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Symmetries of Weyl superconductors with different pairings

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We examine the Bogoliubov–de Gennes Hamiltonian and its symmetries for a time-reversal symmetry broken three dimensional Weyl superconductor. In the limit of vanishing pairing potential, we specify that this Hamiltonian is invariant under two sets of continues symmetries, i.e. the *U*(1) gauge symmetry and the $U(1)_A$ axial symmetry. Although a pairing of the Bardeen–Cooper–Schrieffer type spontaneously breaks both of these symmetries, we show that a Fulde–Ferrell–Larkin–Ovchinnikov type pairing spontaneously breaks only the *U*(1) gauge symmetry (that is then restored via the well-known scalar phase mode of superconductivity). Consequently, in the former case, two Nambu–Goldstone modes are required in the system to restore the broken symmetries. We indicate that one of these two modes is an emergent pseudo-scalar phase mode. We also demonstrate that such a phase mode leads to a pseudo-Meissner effect.

A few decades ago, Nambu[1–3] suggested that, in the Bardeen–Cooper–Schrieffer (BCS) formalism of superconductivity[4], a massless scalar collective mode (later called the Nambu–Goldstone (NG) mode) should appear to recover the charge conservation. That is because, although the basic Hamiltonian of the theory, which considers only the interactions of the electric charges, is invariant under a *local* continuous *U*(1) gauge symmetry, the mean-field reduced BCS Hamiltonian is not. This is an example of the spontaneous symmetry breaking (SSB) and dynamical gap generating that reflects the non-conservation of the electric charge. In other words, each of the quasi particles introduced by Bogoliubov[5] and Valatin[6], which are the building blocks of the Cooper pairs, does not appear to have a definite charge. Indeed, a theory with broken gauge symmetry cannot describe processes including the electromagnetic field (like the Meissner effect[7]), and as Nambu observed, the SSB of the gauge symmetry needs to be restored by an NG mode. To restore the symmetry, one has to take into account the radiative corrections coming from the NG mode to the vertex diagram. After considering this contribution, the modified vertex[8] (or the *dressed* vertex) satisfies the Ward identity, and thus the symmetry is restored via a scalar NG mode. Such a massless NG mode (or phase mode) can also be absorbed into the longitudinal component of electromagnetic fields, and gets elevated to the plasma frequency due to the Anderson-Higgs mechanism[9–11], which leads to the Meissner effect.

Moreover, the similarity of the Bogoliubov–Valentin equation to the Dirac equation[12] led Nambu and Jona–Lasinio (NJL) to transform the BCS theory to strong interaction physics[13,14], wherein the global $U(1)_A$ symmetry [The subscript *A* stands for axial symmetry.] as an approximately conserved global symmetry in flavor space is spontaneously broken. Accordingly, the nucleon mass is generated by a SSB of the $U(1)_A$ symmetry, and the produced pion is the pseudo-scalar NG boson of this symmetry breaking[15,16].

On the other hand, the correspondence between high-energy and condensed matter physics has recently reached new levels in the realm of novel quantum materials with the introduction of concepts such as the Dirac and Weyl materials[17,18], which are commonly used to describe elementary particles[19]. The Weyl semimetals (WSMs) were first proposed in the pyrochlore irridates[20] and later in the heterostructures of topological and normal insulators[21]. These objects are very peculiar in the sense that their valences and conduction bands have non-degenerate touching points in the Brillouin zone (called Weyl nodes) whose low-energy excitations obey the Weyl equation and are chiral fermions. It is well-known that the Weyl nodes come in pairs of opposite chirality[22] and are protected via time-reversal (TR) and/or inversion (IR) symmetry[23] while are separated by a constant vector in momentum space. Chirality is thus a defining emergent property of electrons in WSMs.

Furthermore, the experimental observations and theoretical analysis suggest a superconducting phase in WSMs, like MoTe2[24,25] and TaP[26]. In addition to intrinsic superconductivity, WSMs can also become superconductors via the proximity effect, which occurs when one brings a WSM close to a superconductor.

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It is worth mentioning that despite the 'chirality blockade' in a magnetic WSM, where the Andreev reflection between the normal state of the WSM and a superconductor is suppressed, introducing a Zeeman field at the interface provides the necessary chirality switch. Indeed, such action overcomes the blockade and enables the activation of the Andreev reflection, leading to the proximity induced superconductivity in WSMs[27]. Also, the surface states in WSMs, known as Fermi arc states, violate the chirality blockade. The reason lies in the fact that Fermi arcs are not part of the Weyl spectrum[28]. Moreover, it has been reported that with a suitable choice of the Fermi cutoff, the Fermi arcs do not affect the paring amplitude in WSMs[29]. Furthermore, the classification of the induced pairing and the magnitude of each pair amplitude has been evaluated in the chirality blockade regime for both even- and odd-frequency pairing[29]. Considering the even-frequency spin-singlet pairing, the amplitude of the interorbital *s*-wave pairing has been shown to be 2-orders of magnitude larger than the intraorbital *s*-wave.

