



# OPEN A two-stage robust optimization model for emergency service facilities location-allocation problem under demand uncertainty and sustainable development

Hongyan Li<sup>2,3</sup>, Dongmei Yu<sup>1,2,3</sup>✉, Yiming Zhang<sup>2,3</sup> & Yifei Yuan<sup>2,3</sup>

Under the backdrop of frequent emergencies, the rational layout of emergency service facilities (ESF) and the effective allocation of emergency supplies have emerged as crucial in determining the timeliness of post-disaster response. By adequately accounting for potential uncertainties and carrying out comprehensive pre-planning, the robustness of location-allocation decisions can be significantly improved. This paper delves into the ESF network design problem under demand uncertainty and formulates this problem as a two-stage robust optimization model. The presented model defines a generalized budget uncertainty set to capture victims' uncertain demand and minimizes the sum of the costs involved in the two stages. The objective function integrates the input cost in the preparedness phase, the deprivation cost from the victims' perspective and the environmental impact cost responding to sustainable development in the response phase, which respectively correspond to the comprehensive optimization of the deployment of ESF, the distribution of emergency supplies and the implementation of sustainable measures. Subsequently, we employ the column and constraint generation (C&CG) algorithm to solve the proposed model and take the COVID-19 epidemic in Wuhan as a case to verify the effectiveness of the model and algorithm. Finally, we examine the influence of demand uncertainty and environmental impact cost on the optimal solution, yielding valuable managerial insights.

**Keywords** Emergency service facilities, Location-allocation, Two-stage robust optimization, Demand uncertainty, Sustainable development

In recent years, large-scale emergencies such as earthquakes, floods, and epidemics have led to massive human casualties and economic losses worldwide. According to statistics, natural disasters in 2023 resulted in 86,473 deaths, impacted 93.05 million people, and caused direct economic losses of \$202.65 billion globally. In comparison to the average over the past decade, the number of deaths in 2023 was 482% higher, while direct economic losses increased by 23% higher<sup>1</sup>. In light of this alarming data, emergency service facilities (ESF), such as emergency distribution centers, are essential infrastructures that provide vital support for post-disaster response and recovery. Therefore, designing a scientifically sound and efficient ESF network to ensure the rapid delivery of emergency supplies to affected areas has become an urgent issue that needs greater attention.

The emergency management life cycle of emergencies generally includes four phases: mitigation, preparedness, response and recovery. During the preparedness phase, the primary task is to formulate targeted contingency plans (including ESF location and emergency supplies allocation strategies) based on the comprehensive evaluation of the type, intensity and influence scope of potential emergencies. Then, in the response phase, these plans are immediately activated to minimize losses. However, the uncontrollability of emergencies may cause the uncertainties, resulting in the failure of the location-allocation scheme of ESF.

Dönmez et al.<sup>2</sup> have comprehensively summarized the sources of emergency supply chain uncertainties and related research progress, including facility disruption uncertainty<sup>3–5</sup>, demand uncertainty<sup>6,7</sup>, transportation

<sup>1</sup>Ordos Institute of Liaoning Technical University, Ordos 017004, People's Republic of China. <sup>2</sup>School of Business Administration, Liaoning Technical University, Huludao 125105, People's Republic of China. <sup>3</sup>Institute for Optimization and Decision Analytics, Liaoning Technical University, Fuxin 123000, People's Republic of China. ✉email: yudongmei1113@163.com

uncertainty<sup>8,9</sup> and so on. They emphasized that these uncertainties significantly impacted the effectiveness of ESF location-allocation scheme. In particular, demand uncertainty has become a crucial research field in the ESF network design problem due to its universality, complexity and seriousness of its consequences. For instance, the Haiti earthquake in 2010, the Nepal earthquake in 2015, and the COVID-19 pandemic in 2020 all resulted in serious shortages of emergency supplies due to demand uncertainty, causing victims to face severe survival challenges. These examples highlight the necessity of fully considering potential demand uncertainty in order to enhance the robustness of ESF location-allocation scheme.

Moreover, the intensity, scope and other factors of emergencies are intertwined, which have jointly exacerbated the complexity of demand uncertainty. Meanwhile, the geographical proximity of affected areas and the mobility of victims further intensify the correlation of demand fluctuations. Therefore, the primary problem that needs attention is how to combine the unique attributes of ESF location-allocation problem and develop an effective approach to accurately quantify emergency demand uncertainty.

To address this challenge, diversified approaches have been employed to quantify uncertainty, each possessing specific application scenarios and advantages. Specifically, stochastic programming models often adopt the probability distribution method and scenario generation method to delineate uncertainty<sup>10</sup>. The former utilizes predefined probability distribution information to describe uncertainty, whereas the latter simulates uncertainty by creating a series of possible scenarios. In addition, the Conditional Value at Risk (CVaR)<sup>11,12</sup>, which was originally developed to quantify the maximum loss of financial portfolio optimization, has recently been extended to quantify the uncertainty risk in supply chain management problem. On the other hand, robust optimization models primarily use uncertainty set as a means of uncertainty quantification<sup>13</sup>, seeking solutions that perform well even under the worst-case scenarios within the uncertainty set.

Due to the scarcity of historical data in emergencies, it is difficult to accurately fit the probability distribution of uncertain parameter, so the probability distribution estimation method relying on large amounts of historical data is facing challenges. In this context, the robust optimization method emerges as a powerful tool, as it directly constructs uncertainty sets and gives high attention to worst-case scenarios. This method is particularly suitable for emergencies where data is scarce, providing a more conservative but robust decision-making framework. Therefore, we plan to apply the robust optimization method to quantify the uncertainty of emergency demand.

Beyond the quantification of uncertainty, research on the ESF location-allocation problem has extended into diversified fields. Notably, researchers are increasingly integrating social and environmental sustainability goals into their studies, aiming to enhance emergency response efficiency while promoting social and environmental benefits<sup>14,15</sup>. Given the pressing environmental challenges of today, environmental sustainability has become increasingly crucial in supply chain management, especially in the context of emergency response transportation, which often has a potential impact on the ecological environment. Consequently, merging uncertainty and sustainability in the design of ESF networks emerges as a critical and urgent research direction. This integrated approach can significantly enhance the efficient utilization of emergency supplies while mitigating adverse environmental effects, aligning with the growing emphasis on green and sustainable practices in emergency management.

Accordingly, this paper proposes a two-stage robust optimization model tailored to address the ESF location-allocation problem, which anticipates demand uncertainty and environmental sustainability during the response phase. Firstly, we develop a generalized budget uncertainty set to quantify correlated demand uncertainty by exploring the internal relationship between uncertain factors and demand fluctuation. Subsequently, to mitigate the adverse effects of unmet demand, we introduce the concept of deprivation cost, which specifically quantifies the economic value of the pain suffered by victims due to unmet demand. Furthermore, in response to the global call for sustainable development, we not only focus on cost-effectiveness and demand satisfaction, but also specifically incorporate environmental sustainability into the proposed model. This approach ensures that our model addresses both the immediate needs of emergency response and the long-term environmental consequences. Then, we construct a two-stage robust optimization model for ESF location-allocation problem considering demand uncertainty and sustainable development. Finally, we verify the effectiveness of the proposed model through empirical analysis.

The main contributions of this paper are as follows:

- (1) Developing a new budget uncertainty set quantifying the correlated demand uncertainty based on the internal connection between influencing factors and uncertain demand, so as to provide strong support for effective distribution of emergency supplies.
- (2) Integrating environmental impact cost into the objective function, which not only addresses immediate needs but also promotes sustainable development by taking into account long-term environmental consequences.
- (3) Constructing a two-stage robust optimization model for the ESF location-allocation problem under demand uncertainty and reformulating it through duality theory and linearization techniques to ensure precise solvability. The rest of this paper is structured as follows: section “[Literature review](#)” summarizes the relevant literature on the ESF location-allocation problem. Section “[Problem description and mathematical model](#)” includes the problem description and model construction. Section “[Solution algorithms](#)” introduces column and constraint generation (C&CG) algorithm to solve the proposed model, and deduces the solvable form of the proposed model. In section “[Numerical study and analysis](#)”, the applicability of the model and the effectiveness of the algorithm are illustrated by an actual case. Finally, in section “[Conclusions and policy suggestions](#)”, a brief conclusion is presented, along with the prospects for future research.

## Literature review

This paper focuses on the ESF location-allocation problem under demand uncertainty and sustainability. Therefore, in this section, we primarily systematically review the existing research on ESF location-allocation problem considering demand uncertainty, and analyze the limitations of existing research. Subsequently, we focus on the advantages of the robust optimization in tackling demand uncertainty, and review its current research. Finally, we review the relevant research on the ESF location-allocation problem considering sustainability, highlighting its indispensability in the decision-making of the ESF location layout and emergency supplies allocation.

