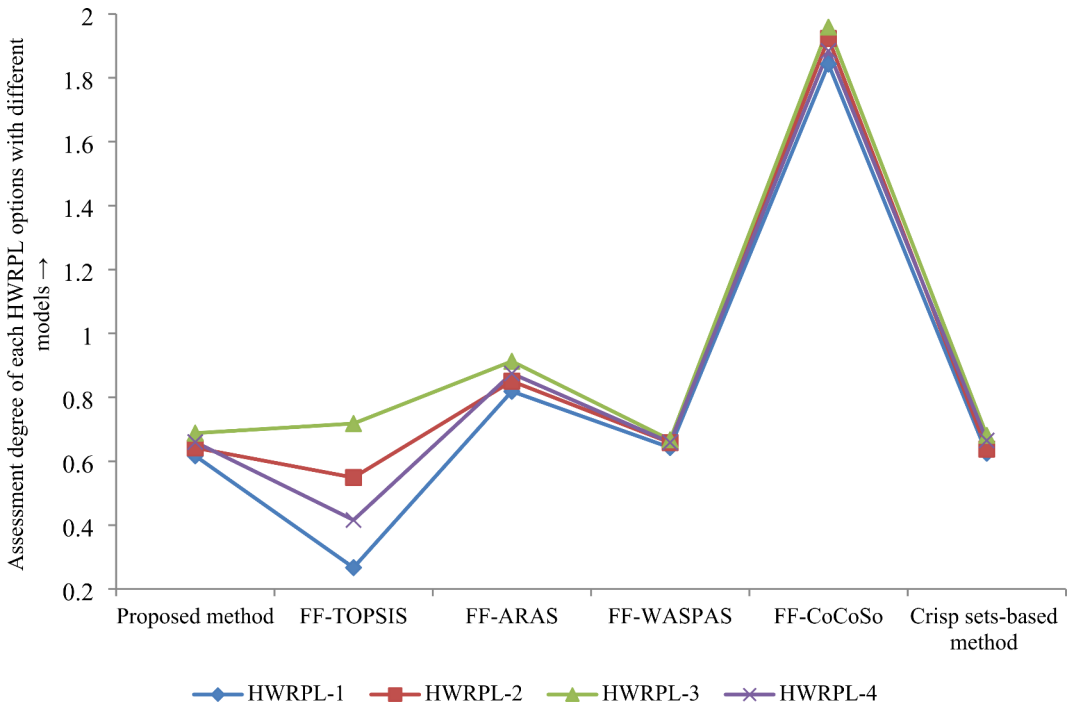
**Fig. 6**. Ranking order for HWRPL assessment with different methods.

a) The score values used by[20,55,56] has counter intuitive problem in some cases, while the developed score function avoids these limitations. Thus, the developed FF-score function can successfully offer the ranks of the FFNs.

b) The FF-distance measure developed in this paper evades the drawbacks of various extant FF-distance measures by[57–59]. Further, the DMEs' weights are computed through the proposed distance FF-distance measure in the presented FF-MEREC-SWARA-MARCOS method. Thus, the proposed model provides more accurate result than existing models.

c) The proposed FF-MEREC-SWARA-MARCOS framework estimates the weight of attributes using FF-MEREC-SWARA approach integrating the objective and subjective weights of attributes, which achieves the weight values of attributes from most favourable ways, whereas in the Mishra and Rani's WASPAS approach, objective weight of attribute is obtained with FF-similarity measure and FF-score rating-based approach, in the Simic et al.'s CoCoSo approach, only objective weight of attribute is taken with similarity measure, in the FF-ARAS, only objective weight of attribute is estimated by MEREC tool and in the FF-TOPSIS model, attribute weight is taken arbitrarily.

d) Comparing with diverse extant models, we find suitable HWRPL-3 (F_3) is the same as the developed hybrid framework. The CUFs of the proposed FF-MEREC-SWARA-MARCOS approach have been estimated with FF-PIR and FF-NIR, while Mishra and Rani's WASPAS and Simic et al.'s CoCoSo models employ averaging and geometric AOs and the FF-ARAS utilizes averaging AO with FF-PIR to obtain final rating of options. Hence, proposed FF-MEREC-SWARA-MARCOS approach is more comprehensive and more flexible. Considering this feature, the proposed FF-MEREC-SWARA-MARCOS approach can be applied more broadly.