Accordingly, the Weyl equation is affected due to the presence of superconducting term in the corresponding Hamiltonian. In this regard, for instance, when a conventional BCS superconductor is placed next to a WSM, an *s*-wave superconductivity is induced in that WSM. The resulting Hamiltonian, namely the Bogoliubov–de Gennes (BdG) Hamiltonian[30,31], includes a pairing term originated from the SSB. However, the nature of such a pairing is under debate[32], as there are generally two candidates to explain this situation. One case is the *s*-wave Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) or intranode pairing, in which electrons are paired up on the same side of the Fermi surface. Another is the conventional BCS or internode pairing, in which the pair consists of electrons on opposite sides of the Fermi surface, and the center of mass of Cooper pairs has zero momentum. Despite some efforts, there are still debates about the preferred pairing state in WSMs. For instance, Cho et al. argued that the FFLO state in the IR-symmetric WSMs has lower-energy compared to the conventional superconducting state via the mean-field calculations[33]. Whereas, other groups have demonstrated that the energy of the BCS state is lower compared to the FFLO state[34,35]. Both types of pairing are allowed when the TR-symmetry is broken but the IR-symmetry is preserved, whilst the FFLO state is the only allowed pairing term when both of the IR-symmetry and TR-symmetry are broken[23]. It is worthwhile to mention that, a minimal model of TR broken WSMs consists of a single pair of the Weyl nodes, while a WSM with broken IR contains four Weyl nodes with total zero chirality[36]. It has already been observed that in a WSM, the chirality can be understood as a topologically protected charge[37–40], and in a 3-dimensional Weyl superconductor (3DWS), the chiral symmetry breaking occurs in addition to gauge symmetry breaking[41]. Indeed, in Ref.[40], the authors have considered the anomalous Hall effect in a topological Weyl superconductor with the broken TR-symmetry and demonstrated the existence of a conserved chiral charge in WSMs. Also, topological superconductivity in WSMs can lead to anomalies in the presence of chiral vortex lines[42].

In this work , we aim to provide a physical insight on the nature of superconductivity in WSMs by focusing on symmetry considerations. Thus, we promote the idea that in a 3DWS, besides the $U(1)$ gauge symmetry, an emergent low-energy $U(1)_A$ symmetry exists, which leads to a new charge for the system, namely a chiral charge. To make this idea clearer, we study the BdG Hamiltonian using a doubled representation of Dirac matrices. Next, we investigate the model when the BCS- and/or the FFLO-type pairings are proximity induced in WSMs. Then, we demonstrate that an emergent pseudo-scalar phase mode appears in WSMs with BCS-type superconductivity, while the FFLO-type lakes this phase mode. In addition, we demonstrate that such a phase mode leads to a pseudo-Meissner effect. The outline of this work is as follows. In Sect. "Chiral invariance and new pseudo-scalar NG mode", we briefly review the SSB in the NJL model, and explain how an NG mode appears to recover a continuous global symmetry. In Sect. "Weyl superconductors and chiral symmetry", we introduce the BdG Hamiltonian for a 3DWS and clearly show that the continuous gauge and axial symmetries are spontaneously broken via the induced pairings. Accordingly, we indicate that, as a result of the SSB of the $U(1)_A$, an emergent pseudo-scalar phase NG mode appears. In Sect. "Pseudo-Meissner effect", we review the Higgs mechanism and argue that the interaction of this pseudo-scalar phase mode with an external pseudo-magnetic can lead to a pseudo-Meissner effect. Finally, we furnish the conclusion in the last section.

Chiral invariance and new pseudo-scalar NG mode

In this section, we review how the the spontaneously broken chiral symmetry in Dirac Lagrangian can be restored by introducing a pseudo-scalar NG mode. The Dirac Lagrangian for a free electron with mass m and momentum p is

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi, \tag{1}$$

where $\bar{\psi} = \psi^\dagger \gamma_0$, the natural units with $\hbar = 1 = c$ is assumed [However, in condensed matter systems, the Dirac Hamiltonian contains the Fermi velocity v_{F} instead of the speed of light.] and the γ_μ's are the Dirac gamma matrices that, in the Weyl or chiral representation, are defined as

$$\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \qquad \text{and} \qquad \gamma_0 = \begin{pmatrix} 0 & \mathbb{I}_{2\times 2} \\ \mathbb{I}_{2\times 2} & 0 \end{pmatrix}. \tag{2}$$

Here, σ_i (where i=1, 2, 3) and $\mathbb{I}_{2\times 2}$ respectively are the Pauli matrices and the unit matrix, and the symbol 0 represents a 2×2 matrix. The γ_5 matrix, that is constructed as $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, is

$$\gamma_5 = \begin{pmatrix} -\mathbb{I}_{2\times 2} & 0 \\ 0 & \mathbb{I}_{2\times 2} \end{pmatrix}. \tag{3}$$

Also, one can construct the charge-conjugation operator $\mathscr{C} = i\gamma_2 K$, with K being the complex-conjugate operator, as

$$\mathscr{C} = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} K, \tag{4}$$

which transforms a particle to an anti-particle, namely $\mathscr{C}\psi \to \psi^{\mathrm{c}}$. Moreover, the γ_μ matrices satisfy

$$\{\gamma_\mu, \gamma_\nu\} \equiv \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\,\eta_{\mu\nu}\mathbb{I}_{4\times 4}, \tag{5}$$

where $\eta_{\mu\nu}$ (with $\mu, \nu = 0, \cdots, 3$) is the Minkowski metric in $(1+3)$ dimensions with the signature -2.

The equation of motion of Lagrangian (1) is the celebrated Dirac Eq.[12]

$$i\,\partial_t\psi = \gamma_0(\boldsymbol{\gamma}\cdot p + m)\psi. \tag{6}$$

By defining the left-handed and right-handed projection operators $\psi_{\mathrm{R}} = (1+\gamma_5)\psi/2$ and $\psi_{\mathrm{L}} = (1-\gamma_5)\psi/2$, the Dirac equation becomes

$$\boldsymbol{\sigma}\cdot p\,\psi_{\mathrm{R}} + m\,\psi_{\mathrm{L}} = \varepsilon\psi_{\mathrm{R}} \qquad \text{and} \qquad -\boldsymbol{\sigma}\cdot p\,\psi_{\mathrm{L}} + m\,\psi_{\mathrm{R}} = \varepsilon\psi_{\mathrm{L}}, \tag{7}$$

with the eigenvalues $\varepsilon = \pm\sqrt{p^2+m^2}$. Obviously when $m = 0$, the right and left chirality sectors are decoupled and the Dirac equation reduces to the Weyl Eq.[19]. In this limit, Lagrangian (1) is invariant against the set of two independent continuous global transformations

$$\psi \to \exp(i\alpha)\psi \qquad \text{hence:} \qquad \bar{\psi} \to \bar{\psi}\exp(-i\alpha), \tag{8}$$

$$\psi \to \exp(i\gamma_5\beta)\psi \qquad \text{hence:} \qquad \bar{\psi} \to \bar{\psi}\exp(i\gamma_5\beta), \tag{9}$$

with α and β as arbitrary constants. Due to the Noether theorem, there are two conserved currents, namely

$$j_\mu = \bar{\psi}\gamma_\mu\psi \qquad \text{and} \qquad j_{5,\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi \tag{10}$$

as the vector and axial vector currents, respectively, which satisfy the continuity equations

$$\partial^\mu j_\mu = 0 \qquad \text{and} \qquad \partial^\mu j_{5,\mu} = 0. \tag{11}$$

This fact corresponds to the conservation of the electron number and the chiral or γ_5 charge, respectively.