### ESF location-allocation problem under demand uncertainty

Emergencies are frequently accompanied by various uncertainties, with demand uncertainty being particularly significant. Demand uncertainty has a direct impact on the basic survival guarantee of victims, and any minor delay may lead to serious consequences. Therefore, it is imperative to explore the ESF location-allocation scheme under demand uncertainty to ensure the efficient emergency response. Currently, researchers have carried out in-depth exploration on the ESF location-allocation problem under demand uncertainty, yielding significant results<sup>16–18</sup>.

In the process of constructing the emergency facility location-allocation model considering demand uncertainty, a classic approach is to assume the demand scenarios. Such as Jia et al.<sup>19</sup> estimated the potential demand scenarios according to the population density distribution, and then put forward a maximum coverage location model for emergency medical service facilities. On this basis, Horner and Downs<sup>20</sup> expanded the scope of demand uncertainty by incorporating different probability levels, and simulated demand scenarios of various scales, thereby making the model closer to the realistic problem. Cavdur et al.<sup>21</sup> more accurately simulated the demand fluctuation for emergency supplies by estimating the specific demand uncertainty scenarios, and further constructed a two-stage emergency facility location model.

Apart from enumerating demand uncertainty scenarios, another approach is to assume that the demand follows a certain probability distribution, and then apply stochastic programming to construct the location-allocation optimization model covering expected cost. For example, Dalal and Üster<sup>22</sup> considered the demand uncertainty caused by the location and intensity of disasters, and then proposed a stochastic programming model by depicting the demand uncertainty with discrete scenarios. To simplify the solution process for the facility location problem under demand uncertainty, Zhang et al.<sup>23</sup> utilized the inverse uncertainty distribution theory to transform the original stochastic model into an equivalent deterministic model. Furthermore, Peng et al.<sup>24</sup> constructed a two-stage stochastic programming model based on the probabilistic constraints of emergency demand to deal with the uncertainty of emergency demand. After that, Wang et al.<sup>25</sup> expanded the uncertainty, further studied the facility location problem under the double uncertainty of demand and transportation, and constructed a dynamic scenarios-based two-stage stochastic programming model. In addition, Ghouschi et al.<sup>26</sup> introduced fuzzy random parameters to define uncertain demand parameters, and proposed a multi-objective mixed integer linear programming model.

On the other hand, Guo et al.<sup>27</sup> employed the CVaR method to quantify the risk of uncertainties surrounding the occurrence time and demand of a perishable emergency supplies inventory system, and constructed a comprehensive risk-averse model with considering perishable emergency supplies replacement strategy. Hu et al.<sup>28</sup> considered the uncertainties in both demand and transportation time, and adopted the average CVaR method to quantify the risks associated with these uncertainties. On this basis, they constructed a risk-averse stochastic programming model. The above research has generally assumed that the probability distribution of demand is completely known. Wang et al.<sup>29</sup> further relaxed this assumption by designing corresponding fuzzy sets based on imprecise probability distribution of demand, and proposed a two-stage distribution robust mean-CVaR optimization model combined with risk-averse criterion.

In addition, there are still some research on the ESF location-allocation problem by using the strategy of simulating demand scenarios, see literature<sup>30</sup> for details. However, Klibi et al.<sup>31</sup> highlighted that selecting typical uncertain scenarios and their respective probabilities is a formidable task. They pointed out that when dealing with uncertainty, stochastic programming based on scenario generation struggles to comprehensively capture the entire range of uncertain scenarios. Insufficient selected scenarios may limit the evaluation scope of decisions, thereby affecting the accuracy and reliability of decisions.

### ESF location-allocation problem under demand uncertainty based on robust optimization

In view of the limitations of stochastic programming in selecting uncertainty scenarios and their probability distribution, researchers turned to utilizing robust optimization to model ESF location-allocation problem under demand uncertainty. For example, by introducing demand uncertainty into the shelter location-allocation problem, Eriskin and Karatas<sup>32</sup> reformulated the deterministic mixed integer linear programming version of the problem as a robust model, which significantly improved the disaster preparedness level in the affected areas. After that, Basciftci et al.<sup>33</sup> and Wang et al.<sup>34</sup> further described the demand uncertainty by constructing the fuzzy set of demand, and put forward the robust optimization model of facility location under demand uncertainty, which provided a new perspective for addressing the demand uncertainty problems.

Aiming at the uncertainty problems, the robust optimization does not depend on the specific probability distribution, but describes these uncertain parameters in the form of uncertainty sets, which is more in line with the nature of ESF location-allocation problem and data characteristics. In the robust optimization model, the common uncertainty sets include box uncertainty set, ellipsoid uncertainty set, polyhedral uncertainty set<sup>35,36</sup>. Sun et al.<sup>37</sup> combined the box uncertainty set with the damage severity score, thus describing the demand uncertainty about casualties more accurately. Since the box uncertainty set is too conservative, Zhang and Jiang<sup>38</sup> further applied the ellipsoid uncertainty set to describe the demand uncertainty and established the location

model of emergency medical service facilities. However, the introduction of ellipsoid uncertainty set increases the number of nonlinear constraints and the difficulty of solving model. In order to avoid this problem, Chen and Fu<sup>39</sup> introduced the polyhedral uncertainty set to describe the uncertainty of the number of victims after the disaster, which ensured the accuracy of the model and reduced the complexity of the solution. Similarly, Ryu and Park<sup>40</sup> introduced budget uncertainty set to describe demand uncertainty, and achieved better expected results.

From the aforementioned details, it is not difficult to find that although the technology of describing demand uncertainty by using uncertainty sets has matured, it is still stylized in the precise quantification of demand uncertainty. Furthermore, there is a lack of in-depth exploration of the underlying causes and mechanisms of demand uncertainty. In order to improve the accuracy of demand uncertainty quantification, Zhang et al.<sup>41</sup> integrated rolling horizon optimization method and uncertainty budget adjustment strategy to describe demand uncertainty, and constructed a multi-cycle, multi-level robust optimization model for facility deployment and resource allocation, which effectively improved the ability to cope with time-varying demand. However, it is worth noting that although the traditional robust optimization model is simple and intuitive, it tends to be overly conservative in decision-making. In order to break through this limitation, Ben-Tal et al.<sup>42</sup> introduced the two-stage robust optimization model, which enabled decision makers to make targeted optimization for different uncertain factors in different stages, and helped to balance the cost and risk of location and allocation more finely under uncertain environments. On this basis, in the design of humanitarian logistics network, Qi et al.<sup>43</sup> proposed a two-stage robust optimization framework to solve the service-oriented location and inventory reservation problem with uncertain demand and third-party supply, which provided a new perspective for emergency management with uncertain demand. More recent studies on the ESF location-allocation problem under demand uncertainty can be found in<sup>44–46</sup>.

