



OPEN Collaborative optimization of layout and cutting scheduling for large-scale customized metal structural parts

Ronghua Meng^{1,2,6}, Jiayi Wang^{1,2,6}, Chongchong Xiang^{1,2,6}, Xuan Du^{1,2,6}, Wenhui Zeng^{3,4,6}, Chaoyan Jiang⁵✉, Zhengjia Wu^{1,2} & Feng Guan^{1,2}

Exploring the collaborative optimization mechanism of layout and cutting scheduling is a key scientific problem that enterprises must solve urgently. To balance the makespan, delay penalty, and sheet utilization rate for large scale customized metal structural parts, a two-stage model is proposed to cooperatively optimize the layout and cutting scheduling of rectangular metal structural parts, considering the processing characteristics. A layout model is established at the first stage to maximize the sheet utilization. Focusing on the sequence of sheet metal processing and machine selection, a mathematical model is formulated to minimize the makespan and delay penalty at the second stage. The layout problem at Stage 1 can be solved by the Improved Grey Wolf Optimizer (IGWO) algorithm. Based on the layout heuristic strategy, the parts with smaller priority values are laid first. The cutting scheduling problem (CSP) at Stage 2 is solved by the Modified Non-dominated Sorting Genetic Algorithm (MNSGA-II). The production scheduling heuristic strategy of the smaller priority values cutting first is formulated to generate the initial population. The experiment results show the effectiveness of the proposed two-stage model and the improved algorithms.

Keywords Large-scale customized, Cutting scheduling, Layout priority, IGWO, MNSGA-II

With China development of large and medium-sized machinery and equipment manufacturing, metal structural components have become key components of hydraulic equipment, ensuring the growth of water conservancy and hydroelectric infrastructure. However, the manufacturing process for structural components is complex, characterized by a fractal-deformation-configuration manufacturing characteristic, with limited flexibility for dynamic adjustments. Additionally, small-batch metal structural components exhibit the characteristics of bulky volume, difficult transportation, and poor interchangeability between parts, which precludes advanced material preparation and leads to extended delivery time. The basic reason for this problem lies in the disconnect between layout planning and production scheduling optimization. In the manufacturing process of structural metal components, layout and cutting scheduling are two closely related processes. The layout determines the matching relationship between parts and raw materials, while scheduling determines the matching relationship between parts and machines, as well as the processing sequence, thus determining the completion time for each part. If layout and cutting scheduling can be optimized in a coordinated manner, the delivery time can be effectively shortened. However, relevant manufacturing companies are currently facing dual losses of delivery delays and soaring costs due to the disconnect between layout planning and production scheduling optimization. In other words, they cannot generate layout and production scheduling schemes that are both material and time efficient with limited raw material inventory and machine production capacity, based on the increasing raw material prices. This paper reviews the existing achievements in the collaborative optimization of layout scheduling in three aspects: layout optimization problems, scheduling problems, and collaborative optimization of layout and scheduling.

¹Hubei Key Laboratory of Hydroelectric Machinery Design and Maintenance, China Three Gorges University, Yichang 44002, China. ²Intelligent Manufacturing Innovation Technology Center, China Three Gorges University, Yichang 44002, China. ³Shaoxing Institute for Advanced Research, Wuhan University of Technology, Shaoxing 312000, China. ⁴School of Mechanical and Electronic Engineering, Wuhan University of Technology, Wuhan 430070, China. ⁵Shanghai Harbor School, Shanghai Maritime University, Shanghai 201306, China. ⁶Ronghua Meng, Jiayi Wang, Chongchong Xiang, Xuan Du and Wenhui Zeng contributed equally to this work. ✉email: dummyjcy@163.com

Many scholars have studied layout problems using deterministic or heuristic methods. Wang et al.¹ introduced a custom branch and binding method to address the layout problem and ensure optimality. Tang et al.² proposed an optimization model to solve the layout of rectangular parts based on the genetic algorithm, aiming at the waste of raw materials and the inefficiency of the traditional layout. For the rectangular parts layout problem, Luo et al.³ and Zeng et al.⁴ improved various algorithms to increase sheet utilization rate. Yuan⁵ developed a common batch optimization problem of rectangular parts, which is addressed with the objective of minimizing the use of raw materials and optimizing the utilization rate of parts. To solve the rectangular parts layout problem in the wood and glass blanking process, Liu et al.⁶ designed a hybrid algorithm based on a cutting and matching algorithm and an improved ant colony algorithm. Li et al.⁷ applied the LHLA with defect constraints is combined with a double-population genetic algorithm to optimize the layout of rectangular parts with defects in the sheet. Zhang et al.⁸ formulated a combined rectangular board layout algorithm based on two-stage nesting and a genetic algorithm, which was proposed by integrating heuristic nesting and optimized sorting. Given the nonlinearity and high complexity of the rectangular parts layout problem, Gao et al.⁹ devised a DPSO-CG hybrid optimization algorithm combining a discrete particle swarm optimization algorithm and a chaotic genetics strategy. The existing rectangular part layout problem is usually studied from the perspectives of sequencing and positioning. Current sequencing strategy research is random and lacks guidance on delivery time.

Research on workshop scheduling problems has proposed many key concepts and methods in the field of scheduling. For parallel machine scheduling problems, Lei et al.¹⁰ used an improved multi-colony artificial bee colony algorithm to solve unrelated parallel machine scheduling problems, with the aim of minimizing makespan and total tardiness simultaneously. Sun et al.¹¹ constructed an integer programming model based on the characteristics of the problem and designed a heuristic column generation algorithm to optimize the model, which aims to decompose the problem and solve the master problem and subproblems based on the initial columns obtained and dynamic programming algorithms. Zhou et al.¹² addressed the unrelated parallel machine scheduling problem by considering both imperfect and perfect preventive maintenance methods. They established a scheduling model with the objectives of minimizing the makespan and the total task delay. Zhang et al.¹³ studied a bi-objective identical parallel machine scheduling problem under uncertainty, in which the first objective is to minimize the total completion time, and the second is to minimize the makespan. Parallel machine scheduling is a commonly used production model in the current manufacturing field.

At present, scholars have already recognized the importance of collaborative optimization between layout and cutting scheduling. Hendry et al.¹⁴ and Poltroniere et al.¹⁵ studied the collaborative optimization of layout and cutting scheduling in copper manufacturing enterprises and coil paper manufacturers, respectively. Yanasse and Lamosa¹⁶ established an integer linear programming model for cutting stock and sequencing problems under particular pattern, and used the Lagrangian method to decouple and modified the subgradient method to solve the dual problem, providing a reference for heuristic algorithms to solve this kind of collaborative optimization problem. Pitombeira-Neto et al.¹⁷ researched how to optimize the relationship between cutting stock and scheduling to meet delivery time in the case of multiple orders. The above literature only considers the balance between layout and cutting scheduling, without considering the requirements of subsequent welding kitting on the delivery time of previous cutting.

Considering the requirements for the delivery time of the preceding cutting stock process due to the subsequent welding kitting of metal structural components, some scholars have studied multi-stage integrated optimization. Sakaguchi¹⁸ investigated the use of a collaborative optimization genetic algorithm to solve layout and scheduling problems in sheet metal processing under different conditions. Subsequently, the research team^{19,20} further studied the optimization of layout and scheduling problems in metal sheet processing (punching, bending, welding, assembly), adopting the dispatching rule of the earliest due date rule to arrange the processing sequence from layout to scheduling. This research further provided the basis for the layout priority proposed in this paper. Melega et al.²¹ reviewed the two stages of development in lot-sizing and cutting stock problem. The second stage considers raw material preparation, cutting stock, and assembly integration, and constructs a three-type integrated model for this problem. Wang et al.²² addressed the characteristics of the metal structure manufacturing systems, constructed a multi-objective job scheduling optimization model with process constraints, with the aim of integrating and optimizing the cutting stock and scheduling problem for structural components.

In summary, although some scholars have achieved certain results in the field of collaborative optimization of layout and scheduling in recent years, there are still many deficiencies that need to be addressed. Firstly, the in-depth exploration of the collaborative optimization mechanism is lacking between layout and scheduling. Cutting scheduling can only passively accept the layout grouping scheme of parts, which may result in the cutting stock time not meeting the requirements of welding kitting time. Secondly, many studies focus solely on the optimal solution for a single optimization objective, such as simply pursuing the maximization of sheet utilization rate or the minimization of makespan, without achieving comprehensive multi-objective optimization. Expressing welding kitting requirements as the part cutting delivery time and prioritizing its requirements during the layout can improve on-time delivery rates, but may be at the expense of significant material utilization.

Addressing the shortcomings of existing research, this paper conducts an in-depth study of the coupling relationship between layout and cutting scheduling in the manufacturing process of metal structural components. It proposes a two-stage framework that considers the collaborative optimization of layout and cutting scheduling, and generates a series of layout schemes based on part delivery priority. Based on the generated layout schemes, reasonable cutting scheduling is performed. This staged optimization method collaboratively optimizes multiple objectives, comprehensively balancing and evaluating sheet utilization rate, makespan, and delay penalty.

The rest of the paper is organized as follows: First, the problem description and models of the layout and cutting scheduling collaborative optimization are given. Second, the Grey Wolf Optimizer algorithm is improved

(IGWO) to solve the layout problem, while the Mon-dominated Sorting Genetic Algorithm (MNSGA-II) is modified to solve the cutting scheduling problem (CSP). Third, experiments on test cases are conducted to confirm the effectiveness of the two-stage model and the algorithm in this paper. Last, the research results and corresponding conclusions are summarized and demonstrated.

Mathematical model of layout and cutting scheduling collaborative optimization problem

The layout and cutting scheduling problem (LCSP) can be described as follows: the given n rectangular parts with known delivery time D , width w , and length h are laid onto m sheets with known width W and length H . A set of layout schemes are selected, and the m sheets under different layout schemes are scheduled to L types of cutting machines with Q sets for processing, and determining the the cutting sequence of each sheet on the machine. Thus, the cutting scheduling results corresponding to different layout schemes are obtained. To solve the problem, a two-stage model is established: the layout model of the first stage and the cutting scheduling model of the second stage.

Stage 1: layout model construction

(1) Symbol explanation of the layout model in Table 1.