Sensitivity investigation
We study changes in CUF ratings and preferences of HWRPLs over changing the weights of diverse attributes from objective and subjective weights using "FF-MEREC-SWARA" model for HWRPLs assessment. The prioritizations of locations of HWRP evaluations are obtained over the objective, the combined and the subjective weights of factors using the FF-MEREC-SWARA models and are discussed in Table 16 and Fig. 7.

Approaches	CUFs for HWRPL assessment				Ranks
	F_1	F_2	F_3	F_4	
FF-MEREC for objective weighting	0.602	0.651	0.705	0.649	$F_3 \succ F_2 \succ F_4 \succ F_1$
FF-SWARA for subjective weighting	0.629	0.636	0.676	0.668	$F_3 \succ F_4 \succ F_1 \succ F_1$
Integrated method	0.617	0.642	0.688	0.66	$F_3 \succ F_4 \succ F_1 \succ F_1$

Table 16. The CUFs for locations assessment with diverse models.
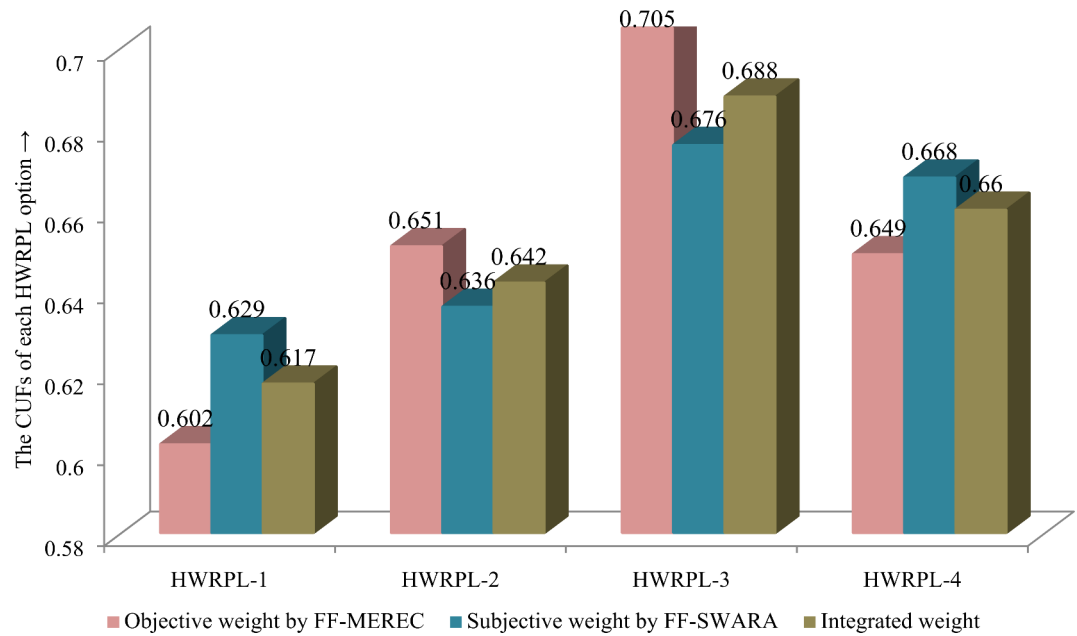
Fig. 7. Changes in CUF ratings for locations assessment with different models.

Apply the FF-MEREC approach, the CUF ratings and preferences of HWRPLs are estimated as $F_1 = 0.602$, $F_2 = 0.651$, $F_3 = 0.705$ and $F_4 = 0.649$ and prioritization of locations is given as $F_3 \succ F_2 \succ F_4 \succ F_1$. Utilizing the FF-SWARA model, the CUF ratings and preferences of HWRPLs are obtained as $F_1 = 0.629$, $F_2 = 0.636$, $F_3 = 0.676$ and $F_4 = 0.668$ and the ranking order of HWRPLs is given as $F_3 \succ F_4 \succ F_2 \succ F_1$. Thus, we conclude that the suitable HWRPL choice considering all types of weight evaluating approach is the same, i.e., HWRPL-3 (F_3). Hence, as per aforementioned study, it is found that the positioning of different significant degree of strategy parameter (ϑ) will improve the performance of proposed FF-MEREC-SWARA-MARCOS methodology.