### ESF location-allocation problem considering sustainability

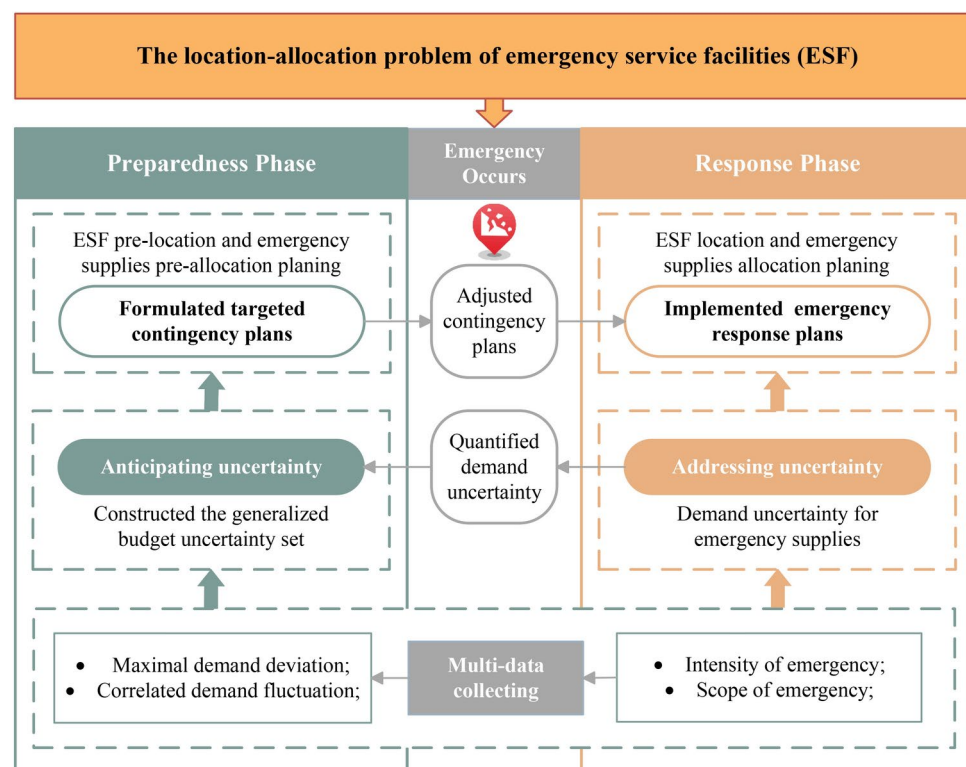
The above reviewed papers about the ESF location-allocation problem have predominantly focused on quantitative analysis of demand uncertainty. To the best of our knowledge, the recent research on this issue has exhibited a diversified trend, delving into and deepening the understanding of this problem from multiple dimensions. For example, regarding the design of blood supply chain during the COVID-19 pandemic, Tirkolaee et al.<sup>47</sup> established a mixed-integer linear programming model under uncertainties of demand, capacity, and blood processing rate, which not only considered the goal of minimizing costs but also incorporated the goal of maximizing the fulfillment of social needs, thereby promoting the sustainable development of the supply chain. Taking into account the aggravation of global climate change, Ahmadi et al.<sup>48</sup> further integrated environmental sustainability goal into the optimization framework of pharmaceutical supply chain, achieving a delicate balance between socio-economic goals and environmental protection. Nayeri et al.<sup>49</sup> put forward a multi-objective mathematical model considering the flexibility and responsiveness of global supply chain, aiming at minimizing the environmental impact and total cost, while maximizing the social impact, and adopted an improved fuzzy robust stochastic method to deal with the uncertainty, taking into account the sustainability, flexibility, responsiveness and global factors comprehensively.

In addition, Kunz et al.<sup>50</sup> and Peretti et al.<sup>51</sup> strongly called for the integration of sustainable practices in emergency relief operations, aiming at reducing the adverse impact on the environment and promoting the development of relief operations in a sustainable direction. Subsequently, Cao et al.<sup>52</sup> integrated sustainability into the design of disaster supply chain, and constructed a multi-objective mixed integer nonlinear programming model, which revealed the close relationship between sustainable development and traditional relief. After emergencies, a large number of emergency supplies are transported to the affected areas, and carbon emissions from transportation inevitably impose a burden on the environment. Consequently, from the perspective of environmental sustainability, Zhang et al.<sup>53</sup> developed a multi-objective optimization model for emergency evacuation path planning problem with the goals of minimizing total cost, total travel time and total carbon emissions. This model provided a more environmentally friendly and efficient scheme for emergency evacuation path planning problem. Furthermore, Oscar et al.<sup>54</sup> considered the carbon emissions cost generated by humanitarian logistics, and drew a remarkable conclusion through comparative experiments: adopting carbon emission reduction measures in the humanitarian logistics will not have any negative impact on the service level provided to victims, but will have a positive effect on reducing the carbon emissions. For urgently needed virus detection equipment under the COVID-19 pandemic, Alizadeh et al.<sup>55</sup> designed a reliable and sustainable stochastic multi-objective model of emergency medical device supply chain considering the greenhouse gas emissions. Thereafter, Boostani et al.<sup>56</sup> and Cao et al.<sup>57</sup> discussed the three dimensions of sustainable development in detail, and proposed a mixed integer multi-objective optimization model for the distribution of relief materials.

In summary, although the ESF location-allocation problem under demand uncertainty have been extensively studied, the accurate quantification of demand uncertainty is still a worthwhile problem, and the research on this problem from the perspective of sustainability is still in the initial stage. Therefore, inspired by previous studies, we construct a generalized budget uncertainty set to quantify correlated demand uncertainty by clarifying the internal relationship between uncertain factors and demand uncertainty. In addition, by adding the environmental impact cost to the objective function, we model this problem as a two-stage robust optimization model for ESF location-allocation considering environmental sustainability under demand uncertainty. In order to highlight the distinctions between our study and the existing studies, Table 1 provides a more detailed literature comparison. Through comparative analysis, we find that our study makes progress in integrating demand uncertainty and sustainability, and successfully realize this integration process in a two-stage robust optimization framework. This measure not only contributes to the rational allocation of emergency supplies, but also promotes the sustainable development of emergency management.

Studies	Demand	Stage		Objectives				Decisions		Model types	
	Uncertainty	Preparedness	Response	Location	Allocation	Deprivation	Environment	Location	Allocation	Stochastic	Robust
Dalal and Üster <sup>22</sup>	✓	✓	✓	✓	✓			✓	✓	✓	
Peng et al. <sup>24</sup>	✓	✓	✓	✓	✓	✓		✓	✓	✓	
Wang et al. <sup>25</sup>	✓	✓	✓	✓	✓	✓		✓	✓	✓	
Hu et al. <sup>28</sup>	✓	✓		✓	✓	✓		✓	✓	✓	
Basciftci et al. <sup>33</sup>	✓	✓	✓	✓	✓	✓		✓	✓		✓
Zhang and Jiang <sup>38</sup>	✓	✓		✓	✓			✓	✓		✓
Ryu and Park <sup>40</sup>	✓	✓		✓	✓			✓	✓	✓	
Zhang et al. <sup>41</sup>	✓	✓		✓		✓		✓	✓		✓
Qi et al. <sup>43</sup>	✓	✓	✓	✓	✓	✓		✓	✓		✓
Cao et al. <sup>52</sup>		✓					✓	✓	✓		
Zhang et al. <sup>53</sup>		✓		✓	✓		✓	✓	✓	✓	
Oscar et al. <sup>54</sup>		✓	✓	✓		✓	✓	✓	✓	✓	
Boostani et al. <sup>56</sup>		✓		✓	✓	✓	✓	✓	✓	✓	
Our study	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓

**Table 1.** The comparison of studies on ESF location-allocation problem.



**Fig. 1.** The framework of the ESF location-allocation problem.

## Problem description and mathematical model

### Problem definition

In large-scale emergencies, the affected areas require large amounts of emergency supplies, such as medicine, food, water, and so forth. Given the huge amount of supplies and their diverse sources, the effective deployment of ESF has become the core element to ensure the rapid and accurate delivery of emergency supplies to affected areas. Note that the demand for emergency supplies in affected areas is not a fixed value. Furthermore, the carbon emissions generated during transportation and waste materials packaging will inevitably have an impact on the environment. Hence, focusing on the preparedness phase, this paper studies the ESF location-allocation problem by integrating demand uncertainty and sustainable development perspective. Figure 1 visually presents the core framework of this study. To enhance the resilience and adaptability of the ESF location-allocation strategy, it is essential to consider potential demand uncertainty in advance. By constructing a generalized



demand uncertainty set based on classical budget uncertainty set and historical data, we can quantify the demand uncertainty more accurately and develop more effective response strategies accordingly.

Model assumptions

The proposed model is supported by the following main assumptions.

- (1) An adequate number of the same type of vehicles are available for the purpose of transporting emergency supplies between ESF and demand points.
- (2) In evaluating the environmental impact cost, we mainly consider the emissions of emergency supplies transportation, and temporarily ignore the emissions generated by facilities construction and personnel activities.
- (3) Nominal demand and correlation of demand points can be estimated based on the population size and the relevant historical disaster data in the affected areas.
- (4) The impact of emergency supplies packaging on the ecological environment can be measured by converting it into the cost of harmless treatment of waste materials packaging.

Notation

The mathematical notations used in this paper are summarized in Table 2.