When the fermion mass term is generated dynamically, for example in the context of the NJL model [This relation is referred to as the gap equation of the NJL model[13,14].], i.e. $m_{\mathrm{f}} \propto \langle\bar{\psi}\psi\rangle$ in modified Lagrangian (1), the conservation relations (11) become

$$\partial^\mu j_\mu = 0 \qquad \text{and} \qquad \partial^\mu j_{5,\mu} = 2i\,m_{\mathrm{f}}\bar{\psi}\gamma_5\psi, \tag{12}$$

which means that the mass term has spoiled the axial or the γ_5 symmetry. Nevertheless, by including the radiative corrections, the vertex function for the axial vector current is not simply given by $\gamma_\mu\gamma_5$, instead by[13,14]

$$\lambda_{5,\mu}(p', p) = \gamma_\mu\gamma_5 - i\frac{2m_{\mathrm{f}}\gamma_5 q_\mu}{q^2}, \tag{13}$$

where p and p' are the initial and final momenta and $q = p' - p$. Therefore, the vertex function now includes an extra term, which indicates the existence of a pseudo-scalar zero-mass state. Hence, by redefining $j^\lambda_{5,\mu} = \bar{\psi}\lambda_{5,\mu}\psi$, we obtain

$$\partial^\mu j^\lambda_{5,\mu} = 0,, \tag{14}$$

which means the chiral symmetry is restored by introducing a pseudo-scalar NG mode.

Weyl superconductors and chiral symmetry

After reviewing the concept of SSB of the $U(1)_A$ symmetry in the context of Dirac equation, we now investigate the relevant symmetries for WSMs after they become 3DWS as a consequence of the SSB. In this regard, we consider a minimal relativistic low-energy model of a WSM with two Weyl nodes of opposite chirality separated by $2\,p_0$ in the momentum space. The nonzero chiral shift p_0 breaks the TR-symmetry in such a model. The Weyl Hamiltonian around the Weyl nodes at momentum $\pm p_0$ (in unit $v_{\mathrm{F}} = 1$) reads

$$\mathcal{H}_{\mathrm{W}}(p) = \begin{pmatrix} H^{\mathrm{W}}_{+} & 0 \\ 0 & H^{\mathrm{W}}_{-} \end{pmatrix} \qquad \text{with} \qquad H^{\mathrm{W}}_{\pm} = \pm\boldsymbol{\sigma}\cdot(p \mp p_0)\,, \tag{15}$$

where p is the momentum of excitations and ± stands for two chiralities of the Weyl nodes. With a proper rotation, the term containing p_0 can be gauged away from this Hamiltonian. In the presence of an external electromagnetic field, such a rotation leads to an induced θ-term[43] in the corresponding action of the field, which is called the axionic field. Such a term is responsible for the anomalous Hall effect and the chiral magnetic effect[44], in which the number of particles of a specific chirality, in the presence of a topologically nontrivial configuration of the background gauge field (like electromagnetic field), is not conserved. This effect is the condensed matter counterpart of the chiral anomaly in high-energy physics. It is also interesting to know that the chiral anomaly has been experimentally observed in the chiral superfluid $3He - A$ with Weyl fermionic quasiparticles[45].

The pairing Hamiltonian for such a system is

$$\mathcal{H}_{\rm BdG}(p) = \begin{pmatrix} \mathcal{H}_{\rm W}(p) - \mu & \Delta \\ \Delta^\dagger & \mu - \mathscr{C}\mathcal{H}_{\rm W}(p)\mathscr{C}^{-1} \end{pmatrix}, \tag{16}$$

with Δ being the superconducting order parameter and μ the electrochemical potential. However, the chemical potential in the Hamiltonian acts as a shift in energy and adjusting its value leads to a quantitative change in the Fermi level of the system, and does not affect the output results considered in this research. Moreover, it is common to consider an undoped WSM corresponding to zero chemical potential, see, e.g., Ref.[29]. Hence, we consider $\mu = 0$ in this work. In the literature, the two chirality sectors have been dealt with separately, see, e.g., Ref.[23]. However, for convenience, we prefer to mix the two chiralities of the Weyl nodes to construct the following Hamiltonians for the internode and intranode pairings as

$$\mathscr{H}_{\rm BCS} = \begin{pmatrix} \boldsymbol{\sigma}\cdot & 0 & \hat{\triangle}_{\rm B} & 0 \\ 0 & -\boldsymbol{\sigma}\cdot & 0 & \hat{\triangle}_{\rm B} \\ \hat{\triangle}_{\rm B}^\dagger & 0 & -\boldsymbol{\sigma}\cdot & 0 \\ 0 & \hat{\triangle}_{\rm B}^\dagger & 0 & \boldsymbol{\sigma}\cdot \end{pmatrix} \quad \text{and} \quad \mathscr{H}_{\rm FFLO} = \begin{pmatrix} \boldsymbol{\sigma}\cdot & 0 & 0 & \hat{\triangle}_{\rm F} \\ 0 & -\boldsymbol{\sigma}\cdot & \hat{\triangle}_{\rm F} & 0 \\ 0 & \hat{\triangle}_{\rm F}^\dagger & -\boldsymbol{\sigma}\cdot & 0 \\ \hat{\triangle}_{\rm F}^\dagger & 0 & 0 & \boldsymbol{\sigma}\cdot \end{pmatrix}, \tag{17}$$

