scientific reports

OPEN

AI simulation models for diagnosing disabilities in smart electrical prosthetics using bipolar fuzzy decision making based on choquet integral

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The integration of AI simulation models within smart electrical prosthetic systems represents a significant advancement in disability disease diagnosis. However, the selection and evaluation of these AI models interpret some multi-criteria decision-making dilemmas because of the presence of uncertainty and bipolarity (positive and negative aspects) of the selection criteria. Current literature lacks the selection and evaluation of AI simulation models that consider both bipolarity and uncertainty of the criteria, while prevailing Choquet integral aggregation operators despite their strong capabilities for handling information relationships, fail to efficiently process bipolar fuzzy information. The existence of this limitation makes it challenging to identify element interactions and non-linear relationships in uncertain environments containing both positive and negative aspects. To overcome these gaps, first, we develop two operators that are the bipolar fuzzy Choquet integral averaging and bipolar fuzzy Choquet integral geometric operators that uniquely integrate dual aspects (bipolarity) with criterion interaction modeling capabilities, fundamentally differing from traditional fuzzy approaches that cannot simultaneously process dual aspects of criterion. Secondly, we design a new multi-criteria decision-making approach using these operators to assess AI simulation models for prosthetic systems, since the criteria involved such as diagnostic accuracy, computational efficiency, and system reliability, have both positive and negative aspects that need to be considered together. Our method was applied in detail to select AI simulation models for smart electrical prosthetic systems and compared with some prevailing methods and standard Choquet integral approaches. This showed that our method is more precise and produces better evaluation results. It introduces a new theoretical basis for bipolar fuzzy Choquet integral aggregation and offers medical professionals a reliable way to pick the best AI simulation models for important prosthetic applications that influence patient outcomes and the functioning of prosthetics.

Keywords Artificial intelligence, Disability, Electrical prosthetic systems, Choquet integral, MCDM methodology

The inclusion of AI simulation models within smart prosthetic systems creates an important advancement in disability diagnosis and treatment through real-time analysis of user movement patterns and neurological signals. Machine learning algorithms in these models analyze sensor data from electrical prosthetics to detect neurological and muscular disorders that impact prosthetic functionality. Through simulated conditions, the AI systems diagnose variations in disability effects which helps medical decision-makers anticipate functional changes in prosthetics to customize individual adjustments. The models track user behavior patterns continuously to spot minor behavioral shifts that signal disability disease development. The technology enhances diagnostic

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precision while enabling prosthetic systems to deliver better support to users who have different levels of disability.

The AI simulation model employs artificial intelligence abilities to manufacture complex system simulations and conduct their analysis. A significant advantage of AI simulation models over traditional simulation approaches is their ability to learn system patterns alongside their ability to adapt to changing conditions and produce new solutions from their training data sets[1–3]. Their unique ability to model complex and dynamic systems and poorly understood systems makes them valuable for modeling tasks beyond traditional methods' capabilities. AI simulation models achieve their predictive capabilities through data learning and inference which produces outcomes that deterministic systems cannot match. Multiple artificial intelligence techniques help build these models. The identification of patterns and relationships inside simulated system data sets is commonly achieved through deep learning and machine learning techniques[4]. Through data analysis, the model acquires system dynamic information regardless of the model's inability to simplify system dynamics by expressing them as mathematical equations. Building use data processing together with configuration data evaluation and environmental factor consideration enables AI algorithms to generate performance predictions for different design alternatives regarding energy efficiency and comfort safety[5]. The assessment results assist building site designers in selecting design choices that lead to optimal performance results.

AI simulation models used for diagnosing disability diseases in smart electrical prosthetic systems represent an important advancement in modern healthcare technology. Advanced artificial intelligence models deliver essential advancements in both technologies used for disability detection and prosthetic device medical solutions. The AI models utilize continuous analysis of disability-related indicators within prosthetic system sensor data which enables them to identify signals that indicate developing diseases or increasing severity. The capability to detect conditions early allows intervention teams to organize timely treatments as well as modify their treatment methods. The analysis of unique movement patterns and neural signals alongside physiological reactions enables these models to create individualized patient profiles that produce optimized prosthetic adjustments and adaptive therapeutic interventions. The implementation of early warning detection AI systems enables health providers to detect upcoming medical complications which enables them to execute preventive healthcare strategies. This technology enables remote patient monitoring to combine complex advanced care services with financial savings decreasing the need for medical office attendance. By using AI models researchers gain better awareness of disease progression in multiple patient groups and this generates crucial medical knowledge about disability conditions. The integration of AI into prosthetic systems creates an end-to-end diagnostic-treatment solution that generates outstanding healthcare services for people with disabilities along with enhanced therapeutic outcomes and life-quality benefits.

The selection and evaluation of AI simulation models for disability disease diagnosis in smart electrical prosthetic systems leads to an MCDM challenge because multiple essential factors operate simultaneously. The evaluation process becomes complicated because it requires the assessment of competing criteria that unite performance metrics with technical requirements clinical aspects implementation aspects and user-specific parameters. When assessing these systems, it is essential to find an equilibrium between diagnostic quality and resource efficiency through examinations involving system dependability allocation and analysis of scalability capabilities and protection requirements together with regulatory specifications. The decision-making process becomes complex due to the interdependent criteria that demand simultaneous optimization of multiple objectives to determine the best AI simulation model for healthcare situations and patient groups.

Bipolar fuzzy theory in MCDM procedures

Bipolar fuzzy set (BFS)[6] stands as a major MCDM procedure advancement because it enables effective analysis of positive and negative aspects of information in complex decision environments. Bipolarity enhances traditional fuzzy sets to create a representation system for determining membership degrees in positive and negative aspects. Bipolar fuzzy TOPSIS utilizes classical TOPSIS principles alongside BFSs to enable effective decision evaluation when working with both positive and negative aspects of criteria[7]. The implementation of BFSs provides substantial benefits to MCDM applications because decision-makers encounter unpredictable and imprecise situations that commonly appear in practical environments. Various research approaches incorporating BFSs have been created to enhance decision accuracy according to recent studies. BFWA operators form the basis of an effective method to combine multiple criteria preferences for precise alternative analysis[8]. The approach performs option ranking by calculating scores from combined positive and negative evaluations to assist decision-makers in finding their best choices. Bipolar fuzzy theory enhances MCDM procedures through complete alternative assessments that handle positive and negative aspects. The dual evaluation standard enhances decision-making frameworks by creating more accurate operational techniques to handle problems across various domains. Recurring research activities within this subject create advanced methods that demonstrate the potential to strengthen upcoming decision-making outcomes.

Research problem and motivation

Selecting and assessing AI simulation models for diagnosing disability diseases in smart electrical prosthetic systems is a challenging MCDM problem that directly influences the health of more than 40 million prosthetic users globally, but existing approaches do not handle the inherent uncertainty and bipolarity in this area well. The evaluation process faces many problems because each selection criterion has both positive and negative sides that must be considered together—for example, accuracy in diagnosis can be high (positive) and can be low (negative) and cost, higher the cost (negative aspects) and lower the cost (positive aspects). In addition, these criteria show that there are strong connections between them, since moderate gains in accuracy and speed can lead to much bigger benefits in diagnostics, security, and connectivity, risks from security and connectivity features tend to multiply rather than add up and reliability and innovation are often substitutes instead of complements. Currently, no one used such comprehensive frameworks for selecting AI simulation models that handle both uncertainty analysis and bipolar criterion assessment which are both important for making decisions in prosthetic systems.

Traditional fuzzy set (FS) methods are not suitable because they cannot handle both the positives and negatives of the same criterion at the same time, while standard MCDM methods that use linear aggregation cannot handle the way some criteria interact such as the relationship between diagnostic accuracy and real-time processing or the way some criteria balance each other such as between system reliability and adaptive learning. While Choquet integral aggregation operators (AOs) are excellent at processing information with variable relationships and at detecting interactions between criteria that other methods cannot, the existing operators struggle to handle bipolar fuzzy information efficiently which means they cannot detect interactions and non-linear relationships in uncertain environments with both positive and negative aspects. Because of this methodological gap, decision-makers are unable to properly weigh the positive and negative aspects of different AI models such as how much precision they offer versus how much they cost to run, how safe they are versus how flexible they are, or how much they can be personalized versus how much they follow standard rules. Consequently, there exists an urgent necessity for interpreting a MCDM approach that can successfully integrate BFS with Choquet integral AOs to create a robust framework capable of coping with the complex, uncertain, and inherently bipolar nature of AI simulation model evaluation criteria, thereby enabling healthcare professionals and biomedical engineers to make evidence-based, optimal selection decisions that maximize diagnostic accuracy while minimizing system limitations and ensuring superior patient outcomes in smart electrical prosthetic applications.

Contribution

The main contribution of this article is given as follows.

- Introducing bipolar fuzzy choquet integral AOs: We interpret two vital mathematical AOs that are the bipolar fuzzy Choquet integral averaging (BFCIA) and the bipolar fuzzy Choquet integral geometric (BFCIG) operators, that signify the first comprehensive integration of BFS with Choquet integral aggregation methods.
- Single framework integration for dual-aspect processing: With our advanced operators, we can handle both the fuzzy measures of criteria and their positive and negative aspects together, so there is no need for different evaluation systems and we can assess all aspects of criteria at once.
- Creating a MCDM methodology: We develop a MCDM that is designed to handle complex problems with bipolarity and uncertain information and aggregated by Choquet integral AOs.
- Practical healthcare application demonstration (case study): We use a detailed case study to prove that our framework can be used in real life by choosing the best AI simulation models for prosthetic systems, highlighting how it addresses tough MCDM problems in healthcare.
- Comprehensive justification via comparative analysis: We prove that our approach performs better than existing theories by comparing them in detail and showing their effectiveness in handling complex medical technology assessment decisions.
- Bridge between theoretical development and clinical practice: We show how advanced methods can be used in practice to help choose the best medical technology for patients.
- Enhanced decision-making precision: Our approach makes it possible to evaluate AI simulation models with great precision by considering both sides of each evaluation criterion and how different factors affect each other.

The flowchart of the contribution is devised in (Fig. 1).

Our bipolar fuzzy method is different from conventional FSs because it allows each criterion to have both positive and negative membership, making it possible to fully represent the ambivalence of AI prosthetic evaluation. Our method is better than intuitionistic fuzzy sets, neutrosophic sets hesitant fuzzy sets, etc. because it models the two-sided nature of decision criteria, rather than handling hesitancy or indeterminacy which is important for prosthetic AI evaluation since each criterion has both positive and negative aspects that must be considered together, unlike other complex fuzzy approaches that address various types of uncertainty but do not capture the bipolarity of criteria. We address the main issue of existing Choquet integral operators which cannot process bipolar information, by developing new operators that keep the Choquet integral's advantages in modeling criterion interactions and add the ability to handle both positive and negative membership degrees, making it possible to detect both synergistic and antagonistic effects between criterion benefits and drawbacks that traditional methods cannot detect.