Implications of the proposed work

This work introduces an innovative approach to select the most suitable site for HWRP construction that approves to enhance the sustainability pillars on FFSs settings. The developed methodology is based on the FF-distance measure, the FF-MEREC, the FF-SWARA and the FF-MARCOS methods called the "FF-MEREC-SWARA-MARCOS" with Fermatean fuzzy information. A case study of HWRPLs selection is taken to validate the results and reasonableness of developed framework. This methodology not only prioritizes the locations with diverse sustainability perspectives, but also recognizes the significance values of DMEs using novel formula and the criteria using integrated weighting tool. Furthermore, this paper discusses new FF-distance measure and FF-score function for FFSs and verifies their usefulness over the formerly proposed FF-distance measures and FF-score/accuracy functions.

Moreover, the comparative assessment with extant procedures namely, the FF-TOPSIS, the FF-ARAS, the FF-WASPAS and the FF-CoCoSo has also shown to elucidate the reasonableness of proposed approach. The findings show that HWRPL-3 (F_3) is the best choice for constructing the HWRP, whereas the preferences of HWRPLs, determined with the proposed approach and extant models, are slightly vary. We observe that the variation in the prioritizations of HWRPLs is owing to the following causes. The proposed approach provides significance to DMEs' preferences in the assessment of alternatives and factors. Based on the introduced method, group of DMEs focuses not only in the beneficial factors but also studies the non-beneficial factors. Additionally, sensitivity investigation over different ratings of coefficient 'ϑ' has implemented to validate the permanence of proposed approach and thus, we form that HWRPL-3 (F_3) is the optimal choice for HWRP establishment.

This work recommends stakeholders/representatives to realize the performance of HWRPL options with diverse features of sustainability under uncertainty setting. The proposed FF-MEREC-SWARA-MARCOS framework has subsequent implications for experts and researchers:

- The most optimal HWRPL candidate executes better over economic, social, environmental and risk aspects of sustainability with least cost and positive impacts on environment.
- Executives and stakeholders can utilize data discussed in this work to assist their decision for locating appropriate HWRPs.
- The proposed FF-MEREC-SWARA-MARCOS approach not only appraises the importance ratings of aforesaid factors but also deals vagueness and fuzziness obtained in the procedure of HWRPLs evaluation.

Conclusions

The current study aims to propose hybrid MCDM framework for assessing recycling plant locations for household waste from sustainable perspectives. For this purpose, based on literature review and experts' knowledge, 10 criteria were selected from sustainability perspectives including economic, social, environmental and risk dimensions. This methodological framework has been incorporated the proposed FF-distance measure-based model, proposed FF-score function, the FF-MEREC model, the FF-SWARA model and the FF-MARCOS method from Fermatean fuzzy scenario called the "FF-MEREC-SWARA-MARCOS" model. In this context, to rank the FFNs, new FF-score function has been introduced, which evades the drawbacks of extant FF-score functions[20,55,56]. Also, to quantify the distance between FFSs, new FF-distance measure has been proposed with some elegant axioms. The developed FF-distance measures can deal the weaknesses of extant FF-distance measures Ashraf et al.'s[57] distance measures, Ganie's[58] distance measures and Kirisci's[59] distance measure) between FFSs. Further, developed hybrid framework has been used on a case study of household waste recycling plant location (HWRPL) selection problem, which confirms the applicability and efficacy. Comparison and sensitivity investigation have been made to reveal validity of obtained outcomes. Based on the comparison with extant approaches, we have found that the presented model is simple, easy-to-use and reliable in order to tackle realistic decision-making problems. This work makes an innovative contribution in the form of a hybrid FF-MEREC-SWARA-MARCOS decision support system, which can be used by the household waste recycling plant construction companies for the selection of location sustainable from sustainability perspective. Furthermore, the proposed hybrid system makes realistic contributions for outlining policies and strategies for solid waste management activities and household waste recycling plant construction. The limitation of the developed approach is as (1) all criteria are assumed to be independent. In effect, there are interrelationships among criteria in realistic group decision-making problems, and (2) the assessment ranking procedures should include more decision experts to provide more concise and valid outcomes.

In future, we will consider the technological aspects of criteria and stakeholders' preferences during the evaluation of household waste recycling plant sites. Furthermore, we will extend developed approach on diverse fuzzy settings such as "Fermatean rough fuzzy sets (FRFSs)", "interval-valued hesitant rough fuzzy sets (IVHRFSs)" and "complex fuzzy sets (CFSs)" to tackle more vagueness of DMEs' subjective decisions.

Data availability

All data generated or analyzed during this study are included in this published article.

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