where

$$\hat{\triangle}_{\rm B} \equiv \begin{pmatrix} \triangle_{\rm B} & 0 \\ 0 & \triangle_{\rm B} \end{pmatrix} \quad \text{and} \quad \hat{\triangle}_{\rm F} \equiv \begin{pmatrix} \triangle_{\rm F} & 0 \\ 0 & \triangle_{\rm F} \end{pmatrix}, \tag{18}$$

with scalars $\triangle_{\rm B}$ and $\triangle_{\rm F}$ that represent the *s*-wave BCS-like and the FFLO-like pairings, respectively. These Hamiltonians act in the space of the Nambu spinors

$$\Phi = \begin{pmatrix} \Psi \\ \Psi^{\rm c} \end{pmatrix}, \tag{19}$$

where Φ is a generic solution,

$$\Psi = \left(\psi_-^\uparrow, \psi_-^\downarrow, \psi_+^\uparrow, \psi_+^\downarrow\right)^{\rm T}, \tag{20}$$

and the charge conjugation operator is defined in (4).

As stated, Hamiltonians (17) have the advantage of accommodating both chiralities in a doubled representation. However, we intend to write these Hamiltonians in a covariant manner. Hence, for this purpose, we define the set of matrices [Similar set of matrices has also been defined in Refs.[46,47].]

$$\Gamma_0 \equiv \begin{pmatrix} -\gamma_0 & 0_{4\times4} \\ 0_{4\times4} & -\gamma_0 \end{pmatrix}, \ \Gamma_i \equiv \begin{pmatrix} \gamma_i & 0_{4\times4} \\ 0_{4\times4} & -\gamma_i \end{pmatrix}, \ \Gamma_5 \equiv \begin{pmatrix} \gamma_5 & 0_{4\times4} \\ 0_{4\times4} & -\gamma_5 \end{pmatrix}, \ \Gamma_6 \equiv \begin{pmatrix} \mathbb{I}_{4\times4} & 0_{4\times4} \\ 0_{4\times4} & -\mathbb{I}_{4\times4} \end{pmatrix}, \tag{21}$$

where the Γ_μ set of matrices form the basis of the Clifford algebra with

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\,\eta_{\mu\nu}\mathbb{I}_{8\times8}. \tag{22}$$

Accordingly, we obtain $\Gamma_5 = i\Gamma_0\Gamma_1\Gamma_2\Gamma_3$ and

$$[\Gamma_6, \Gamma_a] = 0 \quad \text{for} : a = 0, \cdots, 3, 5, \qquad \{\Gamma_5, \Gamma_\mu\} = 0. \tag{23}$$

To deal with the pairing part of the BdG Hamiltonian, we also define

$$\Gamma_7 \equiv \begin{pmatrix} 0_{4\times4} & \mathbb{I}_{4\times4} \\ \mathbb{I}_{4\times4} & 0_{4\times4} \end{pmatrix} \quad \text{and} \quad \Gamma_8 \equiv \begin{pmatrix} 0_{4\times4} & \gamma_0 \\ \gamma_0 & 0_{4\times4} \end{pmatrix}, \tag{24}$$

whose commutation relations with matrices (21) are

$$\begin{aligned} \{\Gamma_7, \Gamma_a\} &= 0 \quad \text{for} : a = 1, 2, 3, 5, 6, \qquad [\Gamma_7, \Gamma_{0,8}] = 0, \\ &= 0 \quad \text{for} : a = 0, \cdots, 3, 5, \qquad \{\Gamma_8, \Gamma_6\} = 0. \end{aligned} \tag{25}$$

Using these matrices, Hamiltonians (17) can be written as

$$\mathcal{H}_{\rm BCS} = \Gamma_0\Gamma_i p^i - \Gamma_0\Gamma_8\Delta_{\rm B} \qquad \text{and} \qquad \mathcal{H}_{\rm FFLO} = \Gamma_0\Gamma_i p^i - \Gamma_0\Gamma_7\Delta_{\rm F} \tag{26}$$

with

$$\Delta_{\rm B} \equiv \begin{pmatrix} \hat{\Delta}_{\rm B}^{\dagger} & 0 & 0 & 0 \\ 0 & \hat{\Delta}_{\rm B}^{\dagger} & 0 & 0 \\ 0 & 0 & \hat{\Delta}_{\rm B} & 0 \\ 0 & 0 & 0 & \hat{\Delta}_{\rm B} \end{pmatrix} \qquad \text{and} \qquad \Delta_{\rm F} \equiv \begin{pmatrix} \hat{\Delta}_{\rm F}^{\dagger} & 0 & 0 & 0 \\ 0 & \hat{\Delta}_{\rm F}^{\dagger} & 0 & 0 \\ 0 & 0 & \hat{\Delta}_{\rm F} & 0 \\ 0 & 0 & 0 & \hat{\Delta}_{\rm F} \end{pmatrix}. \tag{27}$$

Based on Refs.[48,49], as the FFLO-like superconducting pairing is given by $\bar{\psi}\Delta_{\rm F}\gamma^0\psi_c$ and the BCS-like pairing is given by $\bar{\psi}\Delta_{\rm B}\mathbb{I}_{4\times4}\psi_c$, hence the Lorentz invariance is satisfied.