Layout of the article

In Sect. 2, we analyze the literature review and in Sect. 3, we devise some basic concepts related to BFS and a concept of fuzzy measure. In Sect. 4, we devise Choquet integral AOs within the framework of BFS and discuss the related properties. In Sect. 5, we devise an MCDM methodology based on the developed Choquet integral AOs and then analyze a case study related to the selection and assessment of AI simulation models. In Sect. 6, we interpret a comparative study of our work with certain prevailing ones. Section 7 has the conclusion.

Literature review

The literature on disability, prosthetics, and rehabilitation encompasses a broad range of research that paves the way to presenting critical interplay between medical advancements for patients, patient-centric approaches, and the psychosocial impact on prosthetic use. A foundational perspective was presented by Kraft et al.[9], who explored the varied experiences of multiple sclerosis patients, where the focus of the interplay between disability

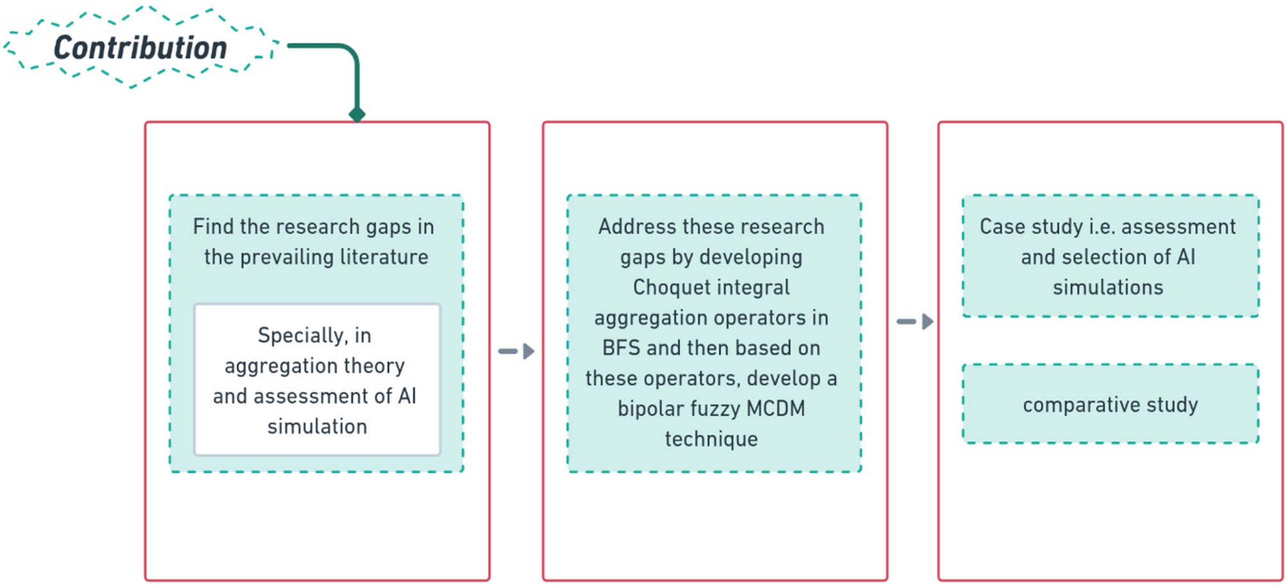

Fig. 1. The flowchart of contribution.

and disease duration was centered on the necessity for tailored rehabilitation services. Their contributions were to bring the patient-centric approach to diagnose and treat complex chronic diseases. Portillo and Sancho[10] advanced microbiological diagnostic techniques for prosthetic joint infections with the advent of state-of-the-art technology, which helps improve diagnostic accuracy. Modern microbiological methodologies used in this study helped reduce the risks from prosthetic implants through effective infection management. Carson[11] specifically targeted penile prosthesis infections, describing new diagnostic, treatment, and prevention approaches. His efforts highlighted the necessity of focused strategies in prosthetic care, specifically both physiological and procedural contributors that impact clinical results. Durstine et al.[12] changed the focus to the advantages of exercise for chronic illness and disability. Such work emphasizes the ability of regular physical activity to augment better physical functioning, reduce disablement, and enhance psychological well-being as a consideration for inclusion into rehabilitation programs. Le Bars et al.[13] conversely investigated disorders in systemic disturbances secondary to using removable prosthetic appliances, serving to illustrate interactions between prosthetic design and generalized health outcomes. Wolfart et al.[14] made further contributions by examining the oral health-related quality of life for shortened dental arches under prosthetic treatment. Results showed improved management of pain, jaw functions, and general patient satisfaction. Psychosocial aspects of prosthetic use were examined by Tamari[15], who reviewed the association of body image with prosthetic aesthetics and disability within the context of Paralympic culture. This study threw light on the cultural and emotional impact of prosthetics and supported the design which integrated functionality with aesthetics to improve self-perception and social integration. Feine et al.[16] furthered the evaluation of implant-supported prostheses by setting firm criteria for the outcome assessment to ensure that the outcome is determined within a standardized framework in evaluating the success of prosthetic treatments. Matsuka et al.[17] conceptualized the oriented diagnostic nomenclature system with patient disability as a focus in prosthetic dentistry as a tool for safe and valid diagnostic practice for a more valid assessment of the patient and individual treatment plan. Psychological adaptation to prosthetic use was the focus of the study by Gallagher and MacLachlan[18] which explored coping mechanisms and adjustment in adults with prosthetic limbs. Nevelsteen et al.[19] addressed the challenges of prosthetic infections in vascular surgery by proposing autogenous reconstruction with lower extremity deep veins as a viable alternative for treating prosthetic infections after reconstructive aortoiliac surgery. Matyokubovna[20] paid special attention to the complications with removable prosthetics - diseases of the oral mucous membrane. The work included preventing and regular monitoring of such risks in prosthetic dentistry. Schiff et al.[21] have made original contributions to prosthetic reconstruction in cases of cognitive disability acquired as a result of brain injury. Their work has added more prosthetic applications to benefit beyond rehabilitation: that is physical to cognitive benefits. In conjunction, these articles are significant inputs into the large and comprehensive database that understands more than just single-dimension benefits through using prosthetics for rehabilitation, impacting health outcomes, or enhancing the quality of life while catering to both physiological and psychological attributes of disability.

MCDM techniques

MCDM has emerged as an essential methodology for dealing with complex decision-making problems involving multiple criteria. Massam[22] provided one of the earliest comprehensive reviews of MCDM techniques, particularly in the field of planning. His work underscored the importance of considering multiple criteria in decision-making processes, especially when these decisions have long-term impacts. Bonissone et al.[23] propose a framework for MCDM and they created a comprehensive framework for MCDM research and applications, especially focusing on adaptability in computational intelligence and also integrating these techniques with artificial intelligence and machine learning to enhance the decision-making outcomes. The versatility of MCDM techniques has increased as decision problems have become more complex. Zavadskas et al.[24] presented a modern review of MCDM and multi-attribute decision-making (MADM) methods by organizing them according to their technology and economic development usage. Taherdoost and Madanchian[25] presented updated MCDM ideas through their research about modern industry trends and sustainability along with fuzzy logic applications. The authors demonstrated how modern technological developments require greater consideration during MCDM technique applications. MCDM has experienced a major advancement through the incorporation of fuzzy logic which enables decision-making under uncertain and imprecise conditions. Traditional MCDM methods gained flexibility through fuzzy logic integration according to Chu and Lin[26] in their work. The researchers at Chen et al.[27] used fuzzy MCDM to identify the most suitable watershed plan for environmental purposes. Their use of fuzzy logic helped deal with the imprecision in data and expert opinions, highlighting the strengths of fuzzy MCDM in complex environmental decision-making. The comparison of different fuzzy MCDM methods was further explored by Zamani-Sabzi et al.[28], who conducted a statistical and analytical assessment of various techniques under fuzzy environments. Wang et al.[29] applied the fuzzy MCDM approach for selecting optimal renewable energy plant locations in Vietnam and showed how to solve the issue of conflicting economic, environmental, and social goals by offering balanced solutions to planning sustainable energy resources. Fuzzy MCDM has been complimented by some hybrid approaches of multiple techniques that have emerged in decision-making tools. Ahmed et al.[30] used a Hybrid Fuzzy AHP/VIKOR method to select the funding strategy for advanced prosthetic and orthotic medical devices for low-income countries. Similarly, Buyukozkan and Mukul[31] have further explored hesitant fuzzy linguistic MCDM methods that can be useful in evaluating smart health technologies. Their research demonstrated how the hesitant fuzzy approach can be applied to model uncertainty in human judgment, particularly in assessing new health technologies. Dhumras and Bajaj[32] proposed an improved EDAS approach to MCDM in robotic agrifarming based on picture fuzzy soft (PFS) Dombi AOs. Dhumras et al.[33] have extensively discussed the application of modified TOPSIS methodology in green supplier selection problems based on R-norm q-rung picture fuzzy information measures and R-norm picture fuzzy discriminant measures were discussed by Singh et al.[34]. In addition, Dhumras et al.[35] developed the TOPSIS/VIKOR approach within the framework of q-rung picture fuzzy that is federated learning-oriented in electronic marketing strategic planning, and Sharma et al.[36] focused on banking site selection and used new picture fuzzy discriminant measures. Similar progress in q-rung orthopair fuzzy hypersoft sets has been made by Khan and colleagues, who proposed ordered aggregation operators to select green suppliers[37], determine tourism carrying capacity[38], and analyze the cryptocurrency market with hypersoft set al.gorithms based on aggregation operators[39]. Al-Sabri et al.[40] have also added to the integration of Pythagorean fuzzy approaches by using cubic fuzzy Einstein AOs in investment management, and Mahapatra et al.[41] have added to dynamic group decision-making in the context of enterprise resource planning selection using two-tuples Pythagorean fuzzy MOORA approaches. Dhumras et al.[42] proposed similarity measures of complex picture fuzzy sets (CPFSs) applicable in pattern recognition. In simple the MCDM literature has thus illustrated how such techniques are critical for making complex decisions across domains. From early development to contemporary developments, the inclusion of fuzzy logic and hybrid methods into MCDM has significantly opened up applications in renewable energy, healthcare, and environmental management, among others. Given the increasingly complicated nature of decision problems, the MCDM will always serve as an efficient tool in researchers' and practitioners' analyses and decision-making strategies in facing multiple-dimensional problems that are getting ever more uncertain with time.

Preliminaries

In this section, we devise some basic concepts related to BFS and a concept of fuzzy measure.