(2) Decision variable:

The decision variable E_{ik} is constrained to be a 0-1 variable. When rectangular part i is placed on sheet k , $E_{ik} = 1$. Otherwise, $E_{ik} = 0$.

$$E_{ik} = \begin{cases} 1 & \text{part } i \text{ is put on the sheet } k \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(3) Objective of the layout problem:

Equation (2) in this model indicates the maximum utilization of sheet metal materials.

$$F = \max \frac{\sum_{i=1}^n w_i h_i}{\sum_{k=1}^{m-1} (W_k \times H_k) + W_m \times H'_m} \tag{2}$$

(4) Constraints of the layout model:

The rectangular parts can not extend beyond the boundaries of the sheet which is positioned by Eqs. (3) and (4). Equation (5) represents that only the horizontal coordinates of the lower left corner are equal or only the vertical coordinates of the lower left corner are equal between two parts within the original sheet, but the vertical coordinates are not equal to each other, and Eq. (6) means that the horizontal and vertical coordinates of the parts within the original sheet are not equal to each other, so Eqs. (5) and (6) indicate that the part within the original sheet do not overlap each other.

Symbol	Description	Symbol	Description
i, j	The i^{th}/j^{th} part $i, j \in \{1, 2, \dots, n\}, i \neq j$	H_k	Length of sheet k
n	Total number of rectangular parts	D_i	The due date of part i
m	Total number of sheets	T_i	The processing time of part i
k	The k^{th} sheet $k \in \{1, 2, \dots, m\}$	x_i	Horizontal coordinate of the lower left corner of part i
x_j	The horizontal coordinate of the lower left corner of part j that is different from the i^{th} part	y_i	Vertical coordinate of the lower left corner of part i
y_j	The vertical coordinate of the lower left corner of part j that is different from the i^{th} part	w_i	Width of rectangular part i
h_i	Length of rectangular part i	H'_m	The maximum height of the layout diagram of the last sheet and $H'_m \leq H_m$
W_k	Width of sheet k	L	Number of machine types
C_k	The processing time of the sheet k	l	Index of machine types, $l = 1, \dots, L$
C_{max}	The maximum processing time of the sheet	B_l	The number of machine type l
st_k	Setup time of sheet k	Q	Number of cutting machines
L_k	Cutting length of sheet k	o	Index of machine numbers, $o = 1, \dots, Q$
v_{ko}	The cutting speed for sheet k processed on machine o	Z_k	Delay penalty of the k sheet
pt_k	Packing time of sheet k	q_1	Unit time delay penalty
q_2	Reasonable delay time	N_I	The number of individuals are selected as the new population of IGWO algorithm
T_I	The maximum number of iterations of the IGWO algorithm	T_N	The maximum number of iterations of the MNSGA-II

Table 1. Symbols and descriptions.

$$1 \leq x_i \leq W, 0 \leq y_i \leq H \quad (3)$$

$$0 \leq x_i + w_i \leq W, 0 \leq y_i + h_i \leq H \quad (4)$$

$$((x_i > x_j) \cup (x_j > x_i + w_i)) \cup ((y_i > y_j) \cup (y_j > y_i + h_i)) \quad (5)$$

$$((x_i > x_j) \cap (x_j > x_i + w_i)) \cap ((y_i > y_j) \cap (y_j > y_i + h_i)) \quad (6)$$

Equation (7) represents a part can only be laid on one sheet.

$$\sum_{k=1}^m E_{ik} = 1 \quad i = 1, 2, \dots, n \quad (7)$$

Stage 2: the model construction of CSP

(1) Assumptions of CSP:

- (1) Each sheet has only one cutting process and can be processed on any machine as long as it complies with certain criteria. Since the collaborative optimization problem for the complete process layout and cutting scheduling for metal structural components is highly complex. To reduce problem complexity, the current study focuses solely on the collaborative optimization problem of the cutting stage and temporarily excludes subsequent production processes such as machining and welding. Therefore, this paper specifies that each sheet undergoes only the cutting process as long as the process constraints regarding the dimensions, specifications, and precision of the material are met. Cutting can be performed on any suitable cutting machine.
- (2) Multiple sheets cannot be processed simultaneously by a single machine. Customized metal structural parts are often large in size and volume, and a single sheet occupies the entire workbench, leaving almost no equipment capable of processing two sheets in parallel. To avoid confusion with composite equipment that can cut multiple sheets simultaneously, this model stipulates that the same cutting machine cannot process two sheets at the same time and that a single cutting machine can only load and cut one sheet at a time.
- (3) There is no interruption during the sheet-cutting procedure. Cutting continuity is a rigid requirement of process constraints. On the same sheet, all parts to be laid must be cut in one go before moving on to the next sheet. There should be no interruptions in between. This is because if the cutting process is paused and then restarted, the laser or plasma cutting equipment needs to be restarted, and the equipment requires multiple preheating cycles, which not only wastes energy but may also cause errors due to repositioning.
- (4) Each sheet can only be cut using the same machine. Each sheet can only be cut using the same machine, i.e., from the start of cutting to the end of cutting, the same sheet must be fixed on the initially selected machine, and it is not allowed to change machines in the middle of the process. Otherwise, re-clamping and recalibration will reintroduce preheating and positioning errors.

(2) Decision variables:

The decision variables R_{ko} , R'_{kl} , and y_{jko} are constrained to be 0-1 variables. when sheet k is processed on machine o or sheet k is processed on type l machines, then $R_{ko}=1$, $R'_{kl}=1$, otherwise $R_{ko}=0$, $R'_{kl}=0$. when sheet j is the preceding workpiece of sheet k on machine o , then $y_{jko}=1$, otherwise $y_{jko}=0$.

$$R_{ko} = \begin{cases} 1 & \text{Sheet } k \text{ is processed on machine } o \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$R'_{kl} = \begin{cases} 1 & \text{Sheet } k \text{ is processed on type } l \text{ machines} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

$$y_{jko} = \begin{cases} 1 & \text{Sheet } j \text{ is the preceding workpiece of sheet } k \text{ on machine } o \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

(3) Objective of CSP:

The objective function (11) represents minimizing the makespan.

$$F1 = \min(C_{max}) \quad (11)$$

Equation (12) represents the delay penalty of the sheet k . The function (13) represents minimizing the delay penalty.

$$Z_k = \begin{cases} 0 & C_k \leq D_i \\ q_1(C_k - \sum_{i=1}^n E_{ik}D_i - q_2) & C_k \geq D_i \end{cases} \quad (12)$$

$$F2 = \min \sum_{k=1}^m Z_k = \min \sum_{k=1}^m q_1(C_k - \sum_{i=1}^n E_{ik}D_i - q_2) \quad (13)$$

(4) Constraints of the cutting scheduling model:

Eqs.(14) and (15) define the completion time of all sheets, and Eq. (16) reveals that each sheet can only be assigned to one kind of cutting machine.

$$C_k = \begin{cases} 0 & k = 0 \\ \sum_{j=1}^m (y_{jmo} C_j + st_k + \frac{L_k}{v_{ko}} + pt_k) & k = 1, 2, \dots, m \end{cases} \quad (14)$$

$$C_{max} \geq C_k \quad (15)$$

$$\sum_{o=1}^Q R_{ko} = \sum_{l=1}^L R'_{kl} = 1 \quad (16)$$

Equation (17) demonstrates that there is only one preorder job when sheet m is cut on the planned machine.

$$\sum_{j=1}^m y_{jko} = R_{ko} \quad o = 1, 2, \dots, Q \quad k = 1, 2, \dots, m, \quad k \neq j \quad (17)$$

Equation (18) shows that there is only one post-order job created when sheet m is cut on the planned machine.

$$\sum_{k=1}^{m+1} y_{jko} = R_{jo} \quad o = 1, 2, \dots, Q \quad k = 1, 2, \dots, m, \quad j \neq k \quad (18)$$

Equation (19) displays that the total quantity of machines and the number of distinct kinds of machines are consistent.

$$\sum_{l=1}^L B_l = Q \quad (19)$$

The solution approach of the two-stage model

Due to the complexity of the aforementioned mathematical model, this paper designs a two-stage solving algorithm. First, generate the population based on the priority of parts, and then utilize the fitness-based lowest horizontal line algorithm (LHLA) to solve the utilization rate of each individual in the population, each individual is arranged in descending order by utilization rate, the first N_I individuals are selected as the new population of IGWO algorithm, and so on, after the IGWO algorithm for each operation, it is necessary to use the fitness-based LHLA to get the new population of the IGWO algorithm, the cycle of iteration finally get a set of better layout scheme and utilization rate, and then use the MNSGA-II to solve the cutting scheduling model, the introduction of the heuristic strategy of the sheet priority values are smaller, the more the first cut to generate the initial population, obtain the integration of the solution to the optimization of layout and cutting scheduling. Through comparative analysis, the selection of suitable layout schemes and the corresponding cutting scheduling results. Figure 1 shows the two-stage solution process.

The problem addressed in this study is the collaborative optimization of layout and cutting scheduling, without considering optimization of the machining and welding stages. To more clearly illustrate the advantages of considering part priority in layout and cutting scheduling on the overall scheduling process time, this study has added integrated layout and cutting scheduling encoding and decoding diagrams as shown in Figs. 2 and 3. The decoding Gantt chart includes whether part priority are considered for the subsequent machining and welding stages.

Assume there are three structural components A, B, and C. Component A consists of parts numbered 1, 2, and 3, B consists of 4, 5, and 6, C consists of 7, 8, 9, and 10.

If the cutting sequence of the sheets is not determined based on part priority, parts 1, 2, 4, 7, and 8 are placed on the same sheet k_1 , while parts 3, 9, 5, 6, and 10 are placed on another k_2 . According to the principle of welding parts in the order of kitting, the determined welding sequence for the parts is 1, 2, 4, 3, 5, 6, 7, 8, 9, 10. As shown in the Gantt chart in Fig. 2, after the cutting of k_1 is completed, it is necessary to wait for sheet k_2 to be cut, as part 3 is laid on the sheet k_2 , and processing of part 3 cannot begin until the cutting of sheet k_2 is completed. Therefore, the welding kitting time for structural component A includes the cutting time for k_1 , the processing time for parts 1, 2, 3, and 4, and the waiting time for sheet k_2 . The welding kitting times for structural components B and C are also correspondingly delayed.

If the cutting sequence of the sheet is determined based on part priority, then parts 3, 1, 2, 4, and 7 are placed on the same sheet k_1 , while parts 8, 9, 5, 6, and 10 are placed on another sheet k_2 . Based on the principle of welding parts in the order of kitting, the determined welding sequence for the parts is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. As shown in the Gantt chart in Fig. 3, once the cutting of sheet k_1 is completed, the processing of parts 1 can begin. It can be seen that the welding kitting time for structural component A includes the cutting time for sheet k_1 and the processing time for parts 1, 2, and 3. Compared to scheduling without considering part priority, this method reduces the waiting time for sheet k_2 , and the welding kitting times for structural components B and C are also correspondingly advanced.