Now, the Lagrangian of the system can be written as

$$\mathcal{L}_{\rm BCS} = \bar{\Phi}(\Gamma_\mu p^\mu - \Gamma_8\Delta_{\rm B})\Phi \qquad \text{and} \qquad \mathcal{L}_{\rm FFLO} = \bar{\Phi}(\Gamma_\mu p^\mu - \Gamma_7\Delta_{\rm F})\Phi, \tag{28}$$

where $\bar{\Phi} \equiv \Phi^\dagger\Gamma_0$. Hence, the corresponding equations of motions are

$$(\Gamma_\mu p^\mu - \Gamma_8\Delta_{\rm B})\Phi = 0 \qquad \text{and/or} \qquad (\Gamma_\mu p^\mu - \Gamma_7\Delta_{\rm F})\Phi = 0. \tag{29}$$

To get a better sense on the role of Γ_5 and Γ_6, we also define the projection operators

$$P_\pm^5 = \frac{1}{2}(\mathbb{I} \pm \Gamma_5) \qquad \text{and} \qquad P_\pm^6 = \frac{1}{2}(\mathbb{I}\pm\Gamma_6), \tag{30}$$

where $\mathbb{I}$ is 8×8 identity matrix, and we have $P_\pm^{5^2} = P_\pm^5,\ P_\pm^{6^2} = P_\pm^6$. In addition, acting these operators on the Nambu spinors (19) yields

$$P_+^5P_+^6\Phi = \frac{1}{4}(\mathbb{I}+\Gamma_5)(\mathbb{I}+\Gamma_6)\Phi = \Psi_+, \qquad P_+^5P_-^6\Phi = \frac{1}{4}(\mathbb{I}+\Gamma_5)(\mathbb{I}-\Gamma_6)\Phi = \Psi_+^c, \tag{31}$$

$$P_-^5P_+^6\Phi = \frac{1}{4}(\mathbb{I}-\Gamma_5)(\mathbb{I}+\Gamma_6)\Phi = \Psi_-, \qquad P_-^5P_-^6\Phi = \frac{1}{4}(\mathbb{I}-\Gamma_5)(\mathbb{I}-\Gamma_6)\Phi = \Psi_-^c. \tag{32}$$

These relations suggest that the operator Γ_5 is related to the chirality and Γ_6 is related to the particle-hole symmetry.

In the limit $\Delta_{\rm B} = 0 = \Delta_{\rm F}$, Hamiltonians (26) will obviously be the same and will be invariant against the set of two independent continuous global transformations

$$\begin{aligned} &\Phi(\mathrm{r},t) \to e^{i\,\Gamma_5\theta/2}\Phi(\mathrm{r},t) \qquad \text{hence:} \qquad \bar{\Phi}(\mathrm{r},t) \to \bar{\Phi}(\mathrm{r},t)e^{i\,\Gamma_5\theta/2}, \\ &\Phi(\mathrm{r},t) \to e^{i\,\Gamma_6\varphi/2}\Phi(\mathrm{r},t) \qquad \text{hence:} \qquad \bar{\Phi}(\mathrm{r},t) \to \bar{\Phi}(\mathrm{r},t)e^{-i\,\Gamma_6\varphi/2} \end{aligned} \tag{33}$$

that can also be written as

$$\begin{aligned} &\Phi \to \left(\Psi_-e^{-i\,\theta/2}, \Psi_+e^{i\,\theta/2}, \Psi_+^ce^{i\,\theta/2}, \Psi_-^ce^{-i\,\theta/2}\right)^{\rm T}, \\ &\Phi \to \left(\Psi_-e^{i\,\varphi/2}, \Psi_+e^{i\,\varphi/2}, \Psi_+^ce^{-i\,\varphi/2}, \Psi_-^ce^{-i\,\varphi/2}\right)^{\rm T}, \end{aligned} \tag{34}$$

where θ and φ are arbitrary constants. The Noether theorem dictates that such invariance leads to the conserved currents

$$J_{5,\mu} = \bar{\Phi}\Gamma_\mu\Gamma_5\Phi \qquad \text{and} \qquad J_{6,\mu} = \bar{\Phi}\Gamma_\mu\Gamma_6\Phi. \tag{35}$$

These currents satisfy the continuity equations

$$\partial^\mu J_{5,\mu} = 0 \qquad \text{and} \qquad \partial^\mu J_{6,\mu} = 0. \tag{36}$$

The $J_{5,\mu}$ (which originates from Γ_5 that is related to the axial symmetry) is the axial current, and in the same vein, the $J_{6,\mu}$ is the electromagnetic current.

It is noteworthy that the continuous global transformation leading to $J_{5,\mu}$ also gives rise to a θ phase shift between the pairings at Fermi surfaces with opposite chiralities. Such a phase shift may lead to interesting observable effects in the Josephson phenomena[50]. Further discussion of this topic is beyond the scope of this work and will be presented in upcoming works. However, it should be noted that our work differs from the Leggett work[51,52] because for the Leggett mode to appear, it is necessary to have an interband pairing term that couples positive and negative chiralities. Whereas this is not the case in Eqs. (29), where we consider either internode pairing, $\triangle_{\rm B}$, or intranode pairing, $\triangle_{\rm F}$, and do not assume both pairings simultaneously.

Now, by utilizing transformations (33) and commutation relations (25), we observe that $\Delta_{\rm B}$ in the left-hand Hamiltonian (26) breaks both of the Γ_5 and Γ_6 symmetries, while $\Delta_{\rm F}$ in the right-hand Hamiltonian (26) breaks only the Γ_6 symmetry. Accordingly, when only $\Delta_{\rm F}$ is present in the system, we can justify that it transforms as $\Delta_{\rm F} \to \Delta_{\rm F}\, e^{i\,\Gamma_6\varphi}$, and hence the conservation relations (36) become

$$\partial^{\mu} J_{5,\mu} = 0 \qquad \text{and} \qquad \partial^{\mu} J_{6,\mu} = -2i\,\Delta_{\rm F}\bar{\Phi}\Gamma_7\Gamma_6\Phi. \tag{37}$$

whereas, when only $\Delta_{\rm B}$ is present, it transforms as $\Delta_{\rm B} \to \Delta_{\rm B}\, e^{i\,\Gamma_6\varphi + i\,\Gamma_5\theta}$, and one has

$$\partial^{\mu} J_{5,\mu} = -2i\,\Delta_{\rm B}\bar{\Phi}\Gamma_8\Gamma_5\Phi \qquad \text{and} \qquad \partial^{\mu} J_{6,\mu} = -2i\,\Delta_{\rm B}\bar{\Phi}\Gamma_8\Gamma_6\Phi. \tag{38}$$

As is obvious, when $\Delta_{\rm B}$ is non-zero, an emergent pseudo-scalar phase mode should appear to restore the broken Γ_5 symmetry.