Definition 1 [6]The following framework displays the notion of BFS

$$\mathcal{I} = \left\{ \left(ɽ\, \partial_{\mathcal{I}}^{\mathcal{P}}(ɽ),\ \partial_{\mathcal{I}}^{\mathcal{N}}(ɽ) \right) \mid ɽ \in \mathcal{Z} \right\}$$

Where $\partial_{\mathcal{I}}^{\mathcal{P}}(ɽ) \in [0,\ 1]$ is analyzed as a positive membership grade and $\partial_{\mathcal{I}}^{\mathcal{N}}(ɽ) \in [-1,\ 0]$ is analyzed as the negative membership grade of each $ɽ \in \grave{Y}$. The set $\mathcal{I} = \left(\partial_{\mathcal{I}}^{\mathcal{P}},\ \partial_{\mathcal{I}}^{\mathcal{N}} \right)$ is devised as a bipolar fuzzy number (BFN)

Definition 2 [43]For two BFNs $\mathcal{I}_1 = \left(\partial_{\mathcal{I}_1}^{\mathcal{P}},\ \partial_{\mathcal{I}_1}^{\mathcal{N}} \right)$ and $\mathcal{I}_2 = \left(\partial_{\mathcal{I}_2}^{\mathcal{P}}, \partial_{\mathcal{I}_2}^{\mathcal{N}} \right)$ and $\acute{\partial} \geq 0$, we have

1.

$$\mathcal{I}_1 \oplus \mathcal{I}_2 = \left(\partial_{\mathcal{I}_1}^{\mathcal{P}} + \partial_{\mathcal{I}_2}^{\mathcal{P}} - \partial_{\mathcal{I}_1}^{\mathcal{P}} \partial_{\mathcal{I}_2}^{\mathcal{P}},\ -\left(\partial_{\mathcal{I}_1}^{\mathcal{N}} \partial_{\mathcal{I}_2}^{\mathcal{N}} \right) \right)$$

2.

$$\mathcal{I}_1 \otimes \mathcal{I}_2 = \left(\partial_{\mathcal{I}_1}^{\mathcal{P}} \partial_{\mathcal{I}_2}^{\mathcal{P}},\ \partial_{\mathcal{I}_1}^{\mathcal{N}} + \partial_{\mathcal{I}_2}^{\mathcal{N}} + \partial_{\mathcal{I}_1}^{\mathcal{N}} \partial_{\mathcal{I}_2}^{\mathcal{N}} \right)$$

3.

$$\acute{\partial}\mathcal{I}_1 = \left(1 - \left(1 - \partial_{\mathcal{I}_1}^{\mathcal{P}} \right)^{\acute{\partial}},\ -\left(\left| \partial_{\mathcal{I}_1}^{\mathcal{N}} \right|^{\acute{\partial}} \right) \right)$$

4.

$$\mathcal{I}_1^{\acute{\partial}} = \left(\left(\partial_{\mathcal{I}_1}^{\mathcal{P}}\right)^{\acute{\partial}}, -1 + \left(1 + \partial_{\mathcal{I}_1}^{\mathcal{N}}\right)^{\acute{\partial}}\right)$$

Definition 3 [44]For a BFN $\mathcal{I} = \left(\partial_{\mathcal{I}}^{\mathcal{P}}, \partial_{\mathcal{I}}^{\mathcal{N}}\right)$,

$$\mathbb{S}(\mathcal{I}) = \frac{1}{2}\left(1 + \partial_{\mathcal{I}}^{\mathcal{P}} + \partial_{\mathcal{I}}^{\mathcal{N}}\right) \quad \mathbb{S}_{\mathcal{B}}(\mathcal{I}) \in [0, 1]$$

is analyzed as score value and

$$\mathbb{H}(\mathcal{I}) = \frac{\partial_{\mathcal{I}}^{\mathcal{P}} - \partial_{\mathcal{I}}^{\mathcal{N}}}{2}, \ \mathbb{H}(\mathcal{I}) \in [0, 1]$$

is analyzed as an accuracy value of $\mathcal{I}$

Definition 4 [45]A function $\mathrm{T} : 2^{\mathcal{Z}} \to [0, 1]$ is interpreted as a fuzzy measure over $\mathcal{Z}$ if

$$\mathrm{T}(\varnothing) = 0, \ \mathrm{T}(\mathcal{Z}) = 1 \text{If } \mathcal{Z}_1, \mathcal{Z}_2 \in P(\mathcal{Z}) \text{ and } \mathcal{Z}_1 \subseteq \mathcal{Z}_2, \text{then } \mathrm{T}(\mathcal{Z}_1) \leq \mathrm{T}(\mathcal{Z}_2).$$

Bipolar fuzzy choquet integral AOs

In this Section, we devise Choquet integral AOs within the framework of BFS and discuss the related properties.

Definition 5 Let a gathering of BFNs $\mathcal{I}_{\jmath} = \left(\partial_{\mathcal{I}_{\jmath}}^{\mathcal{P}}, \partial_{\mathcal{I}_{\jmath}}^{\mathcal{N}}\right), \jmath = 1, 2, \ldots, n$ over $\mathcal{Z}$ and let T be a fuzzy measure over $\mathcal{Z}$. Then the BFCIA operator is evaluated as

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \overset{n}{\underset{\jmath=1}{\oplus}} \left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)\mathcal{I}_{\xi(\jmath)}$$

Where, $(\xi(1), \xi(2), \ldots, \xi(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\xi(\jmath - 1) \geq \xi(\jmath)$, for $\jmath = 2, 3, \ldots, n$, $\mathcal{V}_{\xi(\jmath)} = \left\{y_{\xi(\jmath)} | \jmath \leq k\right\}$ for $k \geq 1$ and $\mathcal{V}_{\xi(\jmath+1)} = \varnothing$.

Theorem 1 *Let a gathering of BFNs* $\mathcal{I}_{\jmath} = \left(\partial_{\mathcal{I}_{\jmath}}^{\mathcal{P}}, \partial_{\mathcal{I}_{\jmath}}^{\mathcal{N}}\right), \jmath = 1, 2, \ldots, n$ *over* $\mathcal{Z}$ *and let* T *be a fuzzy measure over* $\mathcal{Z}$. *Then operating BFCIA operator over* $\mathcal{I}_{\jmath}$ *gives a BFN i.e.*

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \left(1 - \prod_{\jmath=1}^{n}\left(1 - \partial_{\mathcal{I}_{\xi(\jmath)}}^{\mathcal{P}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)}, -\prod_{\jmath=1}^{n}\left|\partial_{\mathcal{I}_{\xi(\jmath)}}^{\mathcal{N}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)}\right) \quad (1)$$

Proof Let $n = 2$. Then to prove that

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2) = \left(1 - \prod_{\jmath=1}^{2}\left(1 - \partial_{\mathcal{I}_{\xi(\jmath)}}^{\mathcal{P}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)}, -\prod_{\jmath=1}^{2}\left|\partial_{\mathcal{I}_{\xi(\jmath)}}^{\mathcal{N}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)}\right).$$

By taking $n = 2$, Eq. (1) is

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2) = \overset{2}{\underset{\jmath=1}{\oplus}} \left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)\mathcal{I}_{\xi(\jmath)} = \left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)\mathcal{I}_{\xi(1)} \oplus \left(\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(3)}\right)\right)\mathcal{I}_{\xi(2)}$$

And we have

$$\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)\mathcal{I}_{\xi(1)} = \left(1 - \left(1 - \partial_{\mathcal{I}_{\xi(1)}}^{\mathcal{P}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)}, -\left(\left|\partial_{\mathcal{I}_{\xi(1)}}^{\mathcal{N}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)}\right)\right)$$

$$\left(\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(3)}\right)\right)\mathcal{I}_{\xi(2)} = \left(1 - \left(1 - \partial_{\mathcal{I}_{\xi(2)}}^{\mathcal{P}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(3)}\right)\right)}, -\left(\left|\partial_{\mathcal{I}_{\xi(2)}}^{\mathcal{N}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(3)}\right)\right)}\right)\right)$$

Then,

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2) = \overset{2}{\underset{\jmath=1}{\oplus}} \left(\mathrm{T}\left(\mathcal{V}_{\xi(\jmath)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(\jmath+1)}\right)\right)\mathcal{I}_{\xi(\jmath)}$$

$$= \left(1-\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(1)})-\mathrm{T}(\mathcal{V}_{\xi\,(2)})\right)}, -\left(\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(1)}}\right|^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(1)})-\mathrm{T}(\mathcal{V}_{\xi\,(2)})\right)}\right)\right)$$
$$\oplus\left(1-\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(2)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(2)})-\mathrm{T}(\mathcal{V}_{\xi\,(3)})\right)}, -\left(\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(2)}}\right|^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(2)})-\mathrm{T}(\mathcal{V}_{\xi\,(3)})\right)}\right)\right)$$
$$= \left(1-\prod\nolimits_{\jmath=1}^{2}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\jmath)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{2}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\jmath)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$

This reveals that Eq. (1) is satisfied for $n=2$. Let Eq. (1) holds for $n=\mathfrak{Q}$, i.e.

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_{\mathfrak{Q}}) = \left(1-\prod\nolimits_{\jmath=1}^{\mathfrak{Q}}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\jmath)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{\mathfrak{Q}}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\jmath)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$

Now to prove that Eq. (1) is satisfied for $n=\mathfrak{Q}+1$. Thus,

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_{\mathfrak{Q}}, \mathcal{I}_{\mathfrak{Q}+1}) = BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_{\mathfrak{Q}}) \oplus \mathcal{I}_{\mathfrak{Q}+1}$$
$$= \left(1-\prod\nolimits_{\jmath=1}^{\mathfrak{Q}}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\jmath)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{\mathfrak{Q}}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\jmath)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$
$$\oplus\left(1-\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\mathfrak{Q}+1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(\mathfrak{Q}+1)})-\mathrm{T}(\mathcal{V}_{\xi\,(\mathfrak{Q}+2)})\right)}, -\left(\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\mathfrak{Q}+1)}}\right|^{\left(\mathrm{T}(\mathcal{V}_{\xi\,(\mathfrak{Q}+1)})-\mathrm{T}(\mathcal{V}_{\xi\,(\mathfrak{Q}+2)})\right)}\right)\right)$$
$$= \left(1-\prod\nolimits_{\jmath=1}^{\mathfrak{Q}+1}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\jmath)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{\mathfrak{Q}+1}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\jmath)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$
$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_{\mathfrak{Q}}, \mathcal{I}_{\mathfrak{Q}+1})$$

Example 1 Let $\mathcal{I}_1=(0.7,\ -0.6)$ and $\mathcal{I}_2=(0.5,\ -0.3)$ be two BFNs over $\mathcal{Z}$. Let T be a fuzzy measure on $\mathcal{Z}$ such as $\mathrm{T}(\mathcal{I}_1)=0.4,\ \mathrm{T}(\mathcal{I}_2)=0.6,\ \mathrm{T}(\mathcal{I}_1, \mathcal{I}_1)=1.0$. Then

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2) = \left(1-\left((1-0.5)^{1-0.6}\times(1-0.7)^{0.6-0.4}\right), -\left(|-0.3|^{1-0.6}\times|-0.7|^{0.6-0.4}\right)\right)$$
$$= \left(1-\left((1-0.5)^{0.4}\times(1-0.7)^{0.2}\right), -\left(|-0.3|^{0.4}\times|-0.7|^{0.2}\right)\right) = (0.4043,\ -0.5578)$$

Properties

The following are the properties that the BFCIA operator satisfy.