It can also be intuitively compared from Fig. 2 and 3 that, compared with considering part priority, the waiting time for layout and scheduling increases significantly when part priority is not considered, and the welding time for structural parts B and C is also delayed accordingly.

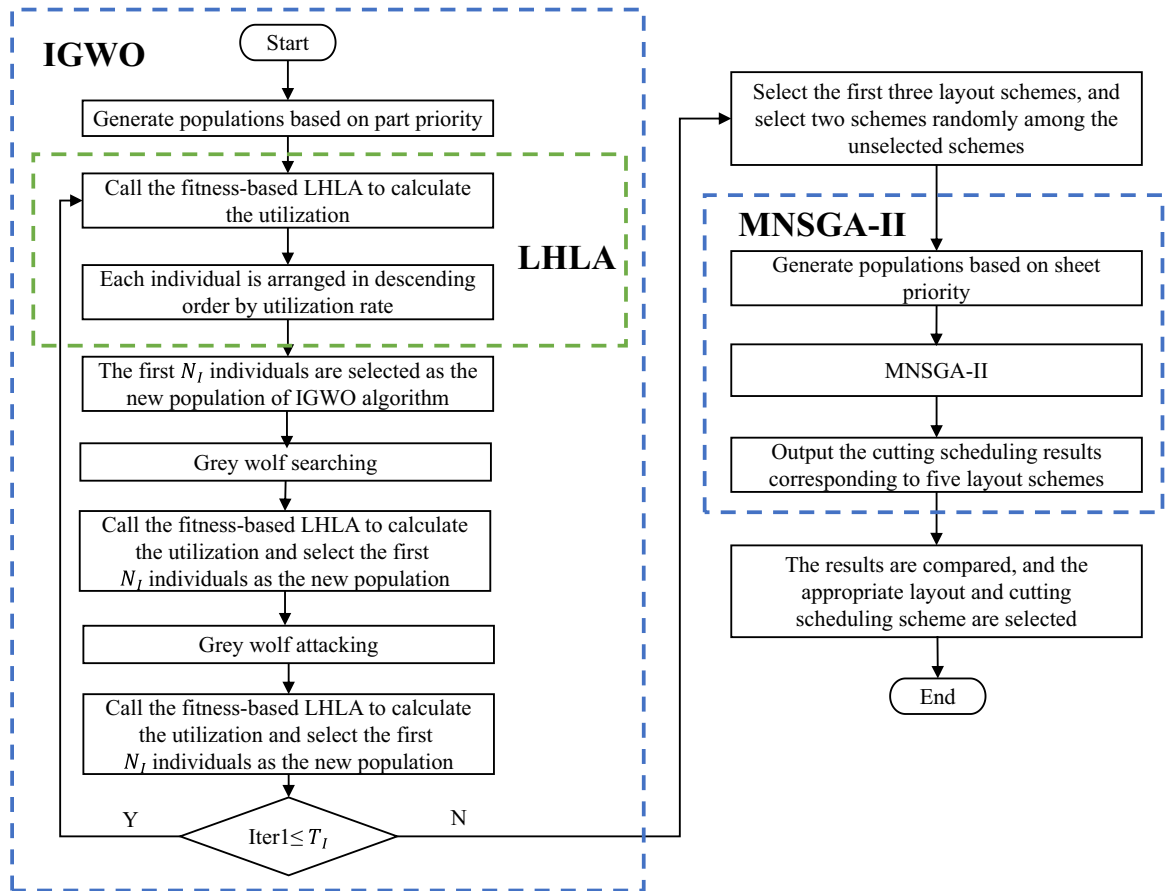


Fig. 1. The flow chart of solving LCSP based on the IGWO and MNSGA-II.

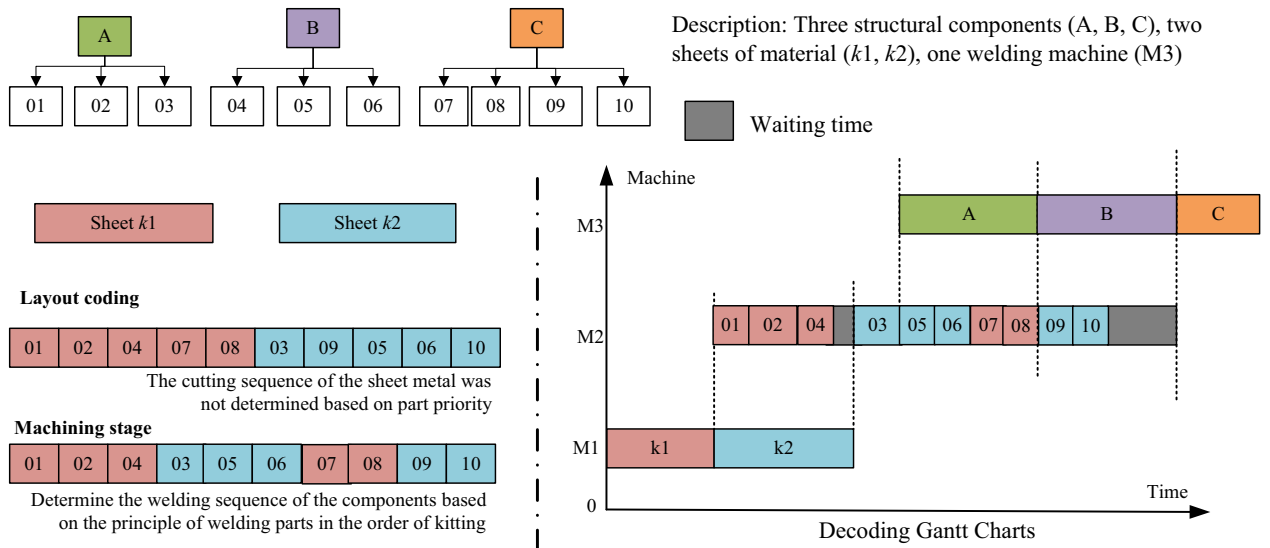


Fig. 2. Layout and cutting scheduling integration coding and decoding schematic diagram without considering part priority.

IGWO algorithm for solving the layout problem IGWO sequencing algorithm

When solving the optimization problem of layout, the rectangular part is the study object. This problem is quite complicated with many variables and large scale. Compared with other swarm intelligence algorithms, the GWO

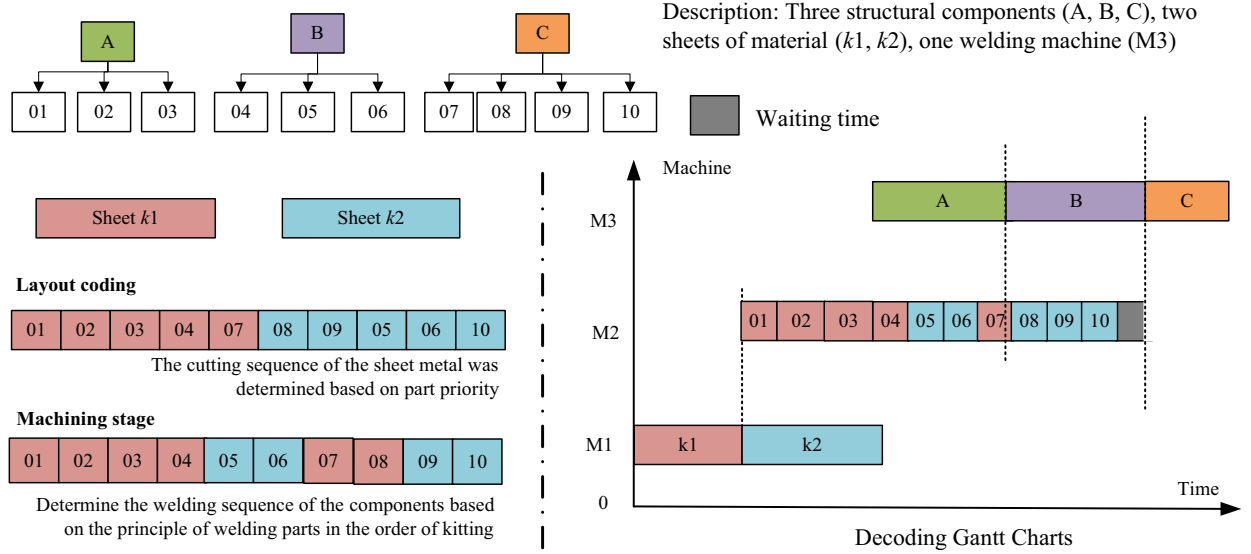


Fig. 3. Schematic diagram of layout and cutting scheduling integration encoding and decoding under part priority consideration.

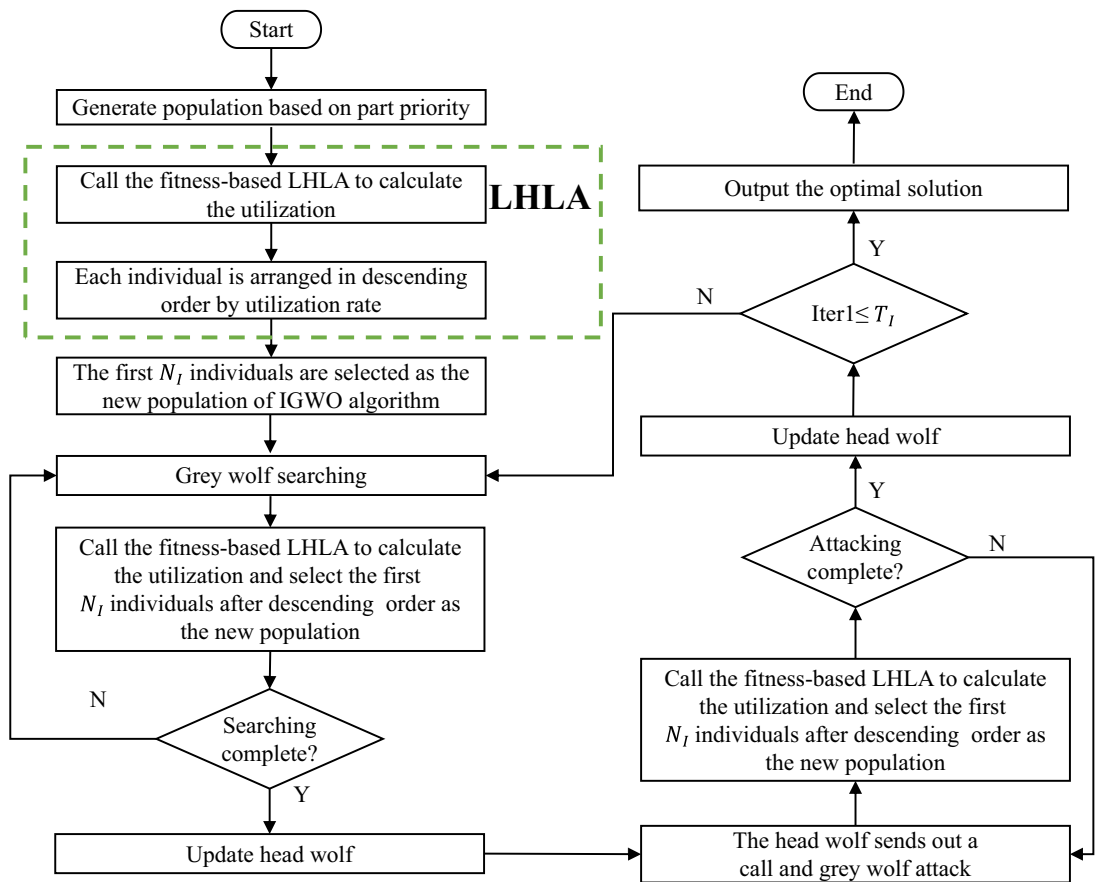


Fig. 4. The flow chart of solving the rectangular parts layout problem based on IGWO.