Following the same procedure that led to relation (13), when only $\Delta_{\rm F}$ is present, the vertex for $J_{5,\mu}$ and $J_{6,\mu}$ currents become

$$\Lambda^{\rm F}_{5,\mu}(p',p) = \Gamma_\mu\Gamma_5 \qquad \text{and} \qquad \Lambda^{\rm F}_{6,\mu}(p',p) = \Gamma_\mu\Gamma_6 + i\frac{2\Delta_{\rm F}\Gamma_7\Gamma_6 q_\mu}{q^2}, \tag{39}$$

and when only $\Delta_{\rm B}$ is the non-vanishing one, we obtain

$$\Lambda^{\rm B}_{5,\mu}(p',p) = \Gamma_\mu\Gamma_5 + i\frac{2\Delta_{\rm B}\Gamma_8\Gamma_5 q_\mu}{q^2} \qquad \text{and} \qquad \Lambda^{\rm B}_{6,\mu}(p',p) = \Gamma_\mu\Gamma_6 + i\frac{2\Delta_{\rm B}\Gamma_8\Gamma_6 q_\mu}{q^2}. \tag{40}$$

Hence for conditions (40), new conserved currents, say $J^{\rm B}_{5,\mu}$ and $J^{\rm B}_{6,\mu}$, satisfy the conservation relations

$$\partial^{\mu} J^{\rm B}_{5,\mu} \equiv \partial^{\mu}\bar{\Phi}\Lambda^{\rm B}_{5,\mu}\Phi = 0 \qquad \text{and} \qquad \partial^{\mu} J^{\rm B}_{6,\mu} \equiv \partial^{\mu}\bar{\Phi}\Lambda^{\rm B}_{6,\mu}\Phi = 0. \tag{41}$$

Also, for conditions (39), we have

$$\partial^{\mu} J^{\rm F}_{5,\mu} \equiv \partial^{\mu}\bar{\Phi}\Lambda^{\rm F}_{5,\mu}\Phi = 0 \qquad \text{and} \qquad \partial^{\mu} J^{\rm F}_{6,\mu} \equiv \partial^{\mu}\bar{\Phi}\Lambda^{\rm F}_{6,\mu}\Phi = 0. \tag{42}$$

These conservation relations demonstrate how the NG modes restore the broken symmetries. Thus, the FFLO-like pairing only breaks the gauge symmetry. Therefore, only a scalar phase mode appears to restore the conservation of this symmetry. Whereas, the BCS-like pairing breaks both of the gauge and the chiral symmetries, and leads to an extra second NG mode.

To be able to detect this new emergent pseudo-scalar NG mode, we recall that a scalar phase mode can interact with an external photon field, leading to the Meissner effect due to the Anderson-Higgs mechanism[9–11]. Actually, as Weinberg stated "a superconductor is simply a material in which electromagnetic gauge invariance is spontaneously broken[53]". In the same vein, we expect a phenomenon similar to the Meissner effect, in which the pseudo-scalar NG mode (created by the SSB of the chiral symmetry via $\Delta_{\rm B}$) gets absorbed by an external pseudo-magnetic field, leading to the field expulsion, and an effect we refer to as pseudo-Meissner.

Such a pseudo-magnetic field can be realized in WSMs in the form of elastic gauge fields constructed with the deformation tensor by coupling the lattice deformations to electronic degrees of freedom[54]. The most interesting feature of these elastic gauge fields is that they are axial pseudo-gauge fields and couple to opposite chiralities with opposite signs. The presence of elastic gauge fields in WSMs was first predicted in Ref.[55] and recently realized experimentally[56]. Further works on the analysis of their physical consequences can be found in Refs.[57–67]. In general, such an axial gauge field can be described as[55,68]

$$A^5_\mu = \xi v_{\mu\nu}\, b^\nu, \tag{43}$$

where

$$v_{\mu\nu}(x) = \frac{1}{2}(\partial_\mu v_\nu + \partial_\nu v_\mu), \tag{44}$$

is the strain tensor, v^μ is the displacement vector, ξ is a Gruneisen parameter and the vector b^μ quantifies the separation of Weyl nodes in momentum space. The pseudo-vector field $\mathbf{A}^5$ can be attributed to an effective axial vector potential[69,70] . Then the axial magnetic and electric fields can be defined as $B^{5,\mu} = \frac{1}{2}\epsilon^{\mu\nu\gamma}\,\partial_{nu}A^5_\gamma$ and $E^5_\mu = -\partial_t A^5_{mu}$, respectively. Apparently, the physical properties of the lattice deformation, such as ripples or strains, determine the size of the emergent pseudo-field in this system[54]. In principle, one can tune the pseudo-field by controlling the lattice deformation, which was previously studied in Dirac materials theoretically[71] and observed experimentally[72].

The pseudo-magnetic (or an axial magnetic) field B^5 is an observable quantity, whose magnitude typically varies from about $0.3\ \ \mathrm{T}$ to about $15\ \ \mathrm{T}$. It couples to fermions of opposite chirality with different sign and is therefore a chiral field. It also gives rise to an unusual dynamics of Cooper pairs, where no Meissner effect is present, see, e.g., Ref.[73]. In fact, this field is completely different from the usual electromagnetic field and does not induce diamagnetic currents (which can destroy superconductivity)[69,70]. Now, taking into account the possibility that the pseudo-scalar phase mode can be absorbed by the $\mathbf{B}^5$ field, the pseudo-Meissner effect is a plausible consequence of the local chiral symmetry breaking in 3DWS. We investigate this case in the next section.