- **Idempotency**: Let a gathering of BFNs $\mathcal{I}_{\jmath}=\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\jmath}}, \partial^{\mathcal{N}}_{\mathcal{I}_{\jmath}}\right)$, $\jmath=1,\ 2,\ \ldots,\ n$ over $\mathcal{Z}$ and let T be a fuzzy measure over $\mathcal{Z}$. if $\mathcal{I}_{\jmath}=\mathcal{I}\ \forall\,\jmath$, then,

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots \mathcal{I}_n) = \mathcal{I}$$

Proof As we have

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \left(1-\prod\nolimits_{\jmath=1}^{n}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi\,(\jmath)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi\,(\jmath)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$

and if $\mathcal{I}_{\jmath}=\mathcal{I}\ \forall\,\jmath$, then we have

$$BFCIA(\mathcal{I}, \mathcal{I}, \ldots, \mathcal{I}) = \left(1-\prod\nolimits_{\jmath=1}^{n}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}}\right)^{\sum_{\jmath=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}, -\prod\nolimits_{\jmath=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}}\right|^{\sum_{\jmath=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)}\right)$$

Since $\sum_{\jmath=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi\,(\jmath+1)}\right)\right)=1$, thus

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots \mathcal{I}_n) = \mathcal{I}$$

- **Monotonicity**: Let two gatherings of BFNs $\mathcal{I}_{\jmath}=\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\jmath}}, \partial^{\mathcal{N}}_{\mathcal{I}_{\jmath}}\right)$, and $\mathcal{I}^{\#}_{\jmath}=\left(\partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\jmath}}, \partial^{\mathcal{N}}_{\mathcal{I}^{\#}_{\jmath}}\right)$ $\jmath=1,\ 2,\ \ldots,\ n$ over $\mathcal{Z}$ and let T be a fuzzy measure over $\mathcal{Z}$. If $\partial^{\mathcal{P}}_{\mathcal{I}_{\jmath}}\le\partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\jmath}}$, $\partial^{\mathcal{N}}_{\mathcal{I}_{\jmath}}\le\partial^{\mathcal{N}}_{\mathcal{I}^{\#}_{\jmath}}$, then

$$BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots \mathcal{I}_n) \le BFCIA\left(\mathcal{I}_1^{\#}, \mathcal{I}_2^{\#}, \ldots \mathcal{I}_n^{\#}\right)$$

Proof As $\mathcal{V}_{\xi(j+1)} \subseteq \mathcal{V}_{\xi(j)}$, then $\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right) \ge 0\,\forall j$. Further, we have $\partial^{\mathcal{P}}_{\mathcal{I}_j} \le \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_j}$, then

$$\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}} \le \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\xi(j)}} \Rightarrow 1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}} \ge 1 - \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\xi(j)}}$$

$$\Rightarrow \left(1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \ge \left(1 - \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow -\prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \le -\prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow 1 - \prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \le 1 - \prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}^{\#}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

Next, we have $\partial^{\mathcal{N}}_{\mathcal{I}_j} \le \partial^{\mathcal{N}}_{\mathcal{I}^{\#}_j}$, then

$$\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}} \le \partial^{\mathcal{N}}_{\mathcal{I}^{\#}_{\xi(j)}} \Rightarrow \left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right| \ge \left|\partial^{\mathcal{N}}_{\mathcal{I}^{\#}_{\xi(j)}}\right|$$

$$\Rightarrow \left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \ge \left|\partial^{\mathcal{N}}_{\mathcal{I}^{\#}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow -\prod_{j=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \le -\prod_{j=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots \mathcal{I}_n) \le BFCIA\left(\mathcal{I}_1^{\#}, \mathcal{I}_2^{\#}, \ldots \mathcal{I}_n^{\#}\right)$$

- **Boundedness**: Let a gathering of BFNs $\mathcal{I}_j = \left(\partial^{\mathcal{P}}_{\mathcal{I}_j}, \partial^{\mathcal{N}}_{\mathcal{I}_j}\right)$, $j = 1, 2, \ldots, n$ over $\mathcal{Z}$ and let T be a fuzzy measure over $\mathcal{Z}$. If $\mathcal{I}^- = \left(\min_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_j}\right\}, \min_j\left\{\partial^{\mathcal{N}}_{\mathcal{I}_j}\right\}\right)$ and $\mathcal{I}^+ = \left(\max_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_j}\right\}, \max_j\left\{\partial^{\mathcal{N}}_{\mathcal{I}_j}\right\}\right)$, then

$$\mathcal{I}^- \le BFCIA(\mathcal{I}_1, \mathcal{I}_2, \ldots \mathcal{I}_n) \le \mathcal{I}^+$$

Proof As $\mathcal{V}_{\xi(j+1)} \subseteq \mathcal{V}_{\xi(j)}$, then $\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right) \ge 0\,\forall j$ and $(\xi(1), \xi(2), \ldots, \xi(n))$ is a permutation of $(1, 2, \ldots, n)$. We have that $\forall j$

$$\min_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\} \le \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}} \le \max_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}$$

$$\Rightarrow \prod_{j=1}^{n}\left(1 - \min_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \ge \prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\ge \prod_{j=1}^{n}\left(1 - \max_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow \left(1 - \min_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\sum_{j=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \ge \prod_{j=1}^{n}\left(1 - \partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\ge \left(1 - \max_j\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\sum_{j=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right) - \mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow 1-\left(1-\min_{j}\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\sum_{j=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \leq 1-\prod_{j=1}^{n}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\leq 1-\left(1-\max_{j}\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}\right)^{\sum_{j=1}^{n}\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow \min_{j}\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\} \leq 1-\prod_{j=1}^{n}\left(1-\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \leq \max_{j}\left\{\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right\}$$

Since we have that

$$\min_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\} \leq \partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}} \leq \max_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}$$

$$\Rightarrow \left|\min_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right| \geq \left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right| \geq \left|\max_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right|$$

$$\Rightarrow \left|\min_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \geq \left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \geq \left|\max_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow -\prod_{j=1}^{n}\left|\min_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \leq -\prod_{j=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \leq -\prod_{j=1}^{n}\left|\max_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$\Rightarrow \min_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\} \leq -\prod_{j=1}^{n}\left|\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right|^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)} \leq \max_{j}\left\{\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right\}$$

If $\mathcal{I}_1=\left(\partial^{\mathcal{P}}_{\mathcal{I}_1},\, \partial^{\mathcal{N}}_{\mathcal{I}_1}\right)$ and $\mathcal{I}_2=\left(\partial^{\mathcal{P}}_{\mathcal{I}_2},\, \partial^{\mathcal{N}}_{\mathcal{I}_2}\right)$ are two BFNS, then $\mathcal{I}_1 \leq \mathcal{I}_2$ iff $\partial^{\mathcal{P}}_{\mathcal{I}_1} \leq \partial^{\mathcal{P}}_{\mathcal{I}_1}$ and $\partial^{\mathcal{N}}_{\mathcal{I}_1} \leq \partial^{\mathcal{N}}_{\mathcal{I}_1}$. Using this we have that

$$\mathcal{I}^{-} \leq BFCIA\left(\mathcal{I}_1,\, \mathcal{I}_2,\, \ldots\, \mathcal{I}_n\right) \leq \mathcal{I}^{+}$$

Definition 6 Let a gathering of BFNs $\mathcal{I}_j=\left(\partial^{\mathcal{P}}_{\mathcal{I}_j},\, \partial^{\mathcal{N}}_{\mathcal{I}_j}\right), j=1,\, 2,\, \ldots,\, n$ over $\mathcal{Z}$ and let T be a fuzzy measure over $\mathcal{Z}$. Then the BFCIG operator is evaluated as

$$BFCIG\left(\mathcal{I}_1,\, \mathcal{I}_2,\, \ldots,\, \mathcal{I}_n\right)=\bigotimes_{j=1}^{n}\left(\mathcal{I}_{\xi(j)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

Where, $(\xi(1),\, \xi(2),\, \ldots,\, \xi(n))$ is a permutation of $(1,\, 2, \ldots,\, n)$ such that $\xi(j-1) \geq \xi(j)$, for $j=2,\, 3,\, \ldots,\, n$, $\mathcal{V}_{\xi(j)}=\left\{y_{\xi(j)} \mid j \leq k\right\}$ for $k \geq 1$ and $\mathcal{V}_{\xi(j+1)}=\varnothing$.

Theorem 2 *Let a gathering of BFNs* $\mathcal{I}_j=\left(\partial^{\mathcal{P}}_{\mathcal{I}_j},\, \partial^{\mathcal{N}}_{\mathcal{I}_j}\right), j=1,\, 2,\, \ldots,\, n$ *over* $\mathcal{Z}$ *and let* T *be a fuzzy measure over* $\mathcal{Z}$*. Then operating BFCIG operator over* $\mathcal{I}_j$ *gives a BFN i.e.*

$$BFCIG\left(\mathcal{I}_1,\, \mathcal{I}_2,\, \ldots,\, \mathcal{I}_n\right)=\left(\prod_{j=1}^{n}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},\, -1+\prod_{j=1}^{n}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right) \quad (2)$$

Proof Let $n=2$. Then to prove that

$$BFCIG\left(\mathcal{I}_1,\, \mathcal{I}_2\right)=\left(\prod_{j=1}^{2}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},\, -1+\prod_{j=1}^{2}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right)$$

By taking $n=2$, Eq. (2) is

$$BFCIG\left(\mathcal{I}_1,\, \mathcal{I}_2\right)=\bigotimes_{j=1}^{2}\left(\mathcal{I}_{\xi(j)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}=\left(\mathcal{I}_{\xi(1)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)} \otimes \left(\mathcal{I}_{\xi(2)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(3)}\right)\right)}$$

And we have

$$\left(\mathcal{I}_{\xi(1)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)}=\left(\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(1)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)},\, -1+\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(1)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(1)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(2)}\right)\right)}\right)$$

$$\left(\mathcal{I}_{\xi(2)}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(2)})-\mathrm{T}(\mathcal{V}_{\xi(3)})\right)}=\left(\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(2)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(2)})-\mathrm{T}(\mathcal{V}_{\xi(3)})\right)},-1+\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(2)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(2)})-\mathrm{T}(\mathcal{V}_{\xi(3)})\right)}\right)$$