algorithm performs a good ability for global optimization, and can obtain better solutions in a shorter time. The GWO is a new swarm intelligence algorithm that simulates wolf predatory behavior and prey distribution, abstracting the three intelligent behaviors of searching, and attacking. The head wolf generation rule is that the winner is the king, and the wolf swarm renewal mechanism is that the strongest should be the survivor.

Sequencing and positioning are two fundamental components of the solution to the rectangular parts layout problem, as mentioned before. The LHLA with decimal encoding, decodes a sequence of rectangular parts to determine where they are in the pattern and calculates the sheet utilization rate. Based on the intelligent behavior of the wolves, the GWO algorithm searches for the optimum sequence of rectangular parts, and the criteria for judging the sequence of rectangular parts is that the larger the sheet utilization obtained by the solution of the fitness-based LHLA, the higher the fitness value is. Figure 4 illustrates the flow of solving the rectangular parts layout problem based on IGWO, where the LHLA is one step in Fig. 4.

Fitness-based lowest horizontal line positioning algorithm

The lowest horizontal line is a straightforward and efficient layout positioning strategy. As shown in Fig. 5, start by setting the initial horizontal line as the bottom edge of the master board. Place parts 1, 2, and 3 in the master board in sequence. At this point, the lowest horizontal line is located at the far right end of the master board. According to the lowest horizontal line rule, part 4 should be placed in this area, but the part will exceed the range of the raw material. Therefore, the lowest horizontal line in this area should be raised to the upper edge of part 3. At this point, the lowest horizontal line is redefined as the upper edge of part 1, allowing the part to be placed in the area above part 1.

This paper adopts the fitness-based LHLA. First, it analyses the comparison between the parts to be laid and the lowest horizontal line, sets appropriate fitness values according to the situation, and selects the parts with the highest fitness values among the parts to be laid. The arrangement of the parts to be laid on the lowest horizontal line and their fitness values are shown in Fig. 6. The fitness-based LHLA in this paper is used to analyze the situation of parts along the lowest horizontal line. Calculate the fitness value and select the part with the highest fitness value to be laid. The sheets have fixed length and width. Arrange the next original sheet and initialize the parameters if none of the remaining parts can be laid. The fundamental flow of the fitness-based lowest horizontal line positioning is as Algorithm 1:

Step 1: Initialization parameters, $i = 1$.

Step 2: Select the i^{th} sheet and initialize the horizontal line sequence.

Step 3: Compare all available parts and select the part with the highest fitness value.

Step 4: If any of the parts can be laid in Step 3, lay out the part onto the sheet and continue to Step 3. Otherwise, move to Step 5.

Step 5: Evaluate whether the horizontal line sequence at this point is a single line in the set. If so, then $i = i + 1$ and go to Step 2. Otherwise, go to Step 6.

Step 6: Progress the horizontal line sequence, update the lowest horizontal line. If the parts are not fully laid, return to step 3. Otherwise, to step 7.

Step 7: Completes the layout process, calculates the utilization rate and outputs the layout results.

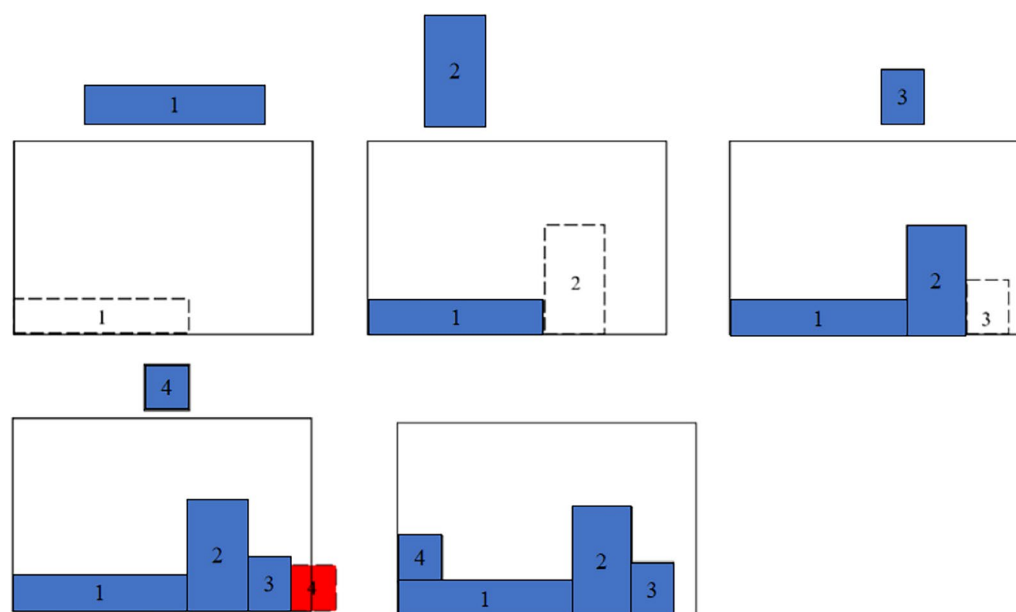


Fig. 5. The positioning process of the fitness-based LHLA.

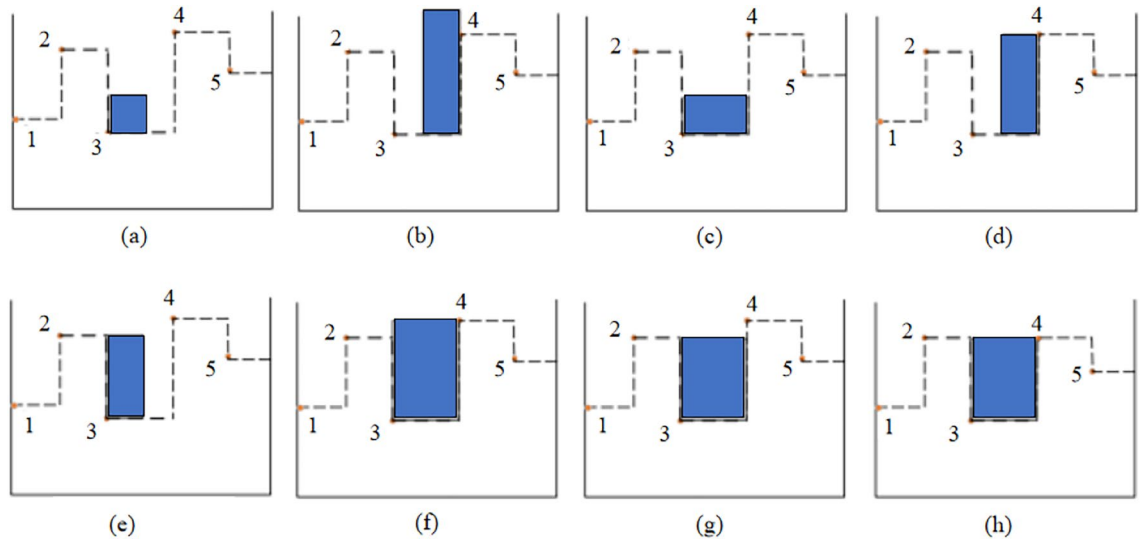


Fig. 6. Layout diagram and corresponding fitness values.

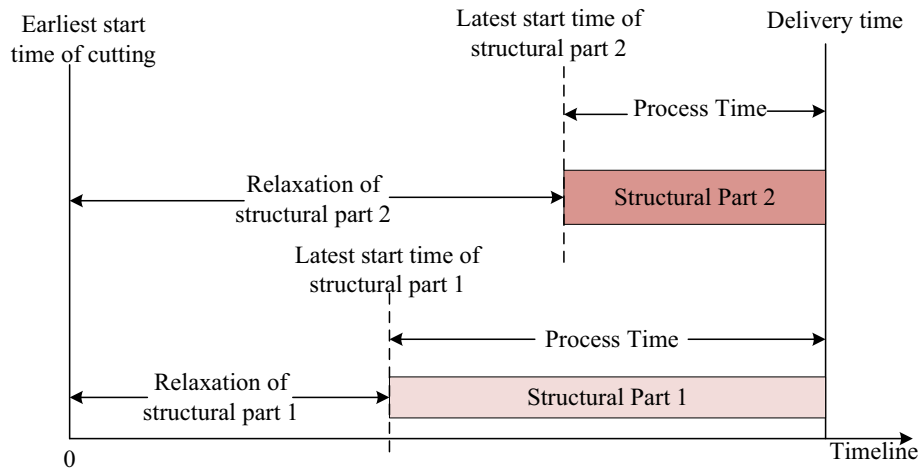


Fig. 7. Plot of structural parts onset time relaxation versus process time.

Algorithm 1. Fitness based lowest horizontal line positioning algorithm

The LHLA among layout positioning algorithms possesses the traits of simplicity and an excellent layout effect. To deal with the rectangular parts layout problem in the current research, the decimal grey wolf sequencing algorithm and the fitness-based LHLA are used to achieve global optimization and local optimization, respectively, to obtain the layout optimization scheme. According to the process of GWO, the encoding and decoding, population initialization, searching, and attacking are improved considering the characteristics of the layout problem.