As another peculiarity of the emergent pseudo-scalar phase mode, we infer that this mode may interact with the axial gauge bosons of the standard model of particle physics[74–76], which also leads to the pseudo-Meissner effect.

Pseudo-Meissner effect

In this section, via the Higgs mechanism, we study the process in which the pseudo-scalar NG mode gets absorbed by the pseudo-magnetic field, and results in the field expulsion. In this regard, we first introduce the Abelian Higgs mechanism, whose Lagrangian is

$$\mathscr{L}_\eta = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(i\partial_\mu - eA_\mu)\eta|^2 - V(\eta), \tag{45}$$

where the field η is a complex classical field over spacetime, e is the unit of the electric charge, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and A^μ is the electromagnetic vector potential. Meanwhile, the potential energy for the field η is

$$V(\eta) = -M^2|\eta|^2 + \frac{1}{2}\lambda|\eta|^4, \tag{46}$$

where M and λ are constants and the Lagrangian $\mathscr{L}_\eta$ remains invariant under the Abelian $U(1)$ gauge transformation $A_\mu \to A_\mu - \partial_\mu\chi(x)/e$ with $\eta \to e^{i\,\chi(x)}\eta$, wherein $\chi(x)$ is a real field without dimension in the natural units.

At this stage, we arrange to have a 'spontaneous breaking of the gauge symmetry' by choosing $<\eta>= v/\sqrt{2}$, where v is the vacuum expectation value of the field η and we set it to be real without loss of generality. The potential energy is minimized when

$$v = \frac{M}{\sqrt{\lambda}}. \tag{47}$$

Then, we fix the gauge such that

$$\eta = \frac{ve^{i\,\chi(x)}}{\sqrt{2}}, \tag{48}$$

where we have ignored an excitation of the potential around its minimum for the sake of convenience. Accordingly, in order to fully fix the gauge, we introduce a new vector potential field

$$A_\mu \to B_\mu = A_\mu - \frac{1}{e}\partial_\mu\chi(x), \tag{49}$$

with which Lagrangian (45) leads to[77]

$$\widetilde{\mathscr{L}_\eta} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}e^2v^2B_\mu B^\mu, \tag{50}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Lagrangian $\widetilde{\mathscr{L}_\eta}$ is interpreted as containing a massive vector field B_μ with the mass $m_{\rm ph} = e\,v$, where the subscript ph stands for photon. The mass of the magnetic field is inversely proportional to its penetration depth, λ, i.e.,

$$\frac{1}{m_{\rm ph}} = \lambda = \sqrt{\frac{m^*}{4\,\mu_0\,e^2\,<\eta>^2}}, \tag{51}$$

where m^* is the effective mass of the Cooper pairs and μ_0 is the permeability of free space.

This model is a Lorentz invariant version of the Landau–Ginzburg (LG) model of superconductivity[11], which is a phenomenological approach to describe the macroscopic properties of superconductors, and it can be said that the NG boson is 'eaten' to become the longitudinal degree of freedom of the photon. We know that in the LG model, the order parameter is a complex scalar field proportional to the BCS pairing potential. We also expect the order parameter to be proportional to the pairing potentials of the model. To integrate our results with the Higgs mechanism, we need to introduce two matrix form order parameters, say $\eta_{\rm B} \equiv \Delta_{\rm B}$ and $\eta_{\rm F} \equiv \Delta_{\rm F}$, where $\Delta_{\rm B}$ and $\Delta_{\rm F}$ are given in relation (27) and hence, $\eta_{\rm F}$ and $\eta_{\rm B}$ transform as $\eta_{\rm F} \to \eta_{\rm F}\,e^{i\,\Gamma_6\varphi}$ and $\eta_{\rm B} \to \eta_{\rm B}\,e^{i\,\Gamma_6\varphi + i\,\Gamma_5\theta}$. Then, besides A_μ, we also use A^5_μ as a pseudo-electromagnetic field, and assume that both of these fields transform as

$$\begin{aligned} &A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\chi \qquad \text{with:} \quad \begin{cases} \eta_{\rm B}(\mathrm{r},t) \to \eta_{\rm B}(\mathrm{r},t)e^{i\,\Gamma_6\chi} \\ \eta_{\rm F}(\mathrm{r},t) \to \eta_{\rm F}(\mathrm{r},t)e^{i\,\Gamma_6\chi}, \end{cases} \\ &A^5_\mu \to A^5_\mu - \frac{1}{g}\partial_\mu\chi \qquad \text{with:} \quad \begin{cases} \eta_{\rm B}(\mathrm{r},t) \to \eta_{\rm B}(\mathrm{r},t)e^{i\,\Gamma_5\chi} \\ \eta_{\rm F}(\mathrm{r},t) \to \eta_{\rm F}(\mathrm{r},t), \end{cases} \end{aligned} \tag{52}$$

where g is a pseudo-charge. Therefore, the related invariant Higgs Lagrangians under these transformations are

$$\mathscr{L}_{\rm B} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{trace}\left(|(\mathbb{I}_{8\times8}i\partial_\mu - e\Gamma_6A_\mu)\eta_{\rm B}|^2\right) - V(\eta_{\rm B}), \tag{53}$$

$$\mathscr{L}_{\rm F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + {\rm trace}\left(|(\mathbb{I}_{8\times 8}i\partial_\mu - e\Gamma_6 A_\mu)\eta_{\rm F}|^2\right) - V(\eta_{\rm F}) \tag{54}$$

