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Global sensitivity analysis of structural seismic demand based on information entropy

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To improve the computational efficiency of global sensitivity analysis (GSA) for complex structures, this study proposed a new importance analysis method (IE) based on the low deviation sequences and orthogonal polynomials to study the influence of parameters' uncertainty on three structural seismic demands. A comparative investigation utilizing nonlinear time history analysis for these seismic demands was conducted using OpenSEES. The variance-based importance analysis method and the Tornado graphic sensitivity analysis method were employed to validate the accuracy of the proposed approach. The results regarding the order of importance are nearly consistent across methods, demonstrating the effectiveness of our proposed method. Notably, the sample size required by this new method is only 1024 to achieve reliable results, which is significantly lower than existing sampling methods that necessitate thousands of samples for effective importance analysis; thus, enhancing overall efficiency. Furthermore, the findings indicate that the influence of representative value of gravity load (M_s) on seismic demands is relatively substantial, whereas the influence of modulus of elasticity of concrete (E_c) is comparatively minor.

Keywords Information entropy, Importance analysis, Orthogonal polynomial estimation, Nonlinear time history analysis, Seismic demand

As is well known, earthquakes significantly impact building structures. The study of uncertainties in evaluating building performance under seismic loading has long been a crucial aspect of structural seismic research[1,2]. In recent years, numerous methods for researching uncertainty have emerged, providing robust support for the advancement of structural seismic studies. For example, Xing et al.[3] investigated the dynamic response of high-rise buildings equipped with one-outrigger systems subjected to two types of seismic hazards using deep neural networks. Ding et al.[4] proposed a probabilistic machine learning (ML) method alongside a probabilistic approach derived from the Poisson binomial distribution model to assess the seismic vulnerability of various building combinations. Kurmi and Haldar[5] evaluated how functional openings in upper fillers affect the seismic performance and failure mechanisms of Un-Reinforced Masonry (URM) infills within open ground storey (OGS) reinforced concrete (RC) buildings across different design levels. Ye and Hua[6] comprehensively considered the uncertainties brought by the records, structural design parameters and modeling in the process of seismic performance evaluation, and established a seismic performance evaluation system for middle and high-rise cold-formed thin-walled steel (CFS) structures considering various uncertainties.

In parameter uncertainty analysis of structural systems, the global sensitivity analysis (GSA, often dubbed importance analysis, IS) approach is garnering growing attention. IS may quantify the impact of the input parameters on the statistical characteristics of the output response in engineering from the whole uncertainty range of the input random parameters[7–10]. The variance-based and moment-independence-based importance analysis methods are frequently employed for IS of structural systems. For instance, using GSA in conjunction with an enhanced Latin hypercube sampling technique, Khaneghahi et al.[11] examine the effects of each random variable and their interactions on the variance of the responses. Xu and Wang[12] developed a moment-independent importance analysis method combined with orthogonal polynomial estimation to investigate the effects of random parameters on structural seismic demands. Ling et al.[13] proposed a novel approach that integrates adaptive Kriging with the multiplicative dimensionality reduction method (M-DRM) to derive the time-dependent global reliability sensitivity index, significantly reducing computational effort.

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It is important to note that different sources of uncertainty are seldom considered comprehensively in structural seismic analysis. Consequently, some researchers have introduced entropy theory and developed a GSA method based on information entropy. For instance, Yazdani et al.[14] examined the impact of uncertainties in ground motion variables and structural parameters on the inelastic response of concrete structures using an entropy-based GSA method. Kala[15] proposed an alternative global sensitivity measure by approximating differential entropy through dome-shaped functionals with non-negative values. Du and Padgett[16] incorporated joint entropy from information theory to quantify the uncertainty associated with unconditional multivariate seismic demands for general multi-response structural systems.

However, the commonly employed IS method necessitates double-layer sampling, which is computationally intensive and not conducive to practical engineering applications. Furthermore, even when utilizing single-layer sampling methods such as simplified Monte Carlo simulations, thousands of samples are required to achieve reliable results in IS, thereby diminishing operational efficiency[17–19]. Given that sampling input random parameters is a critical component of importance analysis, this study develops an orthogonal polynomial estimation (OPE) combined with information entropy to conduct importance analyses regarding seismic demand for structures. The uncertainty of random parameters was simulated using Sobol series, while probability density functions for output responses were approximated via Hermite orthogonal polynomials. Subsequently, the importance index based on information entropy was calculated to provide a comprehensive evaluation of how parameter uncertainty influences structural seismic demands.