Encoding and decoding

The rectangular layout problem is now addressed by decimal encoding, and each rectangular part is given a unique identification integer coding beginning with 1. The decoding algorithm can transform a sorted sequence representing rectangular parts into a layout diagram. Numerous operations are performed on the sequence throughout the entire solution procedure. There are no two identical numbers in the sequence, and no number may be missed, which guarantees its reliability.

Population initialization

The heuristic rule based on layout priority values is designed as follows to generate the initial population:

- (1) The priority equation $p_i = \frac{D_i - T_i}{D_i}$ is designed based on the relationship shown in Fig. 7. Where D_i represents the delivery time of the structural part i . The delivery time is a main constraint in production planning and reflects the customer's expectation. T_i is the process time of part i , which means the total time required from the start of machining to completion. The formula $D_i - T_i$ represents the slack time of the structural part, the time margin available for machining before the delivery time. Define the ratio of slack time to delivery time $\frac{D_i - T_i}{D_i}$ as the layout priority values p_i . If p_i is close to 1, it means that the part owns a long slack time and its priority is lower. while p_i is close to 0, then it means that the part owns a short slack time and its priority is higher. So, the formula relies on only two key parameters (machining time and delivery time), which are easy to understand and calculate. This makes it highly operable in practical applications, especially in production environments that require fast decision making. Priority closely links the urgency of the part with the delivery time, and parts with closer priority can be laid on the same sheet. Then, the whole production layout and scheduling process is optimized. Production efficiency and sheet utilization rate are improved.
- (2) Smaller layout priority values indicate higher urgency for a part, which implies a higher layout priority. All parts are sorted in ascending order of priority values.
- (3) Parts with similar priority values are grouped so that the parts are randomly arranged within the groups to create different individuals.
- (4) The fitness-based LHLA is applied to calculate sheet utilization rate for all individuals. A certain number of individuals with higher sheet utilization rate are selected as the initial population.

Searching for prey

Searching refers to the process of moving a grey wolf coding site rather far to the left or right. For instance, the grey wolf $Z_i = (z_{i1}, z_{i2}, z_{i3}, \dots, z_{in})$ of the first coding site of z_{i1} in grey wolf one unit to the right to become $Z'_i = (z_{i2}, z_{i1}, z_{i3}, \dots, z_{in})$, The subsequent s coding sites in $Q = (1, 2, 3, \dots, n)$ are chosen to move d distances. It would be feasible to express this operation step as follows:

As shown in Fig. 8, the position of the rectangular parts laid on the sheet is changed for each searching operation of the coding sequence, and the utilization rate is recorded each time. For example, parent $Z = (3, 6, 8, 9, 7, 4, 5, 1, 2)$, encoding site $Q = (1, 2, 3, 4, 5, 6, 7, 8, 9)$, $s = 3$, searching step $d = 4$, randomly select the third encoding site, which means that the third, fourth, and fifth encoding sites of Z are moved to the right for 4 steps. The offspring $Z_o = (3, 6, 5, 1, 2, 4, 8, 9, 7)$ is obtained.

Attacking prey

Attacking represents the process of the grey wolf moving and approaching the head wolf. The distance between them means the different values of its corresponding coding. The attacking strategy $R(Z_i, L_1, L_2, d)$ denotes the selection of the first d positions in the i^{th} grey wolf $Z_i = (z_{i1}, z_{i2}, z_{i3}, \dots, z_{in})$ with different from the head wolf coding L_1 , which is replaced by the corresponding positions in L_2 .

Step1: The position of the rectangular parts laid on the sheet is changed for each attacking operation of the sequence, and the utilization rate is recorded. Suppose $Head_Wolf = (10, 9, 5, 7, 8, 3, 6, 4, 2, 1)$, $Grey_Wolf = (10, 9, 6, 7, 3, 5, 8, 4, 2, 1)$, grey wolf carry out the attack operation to head wolf. According to the attacking strategy $R(Z_i, L_1, L_2, d)$, head wolf needs to find the job at the position $L_1 = (3, 5, 6, 7)$. The corresponding values are $L_2 = (5, 8, 3, 6)$, the coding value to carry out the assignment $d=2$.

Step2: Transfer the {5}, {8} of the third and fifth codes of the $Head_Wolf$ to the corresponding coding of $Grey_Wolf$, which are {6}, {3}.

Step3: Since the values of individual coding should not be the same, replace the sixth and seventh codings of the $Grey_Wolf$ with {6}, {3}, thus obtaining the $New_Grey_Wolf = (10, 9, 5, 7, 8, 6, 3, 4, 2, 1)$ as shown in Fig. 9.

MNSGA-II to solve the CSP

For the parallel machine's CSP, the same sheet selects different machines, and the processing times required are different. Different sheets consume different processing times when they are cut on the same machine. The objective of cutting scheduling optimization in this article is to allocate sheets to appropriate machines, optimize sheet scheduling, and obtain the optimal solution for makespan and delay penalty. The NSGA-II performs excellent solving ability in searching multiobjective optimization problems. Our paper improves the NSGA-II to solve the cutting parallel machine scheduling problem, and the solution flow is shown in Fig. 10.

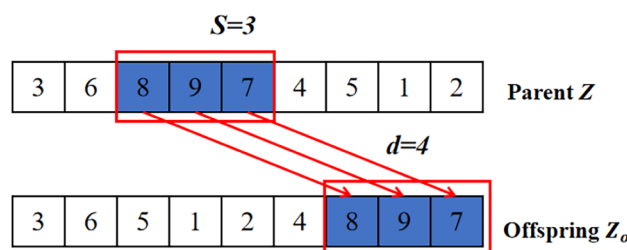


Fig. 8. Search strategy coding diagram.

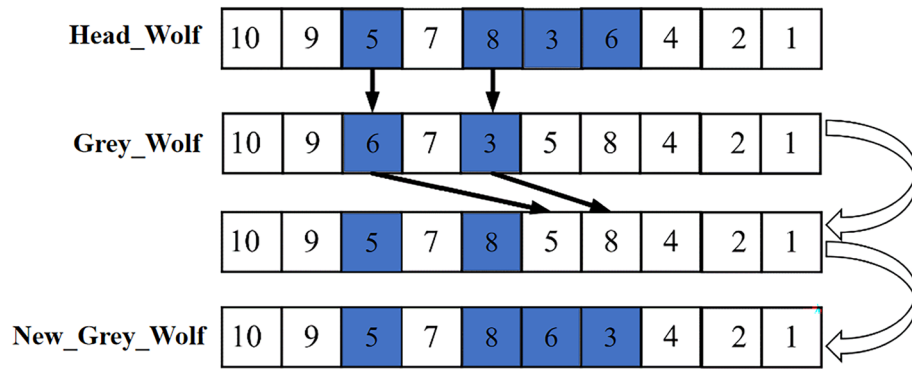


Fig. 9. Attacking operation to accelerate the finding of locally optimal solutions.

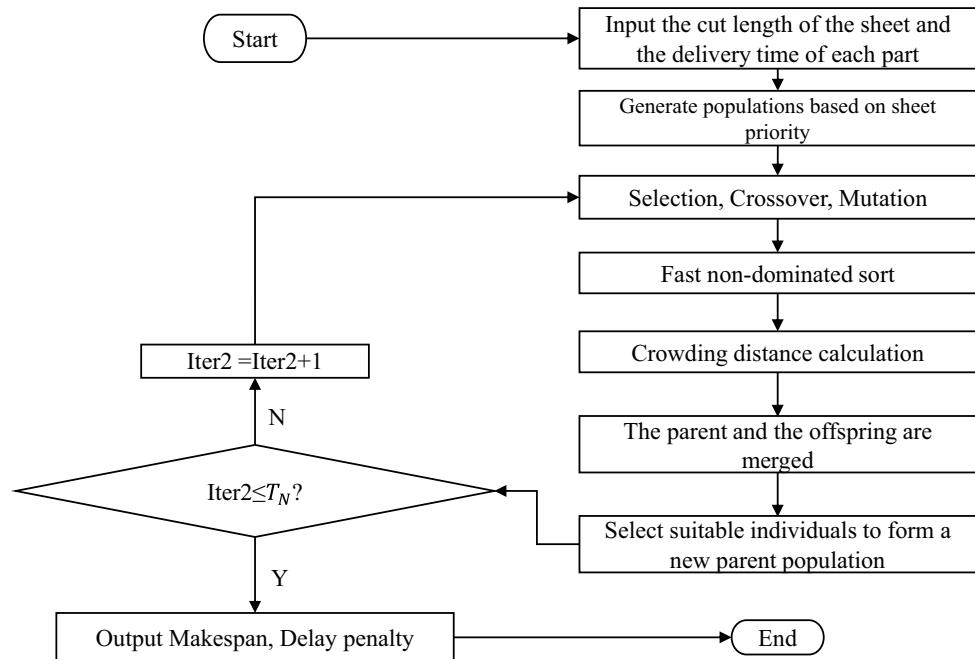


Fig. 10. The flow chart of MNSGA-II to solve the CSP.

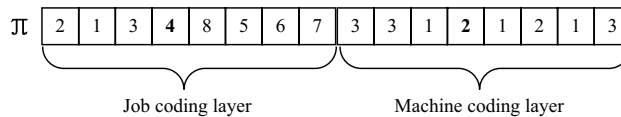


Fig. 11. Coding method.

Encoding and decoding

A chromosome contains information on job sequencing and machine type. The first layer of the chromosome is the job coding, which indicates the machining sequence of the jobs, and the second layer is the machine coding, which represents the machine type selected for the jobs. Each value in the job coding layer of the chromosome represents the number of the job, and the length of the machine coding layer is the same as the length of the job coding layer, corresponding to the type of machine used to process each job. Figure 11 π shows an example of a workable chromosomal coding scheme. The fourth coding site of the job is code 4, which corresponds to a machine code of 2, indicating that job 4 is being processed on the second machine type.

Decoding strategy:

- (1) According to the coding, the job is allocated to the corresponding machine type to process.

- (2) If there are multiple machines of one type, the jobs are assigned to those machines by the minimum start time principle. If more than one machine starts at the same time, the job can be randomly selected for these machines. If several machines of one type are processing jobs at the same time, the new job will be processed to the machine that finished processing the job first.