Then,

$$BFCIG(\mathcal{I}_1,\mathcal{I}_2)=\underset{j=1}{\overset{2}{\otimes}}\left(\mathcal{I}_{\xi(j)}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}$$

$$=\left(\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(1)})-\mathrm{T}(\mathcal{V}_{\xi(2)})\right)},-1+\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(1)})-\mathrm{T}(\mathcal{V}_{\xi(2)})\right)}\right)$$

$$\otimes\left(\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(2)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(2)})-\mathrm{T}(\mathcal{V}_{\xi(3)})\right)},-1+\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(2)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(2)})-\mathrm{T}(\mathcal{V}_{\xi(3)})\right)}\right)$$

$$=\left(\prod_{j=1}^{2}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},-1+\prod_{j=1}^{2}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right)$$

This reveals that Eq. (2) is satisfied for $n=2$. Let Eq. (2) holds for $n=\mathfrak{Q}$, i.e.

$$BFCIG(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_{\mathfrak{Q}})=\left(\prod_{j=1}^{\mathfrak{Q}}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},-1+\prod_{j=1}^{\mathfrak{Q}}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right)$$

Now to prove that Eq. (2) is satisfied for $n=\mathfrak{Q}+1$. Thus,

$$BFCIG(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_{\mathfrak{Q}},\mathcal{I}_{\mathfrak{Q}+1})=BFCIG(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_{\mathfrak{Q}})\otimes\mathcal{I}_{\mathfrak{Q}+1}$$

$$=\left(\prod_{j=1}^{\mathfrak{Q}}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},-1+\prod_{j=1}^{\mathfrak{Q}}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right)$$

$$\otimes\left(\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(\mathfrak{Q}+1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(\mathfrak{Q}+1)})-\mathrm{T}(\mathcal{V}_{\xi(\mathfrak{Q}+2)})\right)},-1+\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(\mathfrak{Q}+1)}}\right)^{\left(\mathrm{T}(\mathcal{V}_{\xi(\mathfrak{Q}+1)})-\mathrm{T}(\mathcal{V}_{\xi(\mathfrak{Q}+3)})\right)}\right)$$

$$=\left(\prod_{j=1}^{\mathfrak{Q}+1}\left(\partial^{\mathcal{P}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)},-1+\prod_{j=1}^{\mathfrak{Q}+1}\left(1+\partial^{\mathcal{N}}_{\mathcal{I}_{\xi(j)}}\right)^{\left(\mathrm{T}\left(\mathcal{V}_{\xi(j)}\right)-\mathrm{T}\left(\mathcal{V}_{\xi(j+1)}\right)\right)}\right)$$

$$BFCIG(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_{\mathfrak{Q}},\mathcal{I}_{\mathfrak{Q}+1})$$

Example 2 Let $\mathcal{I}_1=(0.7,-0.6)$ and $\mathcal{I}_2=(0.5,-0.3)$ be two BFNs over $\mathcal{Z}$. Let T be a fuzzy measure on $\mathcal{Z}$ such as $\mathrm{T}(\mathcal{I}_1)=0.4$, $\mathrm{T}(\mathcal{I}_2)=0.6$, $\mathrm{T}(\mathcal{I}_1,\mathcal{I}_1)=1.0$. Then

$$BFCIG(\mathcal{I}_1,\mathcal{I}_2)=\left((0.5)^{1-0.6}\times(0.7)^{0.6-0.4},-1+\left((1-0.3)^{1-0.6}\times(1-0.6)^{0.6-0.4}\right)\right)$$

$$=\left((0.5)^{0.4}\times(0.7)^{0.2},-1+\left((1-0.3)^{0.4}\times(1-0.6)^{0.2}\right)\right)=(0.7057,-0.2781)$$

Properties

The BFCIG operator hold, idempotency, monotonicity, and boundedness.

Bipolar fuzzy MCDM technique based on choquet integral operators

Consider a situation, where m number of alternatives i.e. $\{\mathfrak{X}_1,\mathfrak{X}_2,\ldots,\mathfrak{X}_m\}$ are available for evaluation based on number of criteria i.e. $\{\tau_1,\tau_2,\ldots,\tau_n\}$ with the help of r number of decision makers i.e. $\{\mathfrak{P}_1,\mathfrak{P}_2,\ldots,\mathfrak{P}_r\}$. These decision-makers have to assess and evaluate the given alternatives based on the provided criteria. For this, they provide their assessment values in linguistic terms and develop a linguistic decision matrix that is $\mathbb{E}^{\ell}=\left[\sigma^{(\ell)}_{jk}\right]_{m\times n}$. Note that $\sigma^{(\ell)}_{jk}$ present the evaluation value of j^{th} alternative based on k^{th} criterion and ℓ^{th} expert. This is a typical MCDM (multi-criteria group decision-making (MCGDM)) problem. To handle this MCDM problem, we are going to develop a bipolar fuzzy CODAS approach as follows.

Step 1 Transform linguistic decision matrices to bipolar fuzzy matrices.

With most of the criteria, uncertainty and bipolarity are involved. Thus, it is important to consider the uncertainties and bipolarities of the criteria of the alternatives. Because of this, the linguistic assessment values provided by the decision-makers must be transformed into bipolar fuzzy numbers and create bipolar fuzzy decision matrices (BFDMs) $\mathbb{E}^{\ell}_{\mathcal{B}}=\left[\mathcal{F}^{(\ell)}_{jk}\right]_{m\times n}$ and

$$\mathbb{E}_{\mathcal{B}}^{\ell} = \left[\mathcal{F}_{jk}^{(\ell)}\right]_{m\times n} = \begin{matrix} \mathfrak{X}_1 \\ \mathfrak{X}_2 \\ \vdots \\ \mathfrak{X}_m \end{matrix} \overset{\begin{matrix} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_n \end{matrix}}{\begin{pmatrix} \left(\partial_{\mathcal{F}_{11}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{11}}^{\mathcal{N},\ell}\right) & \left(\partial_{\mathcal{F}_{12}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{12}}^{\mathcal{N},\ell}\right) & \cdots & \left(\partial_{\mathcal{F}_{1n}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{1n}}^{\mathcal{N},\ell}\right) \\ \left(\partial_{\mathcal{F}_{21}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{21}}^{\mathcal{N},\ell}\right) & \left(\partial_{\mathcal{F}_{22}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{22}}^{\mathcal{N},\ell}\right) & \cdots & \left(\partial_{\mathcal{F}_{2n}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{2n}}^{\mathcal{N},\ell}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\partial_{\mathcal{F}_{m1}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{m1}}^{\mathcal{N},\ell}\right) & \left(\partial_{\mathcal{F}_{m2}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{m2}}^{\mathcal{N},\ell}\right) & \cdots & \left(\partial_{\mathcal{F}_{mn}}^{\mathcal{P},\ell}, \partial_{\mathcal{F}_{mn}}^{\mathcal{N},\ell}\right) \end{pmatrix}}$$

Step 2 Transform all BFDMs into a single BFDM.

This step contains the transformation of all BFDMs into a single BFDM (SBFDM) and this action will be performed by using bipolar fuzzy weighted averaging or bipolar fuzzy weighted geometric operator. The SBFDM is

$$\mathbb{E}_{\mathcal{B}} = \left[\mathcal{F}_{jk}\right]_{m\times n} = \begin{matrix} \mathfrak{X}_1 \\ \mathfrak{X}_2 \\ \vdots \\ \mathfrak{X}_m \end{matrix} \overset{\begin{matrix} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_n \end{matrix}}{\begin{pmatrix} \left(\partial_{\mathcal{F}_{11}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{11}}^{\mathcal{N},\#}\right) & \left(\partial_{\mathcal{F}_{12}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{12}}^{\mathcal{N},\#}\right) & \cdots & \left(\partial_{\mathcal{F}_{1n}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{1n}}^{\mathcal{N},\#}\right) \\ \left(\partial_{\mathcal{F}_{21}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{21}}^{\mathcal{N},\#}\right) & \left(\partial_{\mathcal{F}_{22}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{22}}^{\mathcal{N},\#}\right) & \cdots & \left(\partial_{\mathcal{F}_{2n}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{2n}}^{\mathcal{N},\#}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\partial_{\mathcal{F}_{m1}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{m1}}^{\mathcal{N},\#}\right) & \left(\partial_{\mathcal{F}_{m2}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{m2}}^{\mathcal{N},\#}\right) & \cdots & \left(\partial_{\mathcal{F}_{mn}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{mn}}^{\mathcal{N},\#}\right) \end{pmatrix}}$$

Step 3 Standardize the SBFDM.

To remove the effects of the cost type of criteria, there is a need for standardization of SBFDM. This will be performed by using the formula.

$$\left(\partial_{\mathcal{F}_{jk}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{jk}}^{\mathcal{N},\sim}\right) = \begin{cases} \left(\partial_{\mathcal{F}_{jk}}^{\mathcal{P},\#}, \partial_{\mathcal{F}_{jk}}^{\mathcal{N},\#}\right) & if\ k \in \mathcal{B}_n \\ \left(1 - \partial_{\mathcal{F}_{jk}}^{\mathcal{P},\#}, -1 - \partial_{\mathcal{F}_{jk}}^{\mathcal{N},\#}\right) & if\ k \in \mathfrak{C}_n \end{cases}$$

Where $\mathfrak{C}$ is for the cost kind of criterion and $\mathcal{B}$ is for the benefit type of criterion. The standardized SBFDM (SSBFDM) is given as below

$$\mathbb{SE}_{\mathcal{B}} = \left[\mathcal{F}_{jk}^{\sim}\right]_{m\times n} = \begin{matrix} \mathfrak{X}_1 \\ \mathfrak{X}_2 \\ \vdots \\ \mathfrak{X}_m \end{matrix} \overset{\begin{matrix} \mathcal{T}_1 & \mathcal{T}_2 & \cdots & \mathcal{T}_n \end{matrix}}{\begin{pmatrix} \left(\partial_{\mathcal{F}_{11}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{11}}^{\mathcal{N},\sim}\right) & \left(\partial_{\mathcal{F}_{12}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{12}}^{\mathcal{N},\sim}\right) & \cdots & \left(\partial_{\mathcal{F}_{1n}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{1n}}^{\mathcal{N},\sim}\right) \\ \left(\partial_{\mathcal{F}_{21}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{21}}^{\mathcal{N},\sim}\right) & \left(\partial_{\mathcal{F}_{22}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{22}}^{\mathcal{N},\sim}\right) & \cdots & \left(\partial_{\mathcal{F}_{2n}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{2n}}^{\mathcal{N},\sim}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\partial_{\mathcal{F}_{m1}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{m1}}^{\mathcal{N},\sim}\right) & \left(\partial_{\mathcal{F}_{m2}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{m2}}^{\mathcal{N},\sim}\right) & \cdots & \left(\partial_{\mathcal{F}_{mn}}^{\mathcal{P},\sim}, \partial_{\mathcal{F}_{mn}}^{\mathcal{N},\sim}\right) \end{pmatrix}}$$

Step 4 Fuzzy measure for criteria.