and

$$\mathscr{L}_{5,{\rm B}} = -\frac{1}{4}F_{5,\mu\nu}F_5^{\mu\nu} + {\rm trace}\left(|(\mathbb{I}_{8\times 8}i\partial_\mu - g\Gamma_5 A_{5,\mu})\eta_{\rm B}|^2\right) - V(\eta_{\rm B}), \tag{55}$$

where $F_5^{\mu\nu} = \partial^\mu A_5^\nu - \partial^\nu A_5^\mu$. Analogously, by using the new vector potential (49) and defining $B_5^{\mu\nu}{=}\partial^\mu B_5^\nu - \partial^\nu B_5^\mu$, these Lagrangians lead to

$$\widetilde{\mathscr{L}}_{\eta_{\rm B}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}e^2v^2B_\mu B^\mu, \qquad \widetilde{\mathscr{L}}_{\eta_{\rm F}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}e^2v^2B_\mu B^\mu \tag{56}$$

and

$$\widetilde{\mathscr{L}}_{5,\eta_{\rm B}} = -\frac{1}{4}B_{5,\mu\nu}B_5^{\mu\nu} + \frac{1}{2}g^2v^2B_{5,\mu}B_5^\mu, \tag{57}$$

which indicate that the BCS type of pairing causes the pseudo-magnetic field to acquire the mass $m^p_{\rm ph} = g\,v$, where the superscript p stands for pseudo-magnetic field. In other words, it leads to a pseudo-Meissner effect, which hopefully provides an achievable and quantitatively testable phenomenon to detect the emergent pseudo-scalar NG mode. Similar to the Meissner effect, the mass of the pseudo-magnetic field is inversely proportional to its penetration depth, λ^p, i.e.,

$$\frac{1}{m^p_{\rm ph}} = \lambda^p = \sqrt{\frac{m^*}{4\,\mu_0^p\,g^2\,<\eta_{\rm B}>^2}}, \tag{58}$$

where μ_0^p is the permeability of free space for the pseudo-magnetic field. Assuming that the coupling g and μ_0^p are in the same order as those of the electromagnetic interactions[54], the order of λ^p will be the same as the normal Meissner effect, i.e., λ in relation (51) in topological superconductors[78].

Summary and conclusions

In this work, we have considered a conventional BCS superconductor is placed next to a WSM, where consequently an *s*-wave superconductivity is induced in that WSM due to the proximity effect. To shed light on the nature of the induced pairing, we have considered the continuous symmetries of a 3DWS. In the literature, there have been discussions about whether the superconducting state in that WSM is of the FFLO or BCS type. Here, we have shown that unlike the orthodox BCS superconductors, wherein the $U(1)$ gauge symmetry is the only symmetry of the gapless Hamiltonian (which is then spontaneously broken by the dynamically generated *s*-wave BCS-like pairing), the gapless Hamiltonian of a 3DWS is invariant under two symmetries, namely the $U(1)$ gauge and $U(1)_A$ axial symmetries. To better investigate the issue, we have written the BdG Hamiltonian using the doubled representation of Dirac matrices, which has enabled us to introduce two generators to represent the symmetries of the system. Consequently, two charges appear in the system, i.e. the electric charge and the chiral charge.

Furthermore, we have demonstrated that the dynamical generation of the *s*-wave BCS-like pairing, $\triangle_{\rm B}$, breaks both the symmetries spontaneously, whereas the FFLO-like pairing, $\triangle_{\rm F}$, breaks only the $U(1)$ gauge symmetry. Nevertheless, we have indicated that the conservation of both charges get restored by introducing the NG modes. That is, when only the $U(1)$ gauge symmetry gets broken, the well-known scalar mode recovers the charge conservation. Whereas, when both of the $U(1)$ gauge and $U(1)_A$ axial symmetries are broken, apart from the scalar mode, one needs to introduce an extra new pseudo-scalar phase mode to restore the $U(1)_A$ symmetry as well.

Analogous to conventional superconductors, where the scalar phase mode of superconductivity leads to the Meissner effect in the presence of a normal magnetic field, we have demonstrated that a similar effect appears in 3DWSs, due to the interaction of the pseudo-scalar phase mode with an external pseudo-magnetic. Indeed, the obtained results indicate that when the $U(1)_A$ symmetry is broken by $\Delta_{\rm B}$, the corresponding emergent pseudo-scalar phase mode can get absorbed by an external pseudo-magnetic field (i.e., a $\mathbf{B}^5$ field) that leads to an effect analogous to the Meissner effect. We have referred to this effect as a pseudo-Meissner effect, and it can be tested in future experiments as a key prediction of this work. More specifically, we expect the repulsion of the pseudo-magnetic field inside the WSMs in the superconducting s-wave phase. The pseudo-Meissner effect is consequently achieved by a superconducting surface current that produces a secondary pseudo-magnetic field to compensate for the primary pseudo-magnetic field. Also, the emergent pseudo-scalar NG mode may, in principle, interact with the axial gauge bosons of the standard model of particle physics, and leads to the pseudo-Meissner effect by expelling these bosons from the 3DWS. These predictions are testable in present and forthcoming experiments. As the pseudo-Meissner effect emerges from the BCS-like pairing in WSMs, we propose the pseudo-Meissner effect as a useful tool for distinguishing between the FFLO-like and the BCS-like pairings.

A possible extension of this work is to study the effect of the new phase mode in phenomenon such as the Josephson effect, which we will investigate in upcoming works. Also, a Weyl superconductor originating from a WSM with broken inversion symmetry can lead to various physical effects such as response to external magnetic fields, thermal conductance, and transport properties, which can be investigated in a different work. However, a recent publication has considered the proximity effect on an inversion broken WSM[79].

Data availibility

All data generated or analysed during this study are included in this published article.

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Author contributions

M.Z.A. designed the study, carried out the preliminary calculations and wrote the first draft. The calculations were then cross-checked by M.F. and they completed the manuscript together. Then, M.M. joined the work, and all three contributed to discuss and rewrite the final and revised version of the manuscript.

Additional information

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