Creation of the initial population

Determining the order of sheet processing and machine selection are the two subproblems that make up the CSP. When the layout algorithm adopts the heuristic strategy of part priority to generate the initial population and obtain the layout scheme, the average priority values of parts distributed on the sheet is calculated. The sheet processing order is also determined by the heuristic rule that the smaller the sheet priority, the higher the processing priority. That is, all sheets are sorted according to the determined processing order to obtain the first coding layer sequence, and then randomly generate codes for the corresponding machine. In this way, a certain number of feasible processing sequences are obtained as the initial population.

Select operation

The genetic code determines the start time, the makespan, and the processing time for every job. The fitness values are computed by the makespan and maximum delay penalty objective functions. After fast non-dominated sorting of the combined offspring and parent population, the crowding distances are calculated. The crowding values are then adopted and each individual is chosen by the elite selection retention strategy to generate a new parent population.

Crossover operation

The job coding layer adopts Partial Matching Crossover(PMX) as shown in Fig. 12. In Fig. 12a by randomly selecting the starting positions of several genes in a pair of chromosomes (Parent1 and Parent2 are selected at the same position), in this case, the gene fragments selected in Parent1 are (1,3,4,8,5) and those selected in Parent2 are (1,5,6,7,3), and then interchanging the positions of these two sets of gene fragments, we get Child1'=(2,1,5,6,7,3,6,7) and Child2'=(4,1,3,4,8,5,8,2). However, two invalid chromosomes will be obtained after the crossover and individual genes will be duplicated. In order to repair the chromosomes, a matching relationship can be established for each chromosome within the crossover region, and then this matching relationship can be applied to the duplicated genes within the crossover region to eliminate the conflict. As shown in Fig. 12b, taking the mapping relationship 4-6 as an example, it can be seen that there are two genes 6 in Child1', which are then transformed to gene 4 through the mapping relationship as a way of eliminating the conflict. Finally, all conflicting genes will be mapped to ensure that the new pair of child genes formed is conflict-free, which can be obtained as Child1=(2,1,5,6,7,3,4,8) and Child2=(6,1,3,4,8,5,7,2). Compared with other crossover methods, the PMX effectively handles the constraints of the LCSP, avoids the algorithm falling into a local optimum and gene duplication failure while maintaining the integrity of gene segments in the job processing order, generates offspring, and is computationally efficient.

The machine coding layer obeys the principle of all crossovers, i.e., all the genes of the machine sequences on the chromosomes of the two parents are crossed separately, as shown in Fig. 13. Two sets of random variables α and β containing n random variables between [0,1] are first generated as crossover operators for Child1 and Child2.

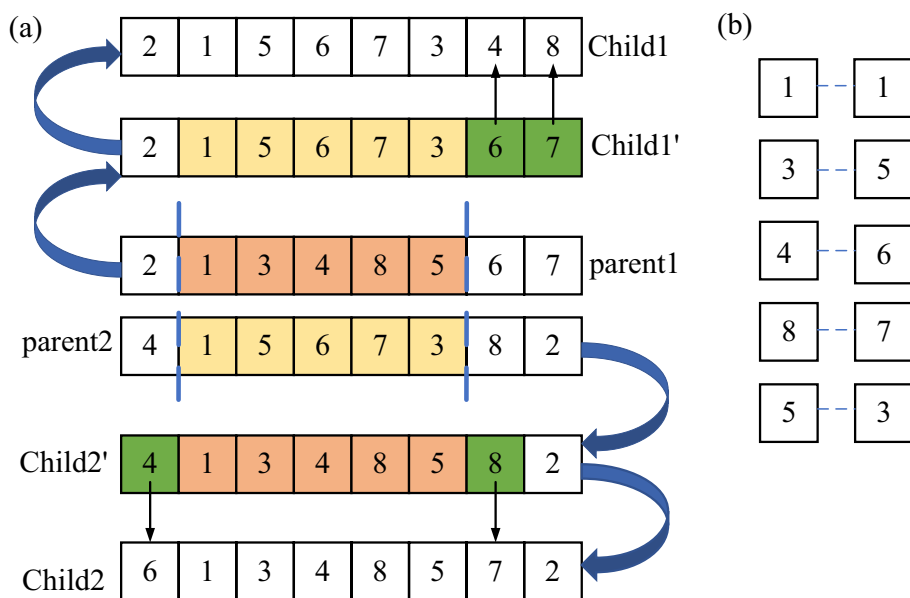


Fig. 12. (a) Job coding layer crossover operation. (b) Establishment of gene mapping relationships in the crossover region.

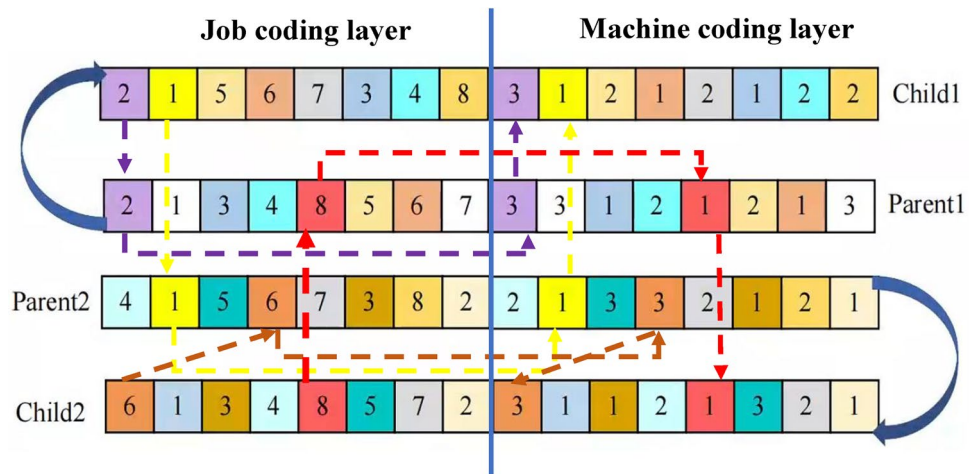


Fig. 13. Machine coding layer crossover operation.

Judge whether each random number in α and β is > 0.5 , if a random number in α is > 0.5 , the corresponding job encoding in Child1 is to select the corresponding machine encoding in Parent2, otherwise the corresponding machine encoding of the job in Parent1 is to be selected. if a random number in β is > 0.5 , the corresponding job encoding in Child2 is to select the corresponding machine encoding in Parent1, otherwise the corresponding job encoding is to be selected in Parent2.

As in this example $\alpha = (0.2, 0.3, 0.1, 0.3, 0.7, 0.1, 0.5, 0.2)$, $\beta = (0.2, 0.6, 0.1, 0.1, 0.7, 0.5, 0.1, 0.9)$, according to the above rule, the first random number in α is taken, $0.2 < 0.5$, and the first gene 2 is found in the sequence of jobs of Child1, Parent1 corresponds to gene 2 in position 1, and the corresponding machine is 3, so the machine corresponding to job 2 in Child1 is 3. When the second random number is taken in α , $0.6 > 0.5$, the second gene 1 is found in the sequence of jobs of Child1, and Parent2 corresponds to gene 1 in position 2, and the corresponding machine is 1, so the machine corresponding to job 1 in Child1 is 1. The other machine genes in α are crossed similarly.

Take the first random number in β , $0.2 < 0.5$, the first gene 6 is found in the sequence of jobs in Child2, Parent2 corresponds to gene 6 is in position 4, and the corresponding machine is 3, so the machine corresponding to job 6 in Child2 is 3. Take the fifth random number in β , $0.7 > 0.5$, the fifth gene 8 is found in the sequence of jobs in Child2, Parent1 corresponds to gene 8 is in the 5th position, the corresponding machine is 1, so the machine corresponding to job 8 in Child2 is 1. The other machine genes in β crossover in a similar manner.

Mutation operation

The job coding layer employs a two-points interchange mutation. Two genetic locations within the chromosome range are randomly selected when an individual is mutated, and then the two locations are exchanged. The term machine coding layer mutation refers to the mutation of the corresponding machine gene of the coding layer of the job in which the mutation occurs. It mutates in such a way that a replacement machine is chosen for the corresponding job from among the alternatives. This mutation approach has unique advantages: Firstly, good structure maintenance, only by exchanging the positions of the two gene loci, it will not damage the overall structure of the gene sequence, nor will it change the coding length, which is in line with the constraint rules of the job coding layer, and ensures the stability of the individual coding structure. Secondly, high search efficiency, as a kind of local mutation operation, it can help the algorithm to explore the better solutions finely in the vicinity of the current solution, and meanwhile, the computing process is simple and fast, which improves the operation efficiency of the algorithm. Third, the balance of solution diversity, which can gradually explore multiple processing solutions on the basis of maintaining the local structure, avoiding excessive destruction of the good gene combinations, so as to better balance the global and local diversity.

Results and discussion

Instance settings and algorithmic parameters

The curved gate arms from a mechanical ship company are used as test data to prove the viability of the proposed two-stage model. The curved gate arms is shown in the Fig. 14. There are 34 distinct parts of curved gate arms. Two sets of arm parts totaling 68 parts comprise the dataset. Provide a delivery time of 200 for the first batch of radial gate arm parts and 250 for the second batch of radial gate arm parts. Unit time delay penalty $q_1 = 3$, reasonable delay time $q_2 = [0, 10]$, $N_I = 100$, $T_I = 80$, $T_N = 100$. The parameters used in the algorithm (e.g. $N_I = 100$, $T_I = 80$, $T_N = 100$) based on similar case scales in relevant literature, extracting experimental settings similar to the scale of the case in this paper. Therefore, this parameter setting was uniformly adopted in the experimental part of this study, enabling the model to reach a convergent solution within a reasonable time frame. The sheets are 2000 x 10000 (mm) in dimension and made of Q355C material. Assume that there are four machines of three types in the cutting process, including two machines of the first type. The cutting speeds are 150, 200, and 250 mm per minute, respectively.



Fig. 14. Physical picture of radial gate.