Model formulation

When making ESF location decisions in the preparedness phase, it is a significant challenge to comprehensively predict and accurately quantify demand uncertainty. Given the advantages of the budget uncertainty set, we utilize it to quantify demand uncertainty, so as to further develop the ESF location model. Taking into account the influence of uncertain factors such as emergency intensity and scope on demand, we further propose the generalized budget uncertainty set that quantifies the correlated demand uncertainty, which is defined as follows:

$$U = \left\{ \tilde{d} \in \mathbb{R}^{|I|} : \tilde{d}_i = \bar{d}_i + z_i \hat{d}_i, z_i \in \{0, 1\}, \hat{d}_i = \left( s / \left( L_i / \sum_{i \in I} L_i \right) \right) \cdot \bar{d}_i, \forall i \in I, \Gamma_L \leq B \cdot z \leq \Gamma_U \right\}, \quad (3.1)$$

where  $\bar{d}_i$  is the nominal demand of the demand point  $i$ , and  $\tilde{d}_i$  is the actual demand of the demand point  $i$ .  $\hat{d}_i$  represents the maximal demand deviation which is determined by the intensity  $s$  and the distance  $L_i$  between demand point  $i$  and the emergency site. Specifically,  $s$  represents increasing intensity of each emergency, which is positively correlated with  $\hat{d}_i$ .  $L_i$ , serves as an indirect indicator of the impact of the emergency scope on the demand uncertainty, which is negatively correlated with  $\hat{d}_i$ .  $\Gamma_L$  and  $\Gamma_U$  are the uncertainty budget vectors that limit the conservatism of decision makers in considering demand uncertainty. Inequality  $\Gamma_L \leq B \cdot z \leq \Gamma_U$  provides the correlation expression of demand fluctuation, where  $B \in \mathbb{R}^{R \times |I|}$  is a Boole matrix, representing the correlation coefficient matrix of demand fluctuation.

Thus, we establish the two-stage robust optimization model with a min-max objective to obtain the ESF location-allocation scheme under demand uncertainty. The model is presented as follows:

Notations	Description
<b>Sets</b>	
$I$	Set of demand points, $i \in I$
$J$	Set of candidate ESF, $j \in J$
<b>Parameters</b>	
$f_j$	The fixed construction cost of opening a ESF at location $j$
$Q_j$	The maximum capacity of ESF at location $j$
$D_i$	The demand at demand point $i$
$\tau$	The unit transportation cost of emergency supplies
$p_i$	The unit deprivation cost of unmet demand at demand point $i$
$\alpha$	The influence coefficient of material packaging on environment
$\beta$	The influence coefficient of transport vehicle on environment
$q$	The maximum capacity of the transport vehicle
$c_{ij}$	The travel distance between demand point $i$ and ESF at location $j$
<b>Decision variables</b>	
$y_j$	Binary variable; equals 1 if a ESF is built at location $j$ and 0 otherwise
$x_{ij}$	Continuous variable; amount of emergency supplies that travels from ESF at location $j$ to demand point $i$
$u_i$	Continuous variable; unmet demand of demand point $i$

Table 2. The notations.

P1

$$\min_y \left\{ \sum_{j \in J} f_j y_j + \max_z g(y, z) \right\} \quad (3.2)$$

$$s.t. \ y_j \in \{0, 1\}, \forall j \in J, \quad (3.3)$$

where  $g(y, z)$  is the second-stage cost:

P2

$$g(y, z) = \min_{x, u} \sum_{i \in I} \sum_{j \in J} \tau c_{ij} x_{ij} + \left( \sum_{i \in I} \sum_{j \in J} \alpha x_{ij} + \sum_{i \in I} \sum_{j \in J} \beta c_{ij} \frac{x_{ij}}{q} \right) + \sum_{i \in I} p_i u_i \quad (3.4)$$

$$s.t. \ \sum_{j \in J} x_{ij} + u_i \geq \bar{d}_i + z_i \hat{d}_i, \forall i \in I, \quad (3.5)$$

$$\sum_{i \in I} x_{ij} \leq Q_j y_j, \forall j \in J, \quad (3.6)$$

$$\Gamma_{L,r} \leq \sum_{i \in I} b_{ri} z_i \leq \Gamma_{U,r}, \forall r \in R, \quad (3.7)$$

$$x_{ij} \geq 0, \forall i \in I, j \in J, \quad (3.8)$$

$$u_i \geq 0, \forall i \in I. \quad (3.9)$$

The objective function (3.2) minimizes the sum of the ESF location cost in the first stage and the worst-case allocation cost in the second stage. The objective function (3.4) details the costs involved in the second stage from three main dimensions: cost-effectiveness, environmental sustainability, and demand satisfaction. Specifically, it includes (a) the cost of transporting emergency supplies, which relates to cost-effectiveness; (b) the cost of harmless treatment of waste materials packaging and carbon emissions during transportation, which corresponds to environmental sustainability; (c) the deprivation cost of unmet demand, which reflects the impact on victims' demand satisfaction. Constraints (3.3) define the binary ESF location decision variables. Constraints (3.5) ensure that the demand of the each demand point is satisfied as much as possible. Constraints (3.6) denote the maximum capacity limitation of each ESF. Constraints (3.7) present the correlation expression of demand uncertainty. Constraints (3.8) and (3.9) define the value range of continuous decision variables  $x_{ij}$  and  $u_i$ .

In above proposed model, the second stage problem is formulated as a bi-level programming problem, whose core lies in the interdependent and interactive nature of the bi-level decisions. Furthermore, the introduction of the demand uncertainty set increases the number of nonlinear constraints in the model, which significantly elevating the complexity and challenge of the solving. To successfully tackle this model, we will illustrate the transformation strategy of the model and present the corresponding algorithmic approach in section “[Solution algorithms](#)”.

## Solution algorithms

In this section, we mainly introduce the reformulation of the proposed model and the solution algorithms.

### Model reformulation

In order to efficiently solve the two-stage robust optimization model proposed in this paper, we intend to adopt a decomposition strategy to disassemble the original model into two parts: the master problem and the subproblem, and then gradually approach the optimal solution of the model by solving master and subproblem alternately. Figure 2 shows the alternately solving framework and its specific process intuitively.

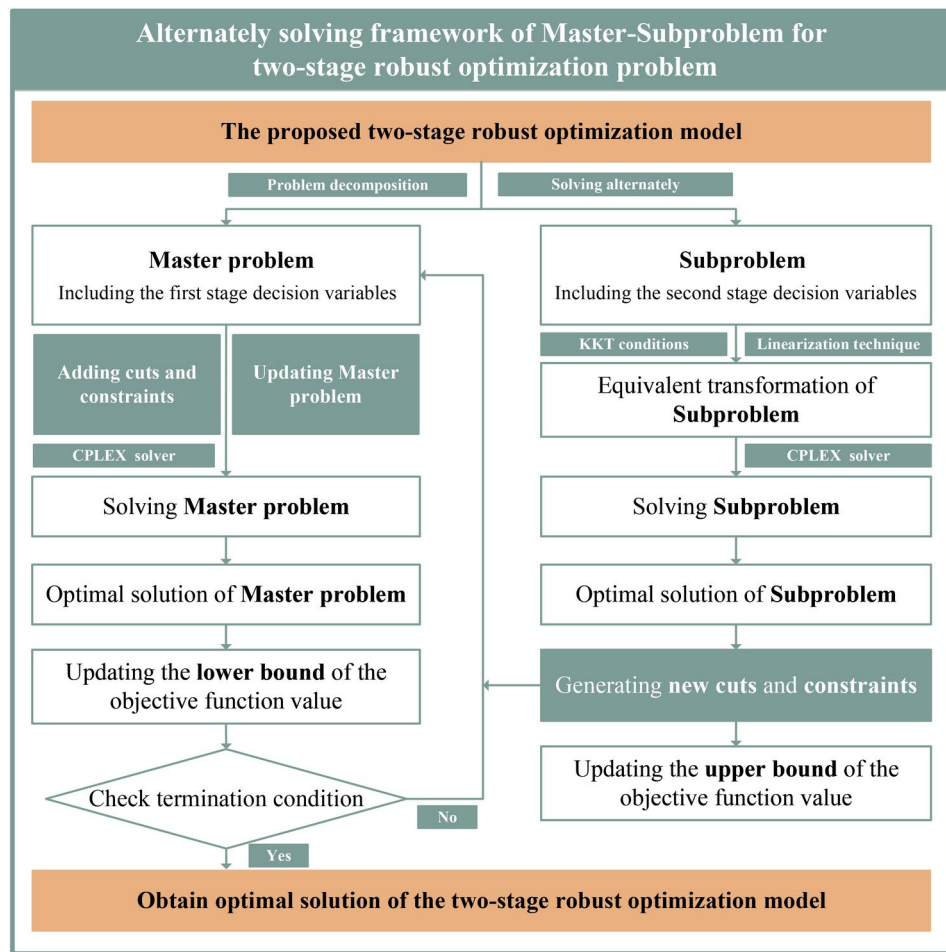
On this basis, we further derive the equivalent master problem and subproblem forms of the proposed model in section “[Model formulation](#)”. The detailed transformation process is as follows.