In this step, consider the fuzzy measure of criteria.

Step 5 Aggregate the SSBFDM.

Aggregate SSBFDM by employing BFCIA (Eq. 1) or BFCIG (Eq. 2) operators to achieve the aggregate value of each alternative.

Step 6 Achieve score values.

Get the score values of the aggregated outcomes of each alternative by using the following formula

$$\mathbb{S}\left(\mathfrak{X}_j\right) = \frac{1}{2}\left(1 + \partial_{\mathfrak{X}_j}^{\mathcal{P}} + \partial_{\mathfrak{X}_j}^{\mathcal{N}}\right)$$

In case of any two same score values, find the accuracy values as follows

$$\mathbb{H}\left(\mathfrak{X}_j\right) = \frac{\partial_{\mathfrak{X}_j}^{\mathcal{P}} - \partial_{\mathfrak{X}_j}^{\mathcal{N}}}{2}$$

Step 7 Rank the alternatives.

In this last step, the alternatives will be ranked by employing score or accuracy values and the finest alternative will be determined.

Figure 2 displays the flowchart of the BF MCDM technique.

Case study

The field of smart electrical prosthetics has evolved through AI-driven capabilities to improve service delivery for people with disabilities in recent years. The systems need precise AI models that both diagnose different disability conditions and modify prosthetic reactions for optimal patient outcomes. The selection and assessment of suitable AI simulation models play an essential role because they determine both patient life quality and the success of prosthetic treatment. Selecting the optimal AI platform represents a challenge because it must process

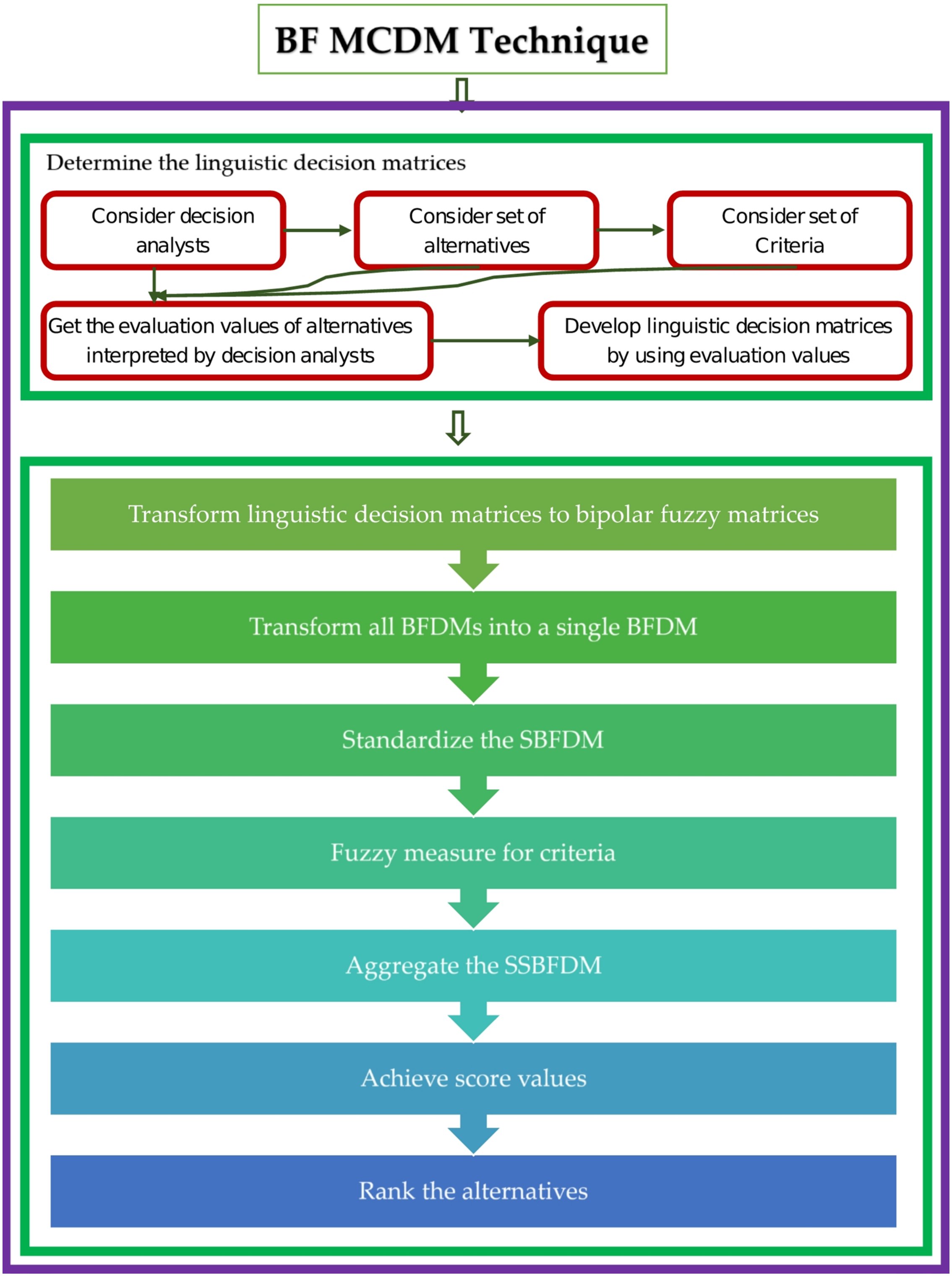

Fig. 2. The flowchart of bipolar fuzzy MCDM method using BFCIA and BFCIG operators.

Notation	AI simulation	Explanation
$\mathfrak{x}_1$	TensorFlow Medical	The open-source framework TensorFlow Medical functions specifically for medical applications to develop smart electrical prosthetic systems. The system uses deep learning models to analyze biosignals obtained from prosthetic devices for real-time disability-related condition diagnosis. Through advanced neural networks, this platform analyzes complex patient data patterns to deliver extensive diagnostic support to healthcare professionals who work with prosthetic systems.
$\mathfrak{x}_2$	BioSignal Analytics Pro	BioSignal Analytics Pro functions as a cloud-based system that specifically processes signals obtained from smart prosthetics. The solution enables the identification of disability-related conditions early through advanced pattern recognition algorithms reviewing continuous prosthesis data. Real-time diagnosis powers are delivered through the platform because its state-of-the-art data processing capabilities help healthcare providers select optimal treatments and adjustments for patients receiving prosthetics.
$\mathfrak{x}_3$	NeuroTech AI	Through the NeuroTech AI platform using neural networks, the system aims to enhance signal processing procedures between prosthetic devices. The system examines motion patterns and neural signals for medical problems and functional disabilities through its analysis process. Machine learning algorithms at an advanced stage assist the system to examine complex neurological patterns which leads to comprehensive data about patient locomotion and prosthetic operation.
$\mathfrak{x}_4$	ProsMed AI	The standalone AI solution ProsMed AI serves only smart prosthetic systems. Designated diagnostic applications in the platform work on actual prosthetic device data to detect and diagnose potential disabilities and related issues. The system operates with both complete monitoring features and adaptive learning systems that boost its ability to detect problems more precisely as time progresses.
$\mathfrak{x}_5$	**SmartLimb Intelligence**	SmartLimb Intelligence is a complete solution that uses AI to maximize the performance of the prosthetic device with real-time biomechanical insights. The system incorporates sensor fusion technology to examine gait patterns, muscle activation signals, and joint dynamics of sophisticated prosthetic limbs. The platform offers predictive maintenance warnings, custom adjustment suggestions, and the progress of rehabilitation through machine learning algorithms to enable prosthetists and physical therapists to offer more efficient care to patients.
$\mathfrak{x}_6$	**ProstheticVision AI**	ProstheticVision AI is a computer vision and deep learning-based system to study the patterns of movement of prosthetic users and the results of their functioning. The platform analyzes video data during a rehabilitation session and sensor data of prosthetic devices to evaluate the progress in mobility and the presence of possible complications. Its adaptive algorithms are trained on the data about each patient to offer personalized therapy recommendations and identify the early signs of prosthetic misalignment or the difficulties in adapting to the prosthesis.
$\mathfrak{x}_7$	**BioAdapt Neural**	BioAdapt Neural is a company that deals with neural signal processing of next-generation myoelectric prosthetics. The platform employs sophisticated signal processing algorithms to identify intricate patterns of muscle activation and convert them into accurate control commands of the prosthesis. Its machine-learning models continually adjust to shifts in the quality of the muscle signals, so they can keep the prosthetics at their best as patients heal and adjust. The system can also offer diagnostic information on nerve activity and muscle condition to facilitate the overall prosthetic rehabilitation programs.

Table 1. The AI simulations.

Notation	Criteria	Explanation
τ_1	Diagnostic Accuracy	Diagnostic Accuracy describes how well the platform detects and diagnoses disability-related conditions by analyzing prosthetic sensor data together with patient movement patterns. The system demonstrates both precise early warning sign detection and reliable consistent diagnostic results regardless of patient profiles.
τ_2	Processing Speed	The platform must demonstrate efficient processing of real-time data streams generated by prosthetic devices through Processing Speed. The system's diagnostic alert response time and its simultaneous processing of multiple data streams together with its speed of delivering actionable insights to healthcare providers form the basis of this criterion.
τ_3	Adaptability	Personal health systems demonstrate adaptability through their ability to learn this new data and handle a range of disability types. The platform demonstrates diagnostic flexibility through individualized patient profiling and develops enhanced accuracy by continuously learning from patient interactions and outcome data.
τ_4	Integration Capability	The Integration Capability section examines the platform's ability to connect with current prosthetic hardware along with medical systems. The criterion examines technical support availability together with documentation quality and platform compatibility with prosthetic devices and healthcare information systems. The evaluation takes into account both the simplicity of installation and upkeep within current medical facilities.

Table 2. The selection criteria of considered AI simulations.

complex bio signals effectively while delivering precise diagnostic results and performing smooth integration with prosthetic devices.

Problem statement

The leading prosthetics research center needs to find the best AI simulation model that will enhance diagnosis features for their advanced electrical prosthesis systems. The chosen platform will determine how well the center diagnoses disabilities and delivers suitable prosthetic solutions. The Center assigned a decision maker that has to select and assess the AI simulation model. The selection process focuses on four different AI platforms which present distinct features and operational trade-offs. These AI simulations are described in (Table 1).