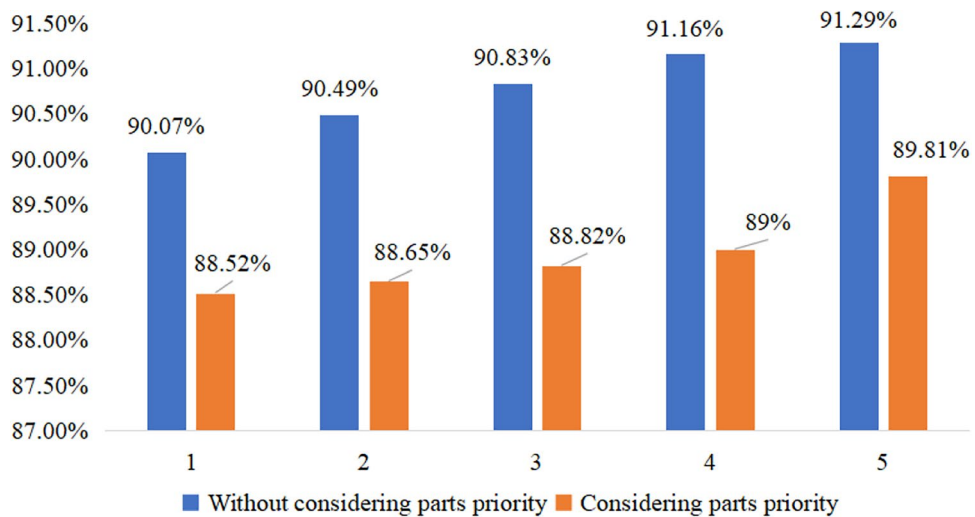


Fig. 15. The sheet utilization rate comparison of whether to consider the part priority.

The sheet utilization rate comparison of whether to consider the part priority

When layout priority is not considered, the parts to be laid are randomly generated into an initial population by the decimal grey wolf sequencing algorithm and the LHLA based on fitness designed in this article. The number of sheets is eight. Through iterative algorithm refinement, a set of layout schemes is obtained. Five layout schemes are selected from this set, the sheet utilization rate for layout schemes 1-1 to 1-5 are shown by the blue bars in Fig. 15.

If guided by the layout priority, wherein smaller layout priority indicates higher urgency for scheduling. The priority values for the 68 parts are calculated based on their delivery and processing time. The number of sheets is eight. By these priority, an initial population is generated through heuristic rules. Subsequently, the IGWO algorithm and the fitness-based LHLA are employed for layout optimization. Through iterative algorithm refinement, a set of layout schemes is obtained, from which five are selected. The sheet utilization rate for layout schemes 2-1 to 2-5 are shown by the orange bars in Fig. 15.

The makespan and delay penalty comparison of whether to consider the part priority.

Processing is performed according to the distribution of sheet metal parts in each layout scheme, and the makespan for the cutting schedule is determined along with the associated delay penalty.

Based on the distribution of sheet metal parts in layout schemes 2-1 to 2-5, the average priority values of parts on each sheet is calculated. This average priority values is used as the processing priority for each sheet, with lower priority values indicating higher processing urgency. Subsequently, a random machine is assigned to each sheet in the determined processing sequence to establish an initial population for the CSP. This initial population is then subjected to the cutting scheduling algorithm to optimize the allocation of sheets to machines, aiming to improve makespan and delay penalty.

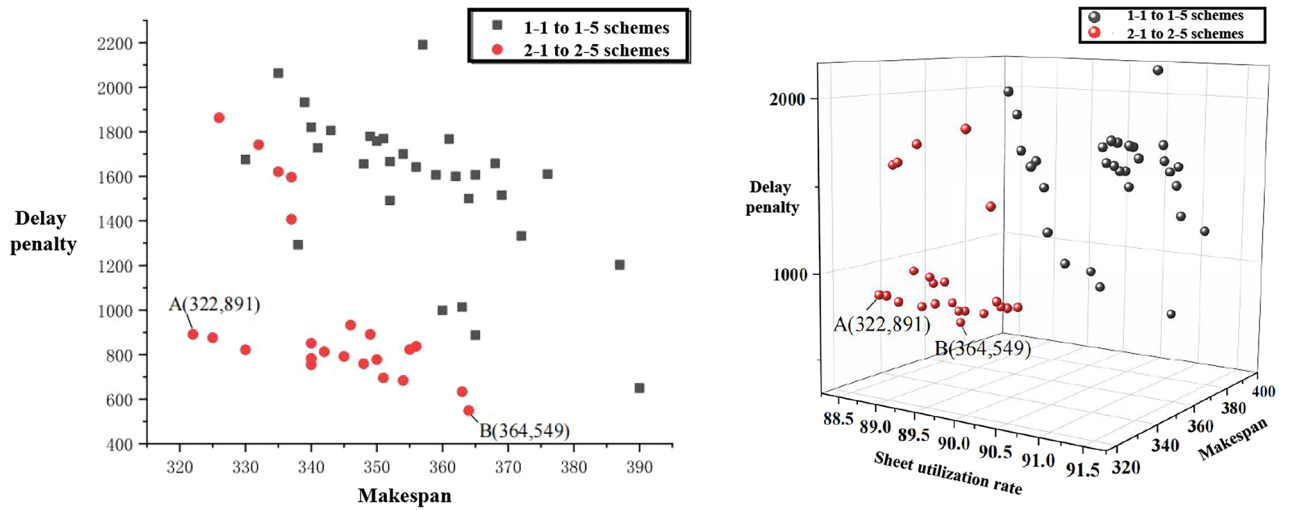


Fig. 16. (a) Pareto distribution of layout schemes. (b) Pareto frontier comparison of layout schemes.

	Layout without considering priority	Layout consider priority	Compare Incremental
Select layout schemes	Layout schemes 1-1 to 1-5	Layout schemes 2-1 to 2-5	
Sheet utilization rate	90.77%	88.96%	-1.81%
Makespan	357	343	-3.9%
Delay Penalty	1563	974	-37.7%

Table 2. Comparison results.

Comparing all Pareto solutions for layout schemes 1-1 to 1-5 with those for layout schemes 2-1 to 2-5, it can be observed from Fig. 16 that the delay penalty is significantly reduced for layout schemes 2-1 to 2-5. The makespan ranges are improved from [330, 390] to [322, 364]. However, the utilization rate of layout schemes 2-1 to 2-5 is lower in comparison to layout schemes 1-1 to 1-5.

In actual production processes, hydraulic engineering metal structural components have a strict delivery time. The delivery time refers to the latest date by which the structural components must be delivered. If the delivery time is exceeded, an additional delay penalty must be paid. The completion time refers to the makespan for all parts in the current production schedule, which determines the starting point for subsequent planned production. The delay is the sum of the differences between the completion time and the delivery time for each part in the current production schedule.

As shown in Table 2. In traditional approaches (not considering layout priority in layout schemes), the production manager would rely on their experience to determine layouts that result in a high sheet utilization rate. Although the layout schemes might achieve a single optimization objective well, such as a high sheet utilization rate, they could lead to poorer outcomes for other objectives, such as makespan and delay penalty.

When using the heuristic strategy proposed in this paper, which employs part priority values to generate initial populations for layout schemes, the sheet utilization rate is a slight sacrifice (by 1.81%) compared to the conventional methods. However, in return, makespan (reduced by 3.9%) and delay penalty (reduced by 37.7%) are improved by the cutting and scheduling optimization. It demonstrates that based on the proposed layout algorithm considering layout priority, the sheet utilization rate is a little trade-off, but the makespan and delay penalty are largely reduced. The optimization of these two objectives—makespan and delay penalty—can maintain the same evolutionary direction in the early stages of iteration, but as the process evolves further, conflicting situations may arise. Whether the production manager chooses the solution with the minimum makespan or the solution with the minimum delay penalty depends on the actual on-site conditions. The production scheduling scheme with the minimum makespan obtained in this study is A[F1=322, F2=891] (layout scheme 2-3), and the production scheduling scheme with the minimum delay penalty is B[F1=364, F2=549] (layout scheme 2-2). The cutting schedule results of A and B are depicted in Fig. 17. Production managers will make decisions based on the following different business scenarios:

- (1) If there are other urgent plans after the current production plan that require the same processing equipment, or if the unit delay penalty is low, solution A (minimum makespan) can be chosen, sacrificing part of the delay penalty to complete the current production plan as soon as possible.
- (2) If no other plans are waiting behind the current production plan, the equipment is not urgently needed, or the unit delay penalty is high. Solution B (minimum delay penalty) should be prioritized.

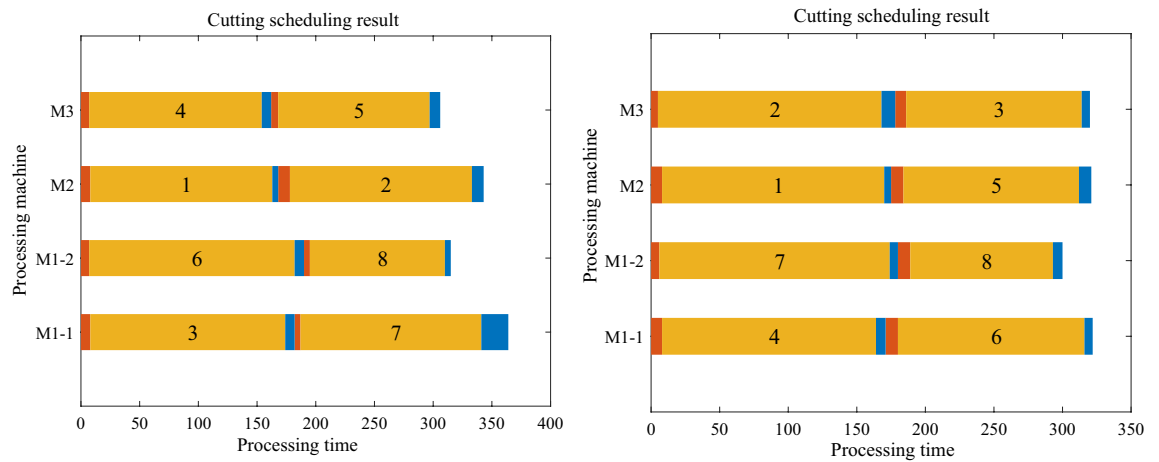


Figure 17. (a) Cutting scheduling Gantt chart of A[F1 = 322 (F2 = 891)]. (b) Cutting scheduling Gantt chart of B[F2 = 549 (F1 = 364)].