The master problem is written as:

MP

$$\min_y \sum_{j \in J} f_j y_j + \eta, \quad (4.1)$$

$$s.t. \ \eta \geq \sum_{i \in I} \sum_{j \in J} \tau c_{ij} x_{ij}^l + \sum_{i \in I} \sum_{j \in J} \alpha x_{ij}^l + \sum_{i \in I} \sum_{j \in J} \beta c_{ij} \frac{x_{ij}^l}{q} + \sum_{i \in I} p_i u_i^l, \quad (4.2)$$



**Fig. 2.** Alternately solving framework for the proposed two-stage robust optimization model.

$$\sum_{j \in J} x_{ij}^l + u_i \geq \bar{d}_i + z_i^l \hat{d}_i, \forall i \in I, \quad (4.3)$$

$$\sum_{i \in I} x_{ij}^l \leq Q_j y_j, \forall j \in J, \quad (4.4)$$

$$y_j \in \{0, 1\}, \forall j \in J.$$

The **MP** seeks to obtain the optimal location decision based on the set of worst cases determined in the subproblem.  $z_i^l$  represents the  $l$ th scenario identified by the subproblem.  $x_{ij}^l$  and  $u_i^l$  represent the decision variable values related to the  $l$ th scenario obtained by the subproblem. Since constraint (4.2) is a subset of demand uncertainty set (3.1), **MP** naturally provides an effective relaxation for the original two-stage robust optimization model. By gradually adding important scenarios (4.2)–(4.4) to the **MP**, a stronger lower bound can be expected.

After obtaining the optimal location decision  $y^*$  of **MP**, we identify important scenarios by solving the following subproblem.



**SP1**

$$\begin{aligned}
\max_z \min_{x,u} & \sum_{i \in I} \sum_{j \in J} \tau c_{ij} x_{ij} + \sum_{i \in I} \sum_{j \in J} \alpha x_{ij} + \sum_{i \in I} \sum_{j \in J} \beta c_{ij} \frac{x_{ij}}{q} + \sum_{i \in I} p_i u_i, \\
s.t. & \sum_{j \in J} x_{ij} + u_i \geq \bar{d}_i + z_i \hat{d}_i, \forall i \in I, \\
& \sum_{i \in I} x_{ij} \leq Q_j y_j^*, \forall j \in J, \\
& \Gamma_{L,r} \leq \sum_{i \in I} b_{ri} z_i \leq \Gamma_{U,r}, \forall r \in R, \\
& x_{ij} \geq 0, \forall i \in I, j \in J, \\
& u_i \geq 0, \forall i \in I.
\end{aligned}$$

The **SP1** is a bi-level optimization problem. To solve it successfully, we utilize the Karush-Kuhn-Tucker (KKT) conditions to transform the above bi-level optimization problem into the single-level optimization problem, which is presented as follows.

**SP2**

$$\begin{aligned}
\max_{z,x} & \sum_{i \in I} \sum_{j \in J} \tau c_{ij} x_{ij} + \sum_{i \in I} \sum_{j \in J} \alpha x_{ij} + \sum_{i \in I} \sum_{j \in J} \beta c_{ij} \frac{x_{ij}}{q} + \sum_{i \in I} p_i u_i \\
s.t. & c_{ij} + \alpha + \beta c_{ij} \frac{x_{ij}}{q} - \pi_i - \theta_j = 0, \forall i \in I, j \in J, \\
& (\bar{d}_i + z_i \hat{d}_i) - \sum_{j \in J} x_{ij} - u_i \leq 0, \forall i \in I, \\
& \sum_{i \in I} x_{ij} - Q_j y_j^* \leq 0, \forall j \in J, \\
& \pi_i (\bar{d}_i + z_i \hat{d}_i - \sum_{j \in J} x_{ij} - u_i) = 0, \forall i \in I,
\end{aligned} \tag{4.5}$$

$$\theta_j \left( \sum_{i \in I} x_{ij} - Q_j y_j^* \right) = 0, \forall j \in J, \tag{4.6}$$

$$\begin{aligned}
\Gamma_{L,r} & \leq \sum_{i \in I} b_{ri} z_i \leq \Gamma_{U,r}, \forall r \in R, \\
x_{ij} & \geq 0, \forall i \in I, j \in J, \\
u_i & \geq 0, \forall i \in I,
\end{aligned}$$

where  $\pi_i$  and  $\theta_j$  are introduced dual variables. It is obvious that constraints (4.5) and (4.6) contain nonlinear terms, which significantly complicate the solving process. To overcome this difficulty, we perform linearization of these constraints (4.5) and (4.6) by using the big-M method, thereby transforming the above model into the following equivalent form.

## SP3

$$\begin{aligned}
\Psi(y^*) = \max_{z, x, u, \pi, \theta, \omega, \nu} & \sum_{i \in I} \sum_{j \in J} \tau c_{ij} x_{ij} + \sum_{i \in I} \sum_{j \in J} \alpha x_{ij} + \sum_{i \in I} \sum_{j \in J} \beta c_{ij} \frac{x_{ij}}{q} + \sum_{i \in I} p_i u_i \\
s.t. & c_{ij} + \alpha + \beta c_{ij} \frac{x_{ij}}{q} - \pi_i - \theta_j = 0, \forall i \in I, j \in J, \\
& (\bar{d}_i + z_i \hat{d}_i) - \sum_{j \in J} x_{ij} - u_i \leq 0, \forall i \in I, \\
& \sum_{i \in I} x_{ij} - Q_j y_j^* \leq 0, \forall j \in J, \\
& -\pi_i \leq M \omega_i, \forall i \in I, \\
& -(\bar{d}_i + z_i \hat{d}_i) + \left( \sum_{j \in J} x_{ij} + u_i \right) \leq M(1 - \omega_i), \forall i \in I, \\
& -\theta_j \leq M \nu_j, \forall j \in J, \\
& \sum_{i \in I} x_{ij} - Q_j y_j^* \leq M(1 - \nu_j), \forall j \in J, \\
& \Gamma_{L,r} \leq \sum_{i \in I} b_{ri} z_i \leq \Gamma_{U,r}, \forall r \in R, \\
& x_{ij} \geq 0, \forall i \in I, j \in J, \\
& u_i \geq 0, \forall i \in I.
\end{aligned}$$

**C&CG algorithm for the proposed model**

The C&CG algorithm was initially proposed by Zeng and Zhao<sup>58</sup>, which was implemented in a master-subproblem framework. Specifically, C&CG algorithm is to solve the master problem to obtain the first stage decisions, and then tackle the subproblem to generate new variables and constraints to be added to the master problem, approximating the optimal solution by iterations. The detailed procedures of the C&CG algorithm to solve the proposed model is given in Algorithm 1.

---

**Step 1.** Initialization: Let  $LB = -\infty$ ,  $UB = \infty$ ,  $\eta = 0$ ,  $k = 0$  and  $\mathbf{o} = \emptyset$ .

**Step 2.** Solving the **MP** to get the location decision  $y_{k+1}^*$  and update  $LB = \mathbf{f}^T y_{k+1}^* + \eta_{k+1}^*$ .

**Step 3.** Solving the **SP3** with given  $y_{k+1}^*$  and updating  $UB = \min\{UB, y_{k+1}^* + \Psi(y_{k+1}^*)\}$ .

**Step 4.** If  $(UB - LB)/UB \leq \epsilon$ , return  $y_{k+1}^*$ . Otherwise, create new variables  $\mathbf{x}^{k+1}$  and add new constraints to **SP3**. Then update  $k = k + 1$ ,  $\mathbf{o} = \mathbf{o} \cup \{k + 1\}$  and continue iteration from **Step 2**.

---

**Algorithm 1.** C&CG algorithm for solving the two-stage location-allocation model**Numerical study and analysis**

In this section, we verify the availability of the proposed model and the effectiveness of the C&CG algorithm through a practical case, and gain some key management insights based on numerical results. In addition, all numerical experiments are conducted on a PC with a 3.0 GHz AMD Ryzen 54600 H CPU and 16 GB RAM.

**Instance set**

We utilize an instance to investigate the ESF location and emergency supplies allocation in Wuhan, Hubei province during the COVID-19 epidemic, and the relevant data comes from Yang et al.<sup>59</sup>. The whole city is divided into 13 demand points of control areas in conformity with the division of large-scale areas. A total of 10 facilities, including large open spaces, stadiums, and convention centers are listed as candidate ESF to deliver emergency supplies for 13 demand points. The distribution of demand points and candidate ESF in Wuhan is presented in Fig. 3, which can be made on <https://dingtuyi.com>. Specifically, the red triangle represents the demand point and the blue circle represents candidate ESF.