For assessing these AI simulations, the decision analyst considered 4 criteria which are interpreted in (Table 2).

The decision analyst assesses these AI simulations based on the considered criteria and provides their assessment values in linguistic terms. (See Table 3)

O is utilized instead of Outstanding, E is utilized instead of Excellent, G is utilized instead of Good, S is utilized instead of Satisfactory, M is utilized instead of Marginal, D is utilized instead of Deficient, and U is utilized instead of Unacceptable in Table 3. To solve this MCDM problem, we use the deduced bipolar fuzzy MCDM technique.

Step 1 As the criteria have uncertainty and bipolarity, thus it is required to transform the linguistic assessment values into the bipolar fuzzy framework. For this, we have the following scale of transformation, which is given in (Table 4).

Alternatives/criteria	τ_1	τ_2	τ_3	τ_4
$\mathfrak{X}_1$	O	E	M	D
$\mathfrak{X}_2$	S	M	U	O
$\mathfrak{X}_3$	E	G	O	D
$\mathfrak{X}_4$	G	S	M	E
$\mathfrak{X}_5$	M	E	G	D
$\mathfrak{X}_6$	U	O	E	G
$\mathfrak{X}_7$	M	D	E	S

Table 3. The assessment values of AI simulation models (Hypothetical data).

Linguistic terms	BFNs
Outstanding ($\boldsymbol{O}$)	$(0.95,\ -0.\ 1)$
Excellent ($\boldsymbol{E}$)	$(0.88,\ -0.22\ )$
Good ($\boldsymbol{G}$)	$(0.79,\ -0.34\ )$
Satisfactory ($\boldsymbol{S}$)	$(0.5,\ -0.54\ )$
Marginal ($\boldsymbol{M}$)	$(0.34,\ -0.67)$
Deficient ($\boldsymbol{D}$)	$(0.26,\ -0.82)$
Unacceptable ($\boldsymbol{U}$)	$(0.15,\ -0.91)$

Table 4. The transformation scale to transform the linguistic terms into bfns.

Alternatives/criteria	τ_1	τ_2	τ_3	τ_4
$\mathfrak{X}_1$	$(0.95,\ -0.\ 1)$	$(0.88,\ -0.22\ )$	$(0.34,\ -0.67)$	$(0.26,\ -0.82)$
$\mathfrak{X}_2$	$(0.5,\ -0.54\ )$	$(0.34,\ -0.67)$	$(0.15,\ -0.91)$	$(0.95,\ -0.\ 1)$
$\mathfrak{X}_3$	$(0.88,\ -0.22\ )$	$(0.79,\ -0.34\ )$	$(0.95,\ -0.\ 1)$	$(0.26,\ -0.82)$
$\mathfrak{X}_4$	$(0.79,\ -0.34\ )$	$(0.5,\ -0.54\ )$	$(0.34,\ -0.67)$	$(0.88,\ -0.22\ )$
$\mathfrak{X}_5$	$(0.34,\ -0.67)$	$(0.88,\ -0.22\ )$	$(0.79,\ -0.34\ )$	$(0.26,\ -0.82)$
$\mathfrak{X}_6$	$(0.15,\ -0.91)$	$(0.95,\ -0.\ 1)$	$(0.88,\ -0.22\ )$	$(0.79,\ -0.34\ )$
$\mathfrak{X}_7$	$(0.34,\ -0.67)$	$(0.26,\ -0.82)$	$(0.88,\ -0.22\ )$	$(0.5,\ -0.54\ )$

Table 5. The bipolar fuzzy decision matrix after the transformation.

Through the scale of transformation, we obtain the BFDM, given in (Table 5).

Step 2 In this case study, there is only one decision-maker, so no need for this step.

Step 3 The criteria is benefit type so no need for standardization in this case.

Step 4 Consider a fuzzy measure of criteria as below.

$$\mathrm{T}(\tau_1) = 0.29,\ \ \mathrm{T}(\tau_2) = 0.47,\ \ \mathrm{T}(\tau_3) = 0.66,\ \ \mathrm{T}(\tau_4) = 0.72$$

$$\mathrm{T}(\tau_1,\ \tau_2) = 0.63,\ \ \mathrm{T}(\tau_1,\ \tau_3) = 0.74,\ \ \mathrm{T}(\tau_1,\ \tau_4) = 0.81,\ \ \mathrm{T}(\tau_2,\ \tau_3) = 0.87$$

$$\mathrm{T}(\tau_2,\ \tau_4) = 0.85,\ \ \mathrm{T}(\tau_3,\ \tau_4) = 0.79,\ \ \mathrm{T}(\tau_1,\ \tau_2, \tau_3) = 0.9,\ \ \mathrm{T}(\tau_1,\ \tau_2, \tau_4) = 0.92\ \ \mathrm{T}(\tau_1,\ \tau_3, \tau_4) = 0.94$$

$$\mathrm{T}(\tau_2,\ \tau_3, \tau_4) = 0.96,\ \ \mathrm{T}(\tau_1,\ \tau_2, \tau_3,\ \tau_4) = 1$$

Step 5 Aggregated the assessment values by using BFCIA and BFCIG operators and the result is presented in (Table 6).

Step 6 The score values of AI simulations are devised in (Table 7).

The score value of $\mathbb{S}(\mathfrak{X}_1)$ will be devised as

Operators	BFCIA operator	BFCIG operator
$\mathfrak{X}_1$	$(0.649,\ -0.456)$	$(0.449,\ -0.617)$
$\mathfrak{X}_2$	$(0.415,\ -0.632)$	$(0.317,\ -0.737)$
$\mathfrak{X}_3$	$(0.762,\ -0.362)$	$(0.579,\ -0.516)$
$\mathfrak{X}_4$	$(0.616,\ -0.476)$	$(0.532,\ -0.519)$
$\mathfrak{X}_5$	$(0.563,\ -0.543)$	$(0.432,\ -0.64)$
$\mathfrak{X}_6$	$(0.753,\ -0.373)$	$(0.509,\ -0.604)$
$\mathfrak{X}_7$	$(0.419,\ -0.633)$	$(0.371,\ -0.68)$

Table 6. The aggregated outcomes of AI simulation models.

Operators	$\mathbb{S}(\mathfrak{X}_1)$	$\mathbb{S}(\mathfrak{X}_2)$	$\mathbb{S}(\mathfrak{X}_3)$	$\mathbb{S}(\mathfrak{X}_4)$	$\mathbb{S}(\mathfrak{X}_5)$	$\mathbb{S}(\mathfrak{X}_6)$	$\mathbb{S}(\mathfrak{X}_7)$
BFCIA	0.597	0.392	0.7	0.57	0.51	0.69	0.393
BFCIG	0.416	0.29	0.541	0.506	0.396	0.453	0.345

Table 7. The score values of AI simulation models.

Operators	Ranking
BFCIA	$\mathfrak{X}_3 \geq \mathfrak{X}_6 \geq \mathfrak{X}_1 \geq \mathfrak{X}_4 \geq \mathfrak{X}_5 \geq \mathfrak{X}_7 \geq \mathfrak{X}_2$
BFCIG	$\mathfrak{X}_3 \geq \mathfrak{X}_4 \geq \mathfrak{X}_6 \geq \mathfrak{X}_1 \geq \mathfrak{X}_5 \geq \mathfrak{X}_7 \geq \mathfrak{X}_2$

Table 8. The ranking of AI simulation models.

$$\mathbb{S}(\mathfrak{X}_1) = \frac{1}{2}(1 + 0.649 - 0.456) = 0.597$$

The rest can be obtained in a similar pattern.

Step 7 Table 8 reveals the ranking of AI simulations.

The findings contained in Fig. 3; Table 8 have major practical implications for healthcare practitioners and biomedical engineers implicated in the process of choosing AI simulation models of smart electrical prosthetic systems. The fact that NeuroTech AI ($(\mathfrak{X}_3)$) was ranked as the best option in both BFCIA and BFCIG operators shows that our bipolar fuzzy Choquet integral method is sound in making intricate decisions where criteria have both positive and negative features. This conclusion indicates that the high level of neural signal processing and motion pattern analysis by NeuroTech AI and good adaptability features can be prioritized over the possible drawbacks of other functions because of the interdependence of evaluation criteria. The near performance of ProstheticVision AI $(\mathfrak{X}_6)$ as the second-best alternative suggests that computer vision-based methods also have a high potential in prosthetics. Notably, the significant gap in the ranking of the traditional methods and our approach indicates the crucial role of addressing criterion interactions and bipolarity in the process of selecting AI models to be used in practice, a factor that may have a serious effect on patient outcomes and the effectiveness of the prosthetic system. These findings will give healthcare decision-makers evidence-based information on how to optimize diagnostic accuracy and reduce system limitations, which will eventually translate to better patient care and prosthetic functionality in clinical practice.

Comparative study

The establishment of superiority and necessity for our proposed method required a comparison with multiple popular existing theories. The comparison encompassed multiple established approaches, including the intuitionistic fuzzy (IF) Choquet integral arithmetic AOs and linked MCDM approach Interpreted by Tan and Chen[46] within the IF framework, IF Choquet integral geometric AOs and corresponding MCDM technique devised by Tan[47], Choquet integral-based AOs and decision-making processes within interval-valued IF set interpreted by Garg et al.[48], Pythagorean fuzzy Choquet integral AOs and MABAC approach, developed by Peng and Yang[49] and Choquet integral AOs within the hesitant fuzzy set (HFS), devised by Wei et al.[50]. We applied existing methodologies and our proposed Choquet integral AOs and MCDM technique within BFSs to analyze the data in (Table 5). The results appear in (Tables 9 and 10, and Fig. 4).