Production economic trade-off analysis

The conclusions of this study on collaborative optimization can be applied to management decisions in actual production. Taking the gate leaf panel of a “9 m × 4.8 m flat sliding steel gate” as an example, converting the 1.8% sheet utilization rate obtained in this study into tangible costs and directly comparing it with the cost savings from a 37.7% reduction in delay penalties demonstrates the advantages of this study’s layout optimization approach over a layout scheme that does not consider layout priority. The specific analysis process is as follows:

- (1) Typical sheet specifications The gate panel uses Q345B hot-rolled medium sheets with a thickness of 16 MM. The standard sheet dimensions are 12 M × 2.5 M (single sheet purchase area of 30 M², single weight of 3.768 T). July 2025 steel mill ex-factory price including tax ≈ 4,200 RMB/ T
- (2) Cost of 1.8% utilization loss Wasted area per sheet: 30 M² × 1.8% = 0.54 M². Corresponding weight: 0.54 M² × 125.6 KG/M² = 68 KG. Direct material loss: 68 KG × 4.2 RMB/KG ≈ 286 RMB/ Per Sheet.
- (3) Cost savings from a 37.7% reduction in penalties for delays Taking 6 sheets from the same batch (approximately 180 M², sufficient for the gate leaf and secondary beam sheets of a 9 M × 4.8 M gate) as an example: The original layout and cutting scheduling plan resulted in an average delay of 2.6 days due to process conflicts. A penalty of 0.5% of the total equipment value is imposed for each day of delay. The contract value for this gate equipment is 480,000 RMB. Original penalty: 480,000 × 2.6 × 0.5% = 6,240 RMB / Batch. After implementing the cutting scheduling adjustments outlined in this paper, the delay penalty is reduced by 37.7%: 6,240 × (1 - 37.7%) = 3,887.52 RMB / Batch.
- (4) Net economic calculation for 6 sheets Total waste for 6 sheets: 286 RMB × 6 = 1,716 RMB/Batch. Savings in delay penalties from 6 sheets: 6,240 - 3,887.52 = 2,352.48 RMB/ Batch. Net savings: 2,352.48 - 1,716 ≈ 636.48 RMB/Batch.

Therefore, for high-value hydraulic gates with strict delivery deadlines, sacrificing 1.8% of the material utilization rate (≈ 1716 RMB/Batch) to achieve a 37.7% reduction in delay penalties (≈ 2,352.48 RMB/ Batch) is a more optimal on-site decision.

Conclusions

In this study, a two-stage model is established to address the collaborative optimization of LCSP for the metal structural parts in the production process. The first stage involves a layout model to maximize the sheet utilization rate, while the second stage focuses on a mathematical model to optimize scheduling, with the aim of minimizing makespan and delay penalty.

Tailored to the characteristics of the research problem, the first-stage layout model is tackled using the IGWO. The processing sequence of the sheet metal is determined based on the heuristic rule of the smaller layout priority values, the earlier scheduling. Moreover, improvements are made to the encoding and decoding, searching, and attacking of the IGWO algorithm to attain more effective layout schemes. For the second stage cutting model, an MNSGA-II is designed. A heuristic rule is incorporated by assigning higher priority to sheet materials with smaller priority values for expedited processing, and it determined the job coding sequence. Subsequently, by allocating each sheet to a machine at random, an initial population of specified size is established. Through operations such as selection, crossover, and mutation, the better cutting scheduling results are obtained.

Parts from the curved gate arms of a mechanical shipbuilding company are regarded as test data. Comparisons are conducted among some important factors (such as sheet utilization rate, makespan, and delay penalty), and whether to consider layout priority values. Based on the proposed two-stage model, the results in Table 2 indicate that when considering layout priority, the makespan and delay penalty are significantly reduced by 3.9% and 37.7% respectively, at the cost of a modest 1.81% decrease in sheet utilization rate. In the future, when

solving the collaborative optimization LCSP, a balanced iterative negotiation mechanism will be established to find a balanced scheme of production efficiency and utilization rate.

Limitations and future work

This study addressed the optimization problem of layout and cutting scheduling using a two-stage model integration approach. Although the interaction between decision variables and priorities effectively balanced the requirements of layout and cutting schedule, resulting in optimal solutions for both layout and cutting schedule, the interaction between layout and cutting scheduling occurred only once during the process, making it difficult to achieve a balanced solution for both layout and cutting schedule. Therefore, future work directions should focus on solving the collaborative optimization problem of layout and cutting scheduling based on master-slave game theory. Specifically, the affiliation relationships between parts and sheets should be adjusted based on the optimization results of scheduling, enabling feedback from the scheduling layer to the layout layer, which in turn will alter the operational costs of the layout layer. By utilizing dynamic game theory between the layout layer and the cutting scheduling layer, multiple interactions between layout and scheduling can be achieved to search for a balanced solution between layout and scheduling. Future research may also explore algorithm optimization strategies to enhance its ability to handle large-scale problems and introduce new features or adjust rules to improve its adaptability to different types of parts.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Received: 18 November 2024; Accepted: 15 September 2025

Published online: 21 October 2025

References

- Wang, A., Hanselman, C. L. & Gounaris, C. E. A customized branch-and-bound approach for irregular shape nesting. *J. Global Optim.* **71**, 935–955. <https://doi.org/10.1007/s10898-018-0637-y> (2018).
- Tang, H.-T. et al. An optimizing model to solve the nesting problem of rectangle pieces based on genetic algorithm. *J. Intell. Manuf.* **28**, 1817–1826. <https://doi.org/10.1007/s10845-015-1067-z> (2017).
- Luo, Q. et al. Improved genetic algorithm based on composite evaluation factors to solve the layout problem of rectangular parts. *Forg. Technol.* **43**, 172–181 (2018) (10.13330/j.issn.1000-3940.2018.02.029).
- Zeng, X. L., Wu, Q. & Yuan, X. H. Adaptive multi-island genetic algorithm optimization of two-dimensional rectangular parts layout problem. *Forg. Technol.* **45**, 53–58 (2020) (10.13330/j.issn.1000-3940.2020.12.009).
- Yuan, X. Q. Batch optimization of rectangular parts. *Ind. Eng. Innov. Manag.* <https://doi.org/10.23977/IEIM.2023.060402> (2023).
- Liu, Y. et al. Research on hybrid solution algorithm for layout problem of rectangular parts with multiple constraints. *J. Syst. Simul.* **36**, 743–755 (2024) (10.16182/j.issn1004731x.joss.22-1258).
- Li, Z. H., Yu, J. F. & Qian, C. H. Research on layout optimization of defective rectangular parts based on dual population genetic algorithm. *Mach. Electron.* **41**, 7–12 (2023).
- Zhang, Z. D., Xue, L. Z. & Wen, C. Y. Application of two-stage heuristic algorithm for rectangle packing. *J. Nanjing Univ. Sci. Technol.* **47**, 767–773 (2023) (10.14177/j.cnki.32-1397n.2023.47.06.005).
- Gao, H. J. et al. Research on DPSO-CG hybrid optimization algorithm for layout optimization problem of rectangular parts. *Mach. Electron.* **42**, 17–22 (2024).
- Lei, D. M. & Yang, H. Multi-colony artificial bee colony algorithm for multi-objective unrelatedparallel machine scheduling problem. *Control Decis.* **32**(1174–1182), 1174–1182. <https://doi.org/10.13195/j.kzyjc.2020.0775> (2022).
- Sun, X. W. et al. Heuristic column generation algorithm for identical parallel machine scheduling problem with deterioration effect. *Control Decis.* **39**, 1636–1644 (2024) (10.13195/j.kzyjc.2022.1615).
- Zhou, Y. et al. Enhanced NSGA-algorithm for unrelated parallel machine scheduling. *Digit. Manuf. Sci.* **22**, 38–42 (2024).
- Zhang, X. C. et al. Minimizing total completion time and makespan for a multi-scenario bi-criteria parallel machine scheduling problem. *Eur. J. Oper. Res.* **321**, 397–406 (2025).
- Hendry, L. C. & Shek, K. K. F. A cutting stock and scheduling problem in the copper industry. *J. Oper. Res. Soc.* **47**, 38–47. <https://doi.org/10.2307/2584250> (1996).
- Poltroniere, S. C. et al. A coupling cutting stock-lot sizing problem in the paper industry. *Ann. Oper. Res.* **157**, 91–104. <https://doi.org/10.1007/s10479-007-0200-6> (2018).
- Yanasse, H. H. & Lamosa, M. J. P. An integrated cutting stock and sequencing problem. *Eur. J. Oper. Res.* **183**, 1353–1370. <https://doi.org/10.1016/j.ejor.2005.09.054> (2005).
- Pitombeira-Neto, A. R. & Prata, B. D. A. A matheuristic algorithm for the one-dimensional cutting stock and scheduling problem with heterogeneous orders. *TOP* **28**, 178–192. <https://doi.org/10.1007/s11750-019-00531-3> (2020).
- Sakaguchi, T. et al. A scheduling method with considering nesting for sheet metal processing. *Int. Symp. Flex. Autom.* **28**, 317–320. <https://doi.org/10.1115/ISFA2012-7197> (2012).
- Sakaguchi, T., Ohtani, H. & Shimizu, Y. Genetic algorithm based nesting method with considering schedule for sheet metal processing. *Trans. Inst. Syst. Control Inf. Eng.* **28**, 99–106. <https://doi.org/10.5687/iscie.28.99> (2015).
- Sakaguchi, T., Matsumoto, K. & Uchiyama, N. Nesting scheduling in sheet metal processing based on coevolutionary genetic algorithm in different environments. *Int. J. Auto Tech.* **12**, 730–738 (2018) (10.20965/ijat.2018.p0730).
- Melega, G., Matsumoto, S. & Jans, R. Classification and literature review of integrated lot-sizing and cutting stock problems. *Eur. J. Oper. Res.* **271**, 1–19. <https://doi.org/10.1016/j.ejor.2018.01.002> (2018).
- Wang, Y. F., Qi, D. Z. & Huang, J. M. Research on process scheduling optimization with multiple layouts. *Mach. Des. Manuf.* <https://doi.org/10.19356/j.cnki.1001-3997.2015.01.039> (2015).

Author contributions

J.Y. W., R.H. M. and C.C. X. proposed the method and implemented it. J.Y. W. and C.C. X. wrote the main manuscript text. X. D. prepared some figures. W.H. Z., C.Y. J., Z.J. W., F. G. supervised the work. All authors reviewed the manuscript.

Funding

This work is financially supported by National Key Research and Development Program of China (No.2023YFE0203200), Regional Innovation and Development Joint Fund Project (U24A20183), the National Natural Science Foundation of China (No.52205575), and Hubei Key Laboratory of Hydroelectric Machinery Design and Maintenance 2024 open fund (2024KJX12).

Declarations

Competing interests

The authors declare no competing interests.

Additional information

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1038/s41598-025-20522-8>.

Correspondence and requests for materials should be addressed to C.J.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Open Access This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License, which permits any non-commercial use, sharing, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if you modified the licensed material. You do not have permission under this licence to share adapted material derived from this article or parts of it. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by-nc-nd/4.0/>.

© The Author(s) 2025