**The impact of uncertainty budget on ESF location-allocation scheme**

The introduction of demand uncertainty set increases the complexity of the proposed two-stage robust optimization model, which makes it a significant challenge to find the optimal solution. Therefore, in order to explore the specific influence of demand uncertainty set on location allocation scheme, we designed a series of numerical experiments under the uncertainty budget constraint  $0 \leq \sum_{i \in I} z_i \leq \Gamma$  obtained from historical data, and systematically studied the influence of the increase of  $\Gamma$  on the ESF location-allocation scheme and the performance of the C&CG algorithm. The results are shown in Table 3.

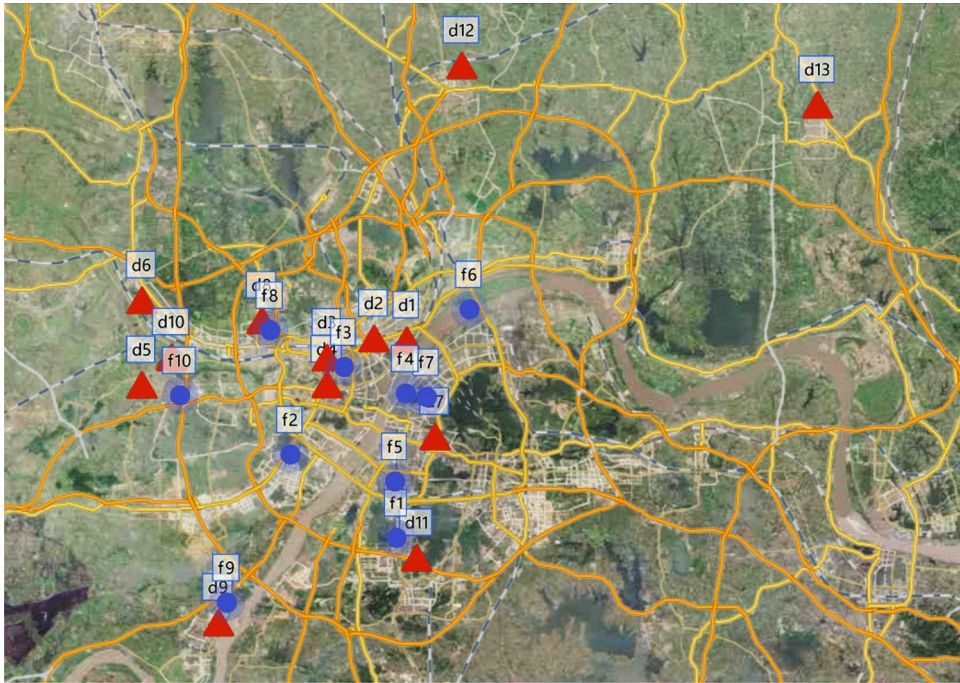


Fig. 3. The nodes distribution of the Wuhan case.

$\Gamma$	CPU time(s)	Iter.	Gap	Facility location strategy	Demand satisfaction rate
1	3.10	2	0.00	[1,2,3,5,6,7]	6 99.99%
2	3.09	2	0.00	[1,2,4,6,7,9]	6 99.58%
3	3.12	2	0.00	[1,3,4,5,8,9,10]	7 100.00%
4	3.09	2	0.00	[1,3,4,5,8,9,10]	7 100.00%
5	3.12	2	0.00	[1,3,4,5,8,9,10]	7 100.00%
6	3.12	2	0.00	[1,3,4,5,8,9,10]	7 100.00%
7	3.11	2	0.00	[1,3,4,5,8,9,10]	7 100.00%
8	3.21	2	0.00	[1,3,4,5,7,8,10]	7 100.00%
9	3.18	2	0.00	[3,4,6,7,8,9,10]	7 100.00%
10	3.11	2	0.00	[1,3,4,6,7,9,10]	7 100.00%
11	3.42	2	0.00	[2,3,4,5,6,7,8]	7 100.00%
12	3.15	2	0.00	[1,2,3,4,6,7,10]	7 100.00%
13	3.16	2	0.00	[1,2,3,4,6,7,9]	7 99.67%

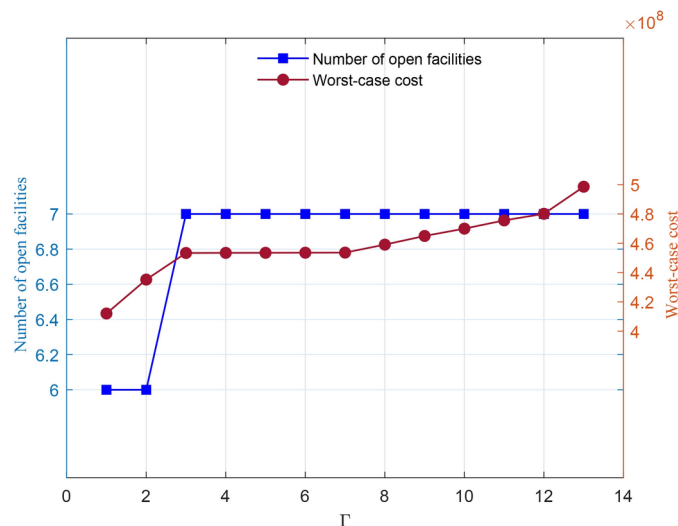
Table 3. The impact of uncertainty budget on the ESF location-allocation scheme.

Table 3 illustrates that when  $\Gamma$  increases from 1 to 13, the CPU time experiences a slight augmentation. Notably, all scenarios can be optimally resolved, with demand satisfaction exceeding 99%. Furthermore, the total number of opening ESF rises from 6 to 7. This phenomenon can be attributed to the fact that when  $\Gamma$  takes a larger value, the total demand increases, necessitating opening additional ESF to mitigate demand uncertainty. However, due to the objective function of the second stage in the model involving the coupling of transportation cost, environmental impact cost and deprivation cost, which comprehensively affects the ESF location decisions. As a result, there are a few scenarios where demand are not entirely met, as it proves more economical to merely penalize unmet demand.

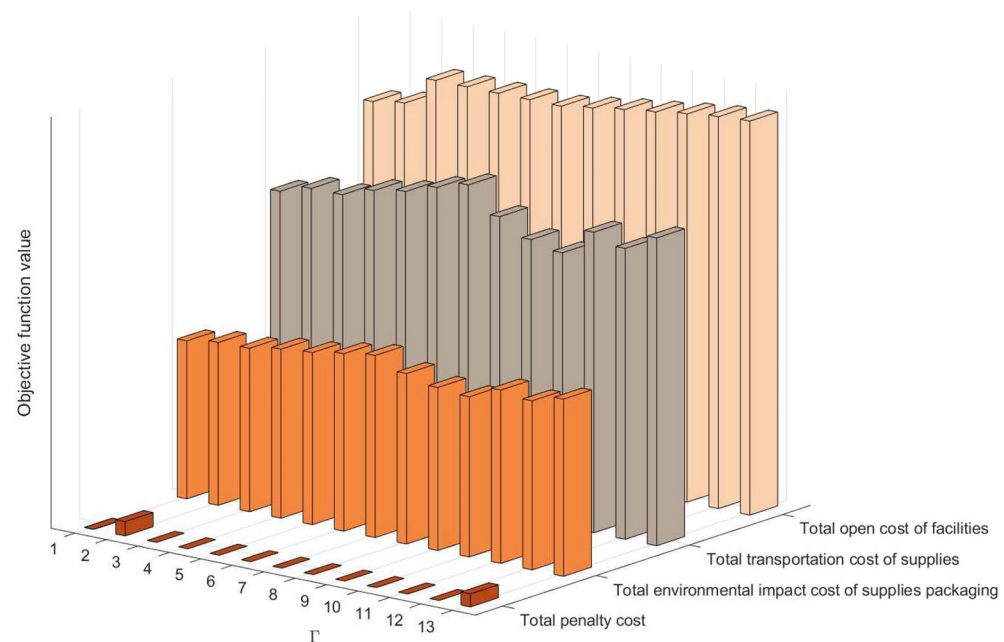
The impact of uncertainty budget on the solutions

In this section, considering that demand is a key factor affecting the solution results of the model, we conducted the experiments to analyze the impact of increasing demand uncertainty on the optimal location decision and the cost function values by adjusting the uncertainty budget  $\Gamma$ . Experiments are conducted by setting  $\Gamma \in [1, 13]$  and the results are shown in Figs. 4, 5 respectively.