The comparative analysis showed that there are serious theoretical and practical limitations of the available aggregation operators and MCDM approaches, especially their inability to process and combine the negative features of evaluation criteria with positive ones. The critical review of the Choquet integral AOs presented

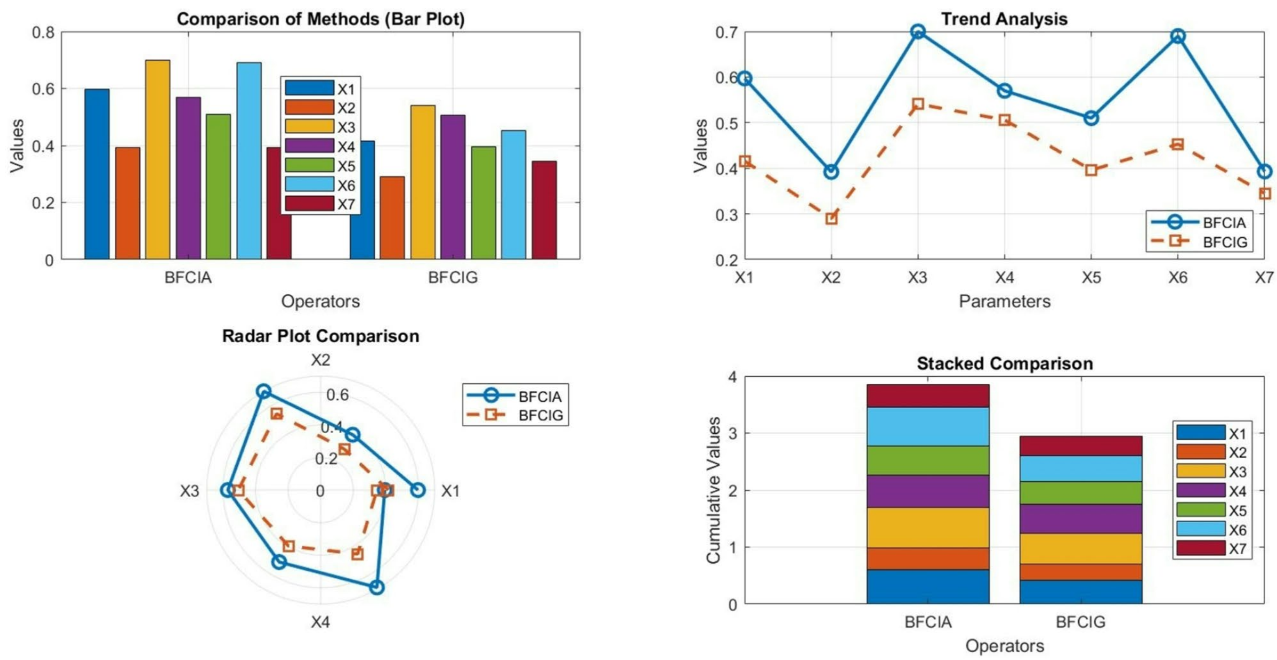

Fig. 3. The graphical interpretation of score values of AI simulation models.

Operators	Technique	$\mathbb{S}(\mathfrak{X}_1)$	$\mathbb{S}(\mathfrak{X}_2)$	$\mathbb{S}(\mathfrak{X}_3)$	$\mathbb{S}(\mathfrak{X}_4)$	$\mathbb{S}(\mathfrak{X}_5)$	$\mathbb{S}(\mathfrak{X}_6)$	$\mathbb{S}(\mathfrak{X}_1)$
Tan and Chen[46]	MCDM	collapse	collapse	collapse	collapse	collapse	collapse	collapse
Tan[47]	MCDM	collapse	collapse	collapse	collapse	collapse	collapse	collapse
Garg et al.[48]	MCDM	collapse	collapse	collapse	collapse	collapse	collapse	collapse
Peng and Yang[49]	MABAC	collapse	collapse	collapse	collapse	collapse	collapse	collapse
Wei et al.[50]	MCDM	collapse	collapse	collapse	collapse	collapse	collapse	collapse
BFCIA	MCDM	0.597	0.392	0.7	0.57	0.51	0.69	0.393
BFCIG	MCDM	0.416	0.29	0.541	0.506	0.396	0.453	0.345

Table 9. The comparison between existing and developed theories.

Operators	Ranking
Tan and Chen[46]	Collapse
Tan[47]	Collapse
Garg et al.[48]	Collapse
Peng and Yang[49]	Collapse
Wei et al.[50]	Collapse
BFCIA	$\mathfrak{X}_3 \geq \mathfrak{X}_6 \geq \mathfrak{X}_1 \geq \mathfrak{X}_4 \geq \mathfrak{X}_5 \geq \mathfrak{X}_7 \geq \mathfrak{X}_2$
BFCIG	$\mathfrak{X}_3 \geq \mathfrak{X}_4 \geq \mathfrak{X}_6 \geq \mathfrak{X}_1 \geq \mathfrak{X}_5 \geq \mathfrak{X}_7 \geq \mathfrak{X}_2$

Table 10. The ranking of the comparison between existing and developed theories.

in the existing literature reveals the essential methodological gap: these operators are not effective in treating bipolarity, which requires the consideration and assessment of both positive and negative membership degrees of criteria at the same time. This shortcoming is especially undesirable when it comes to complex decision-making processes like the choice of an AI simulation model to use in prosthetic systems, where every criterion has both positive and negative aspects, which need to be considered holistically. The theoretical framework of the proposed bipolar fuzzy Choquet integral is the first comprehensive theoretical framework that has been able to allow Choquet integral AOs to aggregate and process genuinely bipolar information structures. The methodological contribution is a solution to the research gap in the literature on AI simulation model selection, in which the past research systematically ignored or poorly addressed the negative aspect of evaluation criteria in the selection procedure. As such, the proposed methodology is the first of its kind to be comprehensive in its

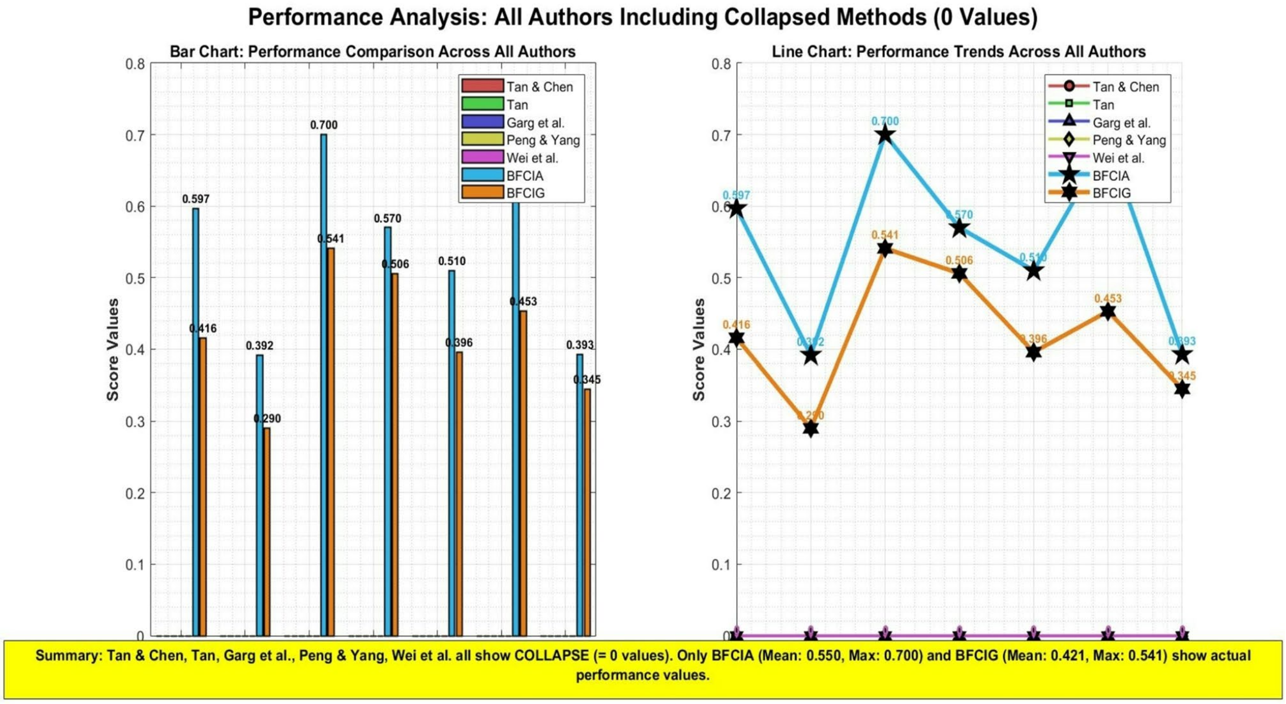

Fig. 4. The graphical structure of prevailing and proposed work.

integration and processing of bipolarity in AI simulation model evaluation, and as such presents a more realistic, theoretically grounded, and practically implementable decision-making structure to these important healthcare technology selection decisions.

Conclusion

This article addressed the important MCDM problem of choosing and ranking AI simulation models to diagnose disability diseases in smart electrical prosthetic systems with innovative BFCIA and BFCIG operators, which is the first mathematical framework to effectively combine BFSs with Choquet integral AOs to assess healthcare technology. The theoretical value of our contribution is threefold: firstly, we constructed operators that are uniquely capable of representing both positive and negative membership degrees and, at the same time, model non-linear interactions among criteria, and thus, it fills the gap between bipolarity representation and non-linear aggregation processes; secondly, we addressed the inherent shortcoming of the existing Choquet integral frameworks that are unable to process bipolar fuzzy information, and thus, we can detect both synergetic and antagonistic effects between criterion positive and negative aspects that cannot be detected using standard methods. Further, we developed a MCDM method by using BFCIA and BFCIG operators within the framework of BFSs and then discussed a case study. The overall validation of the case study showed that NeuroTech AI is the best possible solution with BFCIA 0.7 and BFCIG 0.541, which outperforms others in terms of its high balance between diagnostic accuracy $(0.88,\ -0.22)$ and adaptability $(0.95,\ -0.1)$ and acceptable integration capabilities, which directly leads to a high level of precision in treatment and quality of life of more than 40 million people using prostheses in the world. The revolutionary potential of this study goes well beyond the field of prosthetics to transform the entire field of healthcare technology assessment to transform healthcare decision-making in any area where decision criteria are inherently bipolar and have complex interdependencies, such as medical device selection, treatment protocol optimization, and the design of healthcare systems, by offering clinicians, biomedical engineers, and healthcare policymakers a rigorous mathematically based, evidence-based decision-making framework that can maximize therapeutic benefits and minimize system risks and functional limitations. The comparative analysis that we have conducted proves beyond doubt that the proposed framework is more accurate in evaluation than the traditional MCDM methods, and standard Choquet integral methods since it processes the dual-aspect criteria relationships which the traditional methods cannot process simultaneously.

Future direction

The potential future research directions include: the development of multi-agent extensions that will allow collaborative decision-making in interdisciplinary healthcare teams of prosthetists, physicians, and rehabilitation specialists; the development of scalable architectures that will support enterprise-level healthcare networks with thousands of AI options and dynamic evaluation criteria; the design of adaptive operators that will include real-time learning capabilities to support the dynamic nature of prosthetic technologies and changing patient demographics; to expand Choquet Integral to picture fuzzy[51–53], complex hesitant fuzzy[54,55], and bipolar complex fuzzy sets[56,57].

Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

Received: 7 March 2025; Accepted: 16 July 2025
Published online: 10 August 2025

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Acknowledgements
The authors extend their appreciation to the King Salman center For Disability Research for funding this work through Research Group no KSRG-2024-418.

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Declarations

Competing interests
The authors declare no competing interests.

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