Figure 4 shows the trend of the optimal location decision and the total cost function value as uncertainty budget  $\Gamma$  gradually increases. The location decision aims at effectively responding to the demand at demand



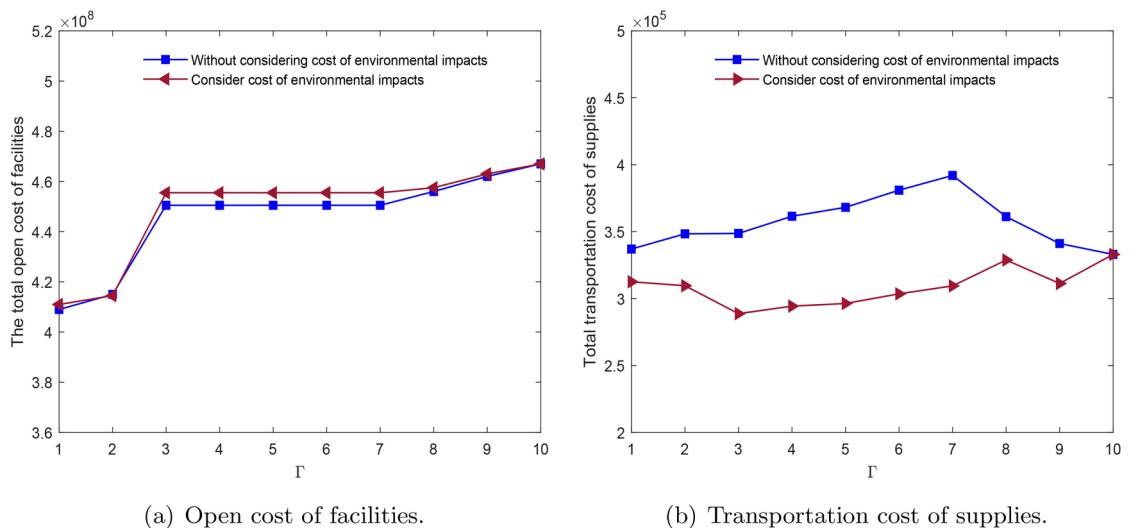
**Fig. 4.** The impact of uncertainty budget  $\Gamma$  on optimal solutions.



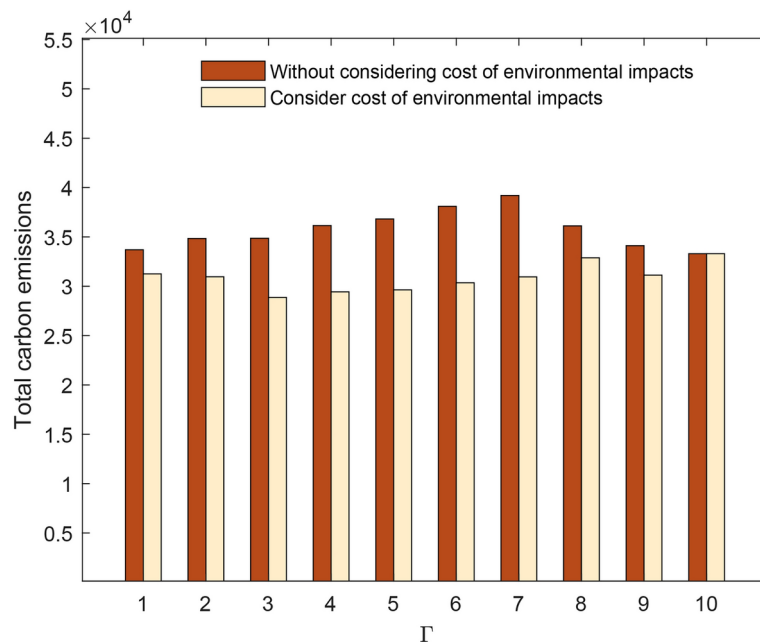
**Fig. 5.** The impact of uncertainty budget  $\Gamma$  on the cost functions.

points, which is inherently related to the total cost function value. Generally, with the increase in the number of ESF, the total cost function value exhibits an upward trend. As shown in Fig. 4, when the  $\Gamma$  is increased from 2 to 3, a new ESF is opened and incorporated into the original location decision, resulting in a significant increase in the total cost function value. However, for  $\Gamma \geq 3$ , the number of ESF remains constant, and the total cost function value increases only marginally. This phenomenon can be attributed to the fact that, during the initial phase of demand fluctuation, the original location decision struggles to fully meet the escalating demand, prompting the system to increase the number of ESF to deal with this situation. Nevertheless, as  $\Gamma$  further increases, the total demand increases slightly from the original, and the existing ESF are sufficient to meet the rising demand. Therefore, the system does not open more ESF, but improves the allocation scheme to achieve the balance between the various cost functions.

Figure 5 shows the dynamic evolution of various cost function values in the model in detail, including open facility cost, transportation cost, environmental impact cost and deprivation cost. Obviously, it can be observed that the cost of opening ESF remains stable when  $\Gamma \geq 3$ . Meanwhile, the transportation cost shows a continuous fluctuation trend, which further proves that when  $\Gamma \geq 3$ , the system can cope with the slight rising demand by fine adjustment and optimization of transportation strategy. In addition, it is worth noting that the



**Fig. 6.** The impact of the introduction of environmental impact cost on the solution.



**Fig. 7.** The impact of the introduction of environmental impact cost on the carbon emission.

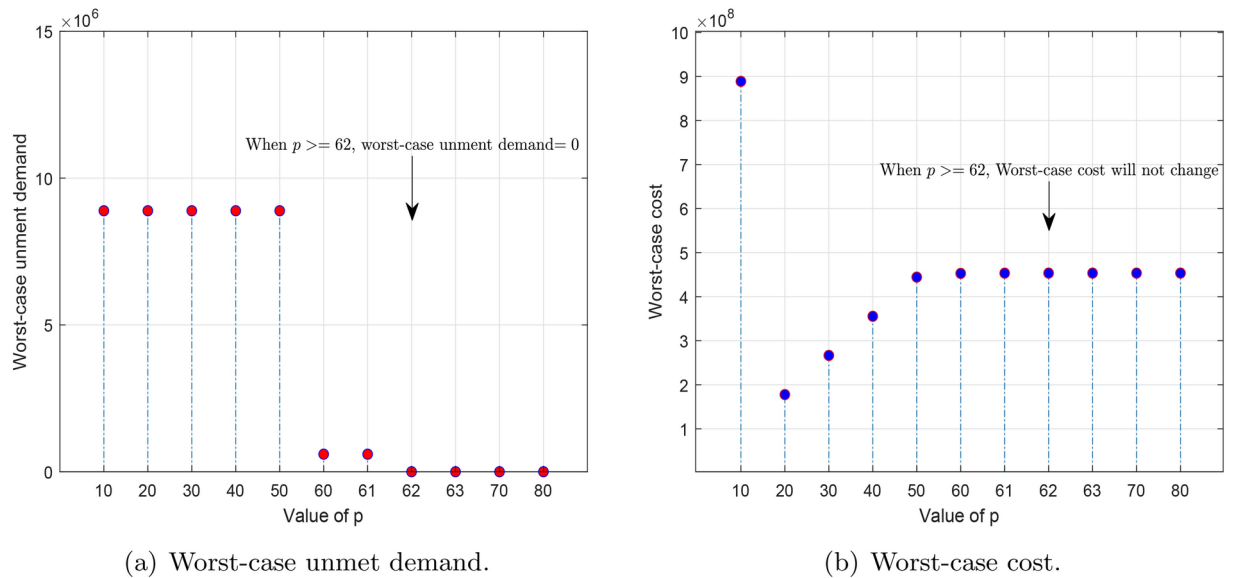
variation trend in environmental impact cost and transportation cost are highly consistent. The reason is that the environmental impact cost includes the social cost caused by carbon emissions during transportation and the cost of harmless treatment of waste materials packaging, both of which are directly affected by the quantity of transported materials and transportation distance, so their variation trends are obviously synchronous.

### The impact of the environmental impact cost on the solutions

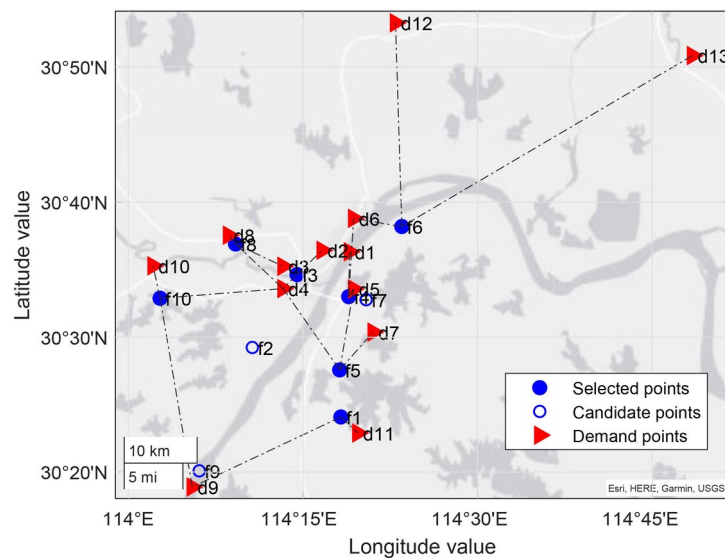
In order to illustrate the impact of the introduction of environmental impact cost on the solution results of the model, we compare the solution results of the proposed model with the model excluding environmental impact cost, as shown in Figs. 6, 7.

Figure 6 shows the difference between the proposed model in this paper and the model without considering the environmental impact cost. As can be seen from Fig. 6, compared with the model without considering the environmental impact cost, the proposed model will not have a significant impact on the location decision, but there are differences in the material allocation scheme. Furthermore, Fig. 7 deeply compares the carbon emissions generated by the two models in the process of material transportation. Obviously, the proposed model has significant advantages in reducing the carbon emissions. This result shows that in emergency management,





**Fig. 8.** Sensitivity analysis of unit deprivation cost  $p$ .



**Fig. 9.** Location-allocation scheme.

although the urgency of emergencies often makes environment sustainability temporarily ignored, the consideration of environmental impact cost will not greatly change the location decision, but can effectively reduce the adverse impact on the environment. Therefore, due to the durability of ESF, it is of great practical value to include the consideration of environmental impact cost in location strategy in the long term.

### The sensitivity analysis of unit deprivation cost

The selection of unit deprivation cost for unmet demand has great influence on ESF location-allocation scheme. Setting a higher unit deprivation cost is conducive to increasing the cost of opening ESF to reduce the unmet demand, but a lower unit deprivation cost results in insufficient ESF and pre-deployed emergency supplies, which may lead to the inability to transport emergency supplies to the demand point. Figure 8 shows the sensitivity analysis of the unit deprivation cost  $p$ .

The results presented in Fig. 8 indicate that when the unit deprivation cost  $p$  is less than 50, the unmet demand remains high and the total cost of the system changes dramatically. This phenomenon arises due to the  $p$  being set too small, resulting in the deprivation cost of increased unmet demand being more economical than the expense of opening additional ESF. Consequently, The total cost increases with the increase of  $p$ . When the  $p$  exceeds 60, the new optimal solution will not be significantly different even if the  $p$  further increases. Therefore, in order to meet the demand as much as possible, it is necessary to set the  $p$  to a value greater than 60.

Selected ESF	Covered demand points
1	9, 11
3	2
4	1, 6
5	4, 5, 7
6	6, 12, 13
8	3, 8
10	4, 9, 10

**Table 4.** Location-allocation scheme.

$ I  =  J $	$\Gamma=2$			$\Gamma=4$			$\Gamma=6$			$\Gamma=8$		
	CPU	Gap	#Iter	CPU	Gap	#Iter	CPU	Gap	#Iter	CPU	Gap	#Iter
10	4.86	0.00	5	6.85	0.00	7	5.68	0.00	6	1.73	0.00	2
15	5.93	0.00	5	11.59	0.00	8	18.03	0.00	11	4.56	0.09	4
20	9.51	0.00	6	25.69	0.00	11	17.80	0.00	8	9.50	0.04	6
25	9.93	0.00	5	9.72	0.00	5	24.27	0.05	8	9.57	0.36	5
30	16.20	0.00	6	8.47	0.00	4	11.36	0.09	5	13.06	0.07	5
35	42.56	0.00	7	35.36	0.01	6	44.84	0.02	7	33.84	0.08	6
40	62.37	0.00	11	65.88	0.08	12	54.65	0.21	11	55.22	0.20	9

**Table 5.** The performance analysis of C&CG algorithm.

Due to the limitation of space, we exemplify the ESF location-allocation scheme for the scenario with  $\Gamma = 6$  in Fig. 9 and Table 4. According to the location-allocation scheme in Fig. 9, seven candidate ESF are opened, including candidate ESF 1, candidate ESF 3, candidate ESF 4, candidate ESF 5, candidate ESF 6, candidate ESF 8 and candidate ESF 10. Demand points 9 and 11 are assigned to candidate ESF 1, demand point 2 to candidate ESF 3, demand points 1 and 6 to candidate ESF 4, demand points 4, 5 and 7 to candidate ESF 5, demand points 6, 12 and 13 to candidate ESF 6, demand point 3 and 8 to candidate ESF 8 and demand points 4, 9 and 10 to candidate ESF 10. We summarize this result in Table 4.

**The performance analysis of the C&CG algorithm**

In order to verify the solution effect of C&CG algorithm for the proposed two-stage robust optimization model, we compare the performance of the algorithm with examples of different scales (each group of examples is randomly selected from 49 nodes in the United States). Table 5 shows the CPU time, iterations and tolerance of C&CG algorithm under different scale examples.

As shown in Table 5, with the gradual increase of the scale of the problem, the CPU time of C&CG algorithm shows an increasing trend. This phenomenon can be attributed to the increase in the number of demand points, which leads to a significant increase in the complexity of demand uncertainty set, and then the constraints to be met in the model increase, thus prolonging the time required for the solution process. However, it is worth noting that the C&CG algorithm can converge in relatively few iterations, which fully demonstrates its efficiency in solving the proposed model. Therefore, the C&CG algorithm has obvious applicability and advantages for the proposed model.

**Conclusions and policy suggestions**

In this paper, we propose a two-stage robust optimization model for the ESF location-allocation problem considering demand uncertainty and sustainability, aiming to provide effective support to victims after emergencies. In the process of model construction, we incorporate correlated demand uncertainty and precisely quantify it through developing a generalized budget uncertainty set. To address the computational challenges associated with solving the proposed model, we derive its equivalent form based on KKT conditions and linearization techniques, and solve it by C&CG algorithm successfully. Finally, with a case study on Wuhan, Hubei province, China, through in-depth numerical experiments and sensitivity analysis of related parameters, we get the optimization framework of ESF location-allocation problem under demand uncertainty. The main conclusions of this study and the corresponding policy suggestions are as follows:

- (1) This study constructs a generalized budget uncertainty set framework, which allows for accurately deriving corresponding demand uncertainty set based on the diverse characteristics of emergencies. This framework not only contributes to the quantification of the specific impacts of demand uncertainty on ESF location and emergency supplies allocation decisions, but also provides a solid theoretical and empirical basis for scientific formulation and implementation of efficient emergency management strategies. Therefore, decision makers should consider customizing uncertainty sets to simulate potential uncertainties, so as to obtain highly robust solutions.

- (2) The sensitivity analysis of uncertainty budget parameter reveals that with the increase of demand uncertainty, more ESF need to be deployed to effectively hedge the potential risk of supplies shortage. This underscores the importance of thoroughly considering and accurately quantifying demand uncertainty during ESF planning to address the diverse situations that may arise during the response phase. Therefore, decision makers need to implement the following strategies: firstly, moderately expanding the number of ESF; and secondly, enhancing facility flexibility. These measures will enhance the effectiveness of emergency response, ensuring timely and adequate provision of necessary services under demand uncertainty.
- (3) By comparing the solutions of the baseline model (without considering environmental impact cost) and the proposed model (incorporating environmental impact cost), we found that the inclusion of environmental impact cost will not significantly affect ESF location decisions, but also effectively reduce carbon emissions. Therefore, decision makers can consider integrating environmental impact cost into the process of ESF location and emergency supplies allocation. By optimizing allocation strategies, we can reduce carbon emissions and promote sustainable development in emergency management.
- (4) Through the sensitivity analysis of the unit deprivation cost of unmet demand, we can accurately determine optimal parameter ranges and their influence on optimal solutions. This guides the optimization of ESF layouts and emergency supply allocation, ensuring dual economic and environmental benefits while meeting affected people's needs. Decision makers should establish and refine the parameter optimization mechanism, leveraging sensitivity analysis to accurately define the range of key parameters, and regularly adjust them to ensure that these parameter settings can reflect the actual needs of current emergency management. In summary, the proposed model offers a comprehensive and sustainable perspective to the ESF location-allocation problem under demand uncertainty. In terms of future research, we can consider the comprehensive influence of more uncertainties such as facility disruption on location decisions.

## Data availability

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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## Declarations

## Competing interests

The authors declare no competing interests.

## Ethical approval

Not applicable.

## Additional information

**Correspondence** and requests for materials should be addressed to D.Y.

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