The variance-based importance analysis method is a classical approach[20], often considered an exact solution and widely employed to validate the accuracy of new methodologies[21–24]. Additionally, the Tornado graphical method has gained popularity in economic decision analysis due to its straightforward principles and ease of calculation. In recent years, it has also been applied in sensitivity analyses concerning seismic losses in buildings[25–27]. Consequently, this study calculates the variance-based importance index for comparative purposes to assess the accuracy of the proposed methodology. Furthermore, this research introduces a novel importance index designed to evaluate the impact of parameter uncertainty on structural seismic demand, thereby providing significant insights for enhancing computational efficiency within the realms of structural and earthquake engineering.

Analytical method

Information entropy importance index

For an output response $Y = g(X)$, when the random variable X is considered as the realized value x_i, the probability density function of Y is denoted as $f_{Y|X_i=x_i}(y)$, and the uncertainty of X_i has been eliminated, so the impact of X_i on the output response Y based on information entropy can be defined as follow[28]:

$$\varepsilon_i = \left| H_Y - H_{Y|X_i=x_i} \right| \tag{1}$$

where, H_Y represents the original entropy of the output response Y, and $H_Y = -\int\limits_{D_Y} f_Y(y)\log f_Y(y)dy$; $H_{Y|X_i} = x_i$ denotes the conditional entropy of the output response, which is obtained by $H_{Y|X_i=x_i} = -\int\limits_{D_Y} f_{Y|X_i=x_i}(y)\log f_{Y|X_i=x_i}(y)dy$.

According to Eq. (1) and in conjunction with the definition of importance index[8], the average impact on the output response can be expressed as follow:

$$\begin{aligned} \overline{\varepsilon_i} &= E\left(\left| H_Y - H_{Y|X_i=x_i} \right|\right) H_Y - H_{Y|X_i=x_i} \Big| \\ &\approx \frac{1}{N_0}\sum_{j=1}^{N_0}\left| H_Y - H_{Y\left|X_i=x_{ij}\right.} \right| \end{aligned} \tag{2}$$

where, D_{X_i} denotes the variation range of random parameters, E(·) represents the mathematical expectation, the sample $X_{ij}(j = 1, 2, \cdots, N_0)$ is generated according to the probability density function of X_i, and $H_{Y\left|X_i=x_{ij}\right.}$ refers to the conditional entropy when X_i assumes the value of $X_{ij}(j = 1, 2, \cdots, N_0)$.

The solution method of information entropy importance index

The explicit expression of the probability density function $f_Y(y)$ is often unknown in practical engineering applications, necessitating the use of estimation methods to obtain it. The estimated value $\hat{f}_Y(y)$ of $f_Y(y)$ can be derived when the sample $y_k(k = 1, 2, \cdots, N)$ of Y is given. Numerous estimation methods exist for obtaining $\hat{f}_Y(y)$, including histogram estimation[29], Orthogonal polynomial estimation (OPE)[30], kernel density estimation[31], maximum entropy principle estimation[32], etc. It is important to note that parameter estimation methods typically require prior knowledge, which may lead to fitting results that significantly deviate from actual conditions. In contrast, non-parametric estimations directly fit the distribution based on the inherent characteristics of the data samples themselves, without making assumptions about the underlying distribution type. This advantage renders non-parametric methods highly valued in both statistical theory and practical applications. Among non-parametric estimation techniques, OPE stands out as a representative method due to its straightforward principles and high precision. It allows for direct fitting of probability distributions according to the characteristics of the data samples with considerable accuracy. Consequently, OPE has been employed in this study to estimate $f_Y(y)$.

According to Feiler[33], if the moment $\mu_1, \mu_2, \cdots \mu_r (r \geqslant 3)$ exists and the characteristic function Φ of the probability distribution meets the condition that $|\Phi|^v \quad (v \geqslant 1)$ is integrable, then the polynomial f_n that obtained by inverse Fourier transform exists when $n \geq v$, where f_n represents the finite term of the characteristic function Φ expanded via Taylor series. Furthermore, the variable *x* satisfy the follow equation when $n \to \infty$:

$$f_n(x) - \phi(x)(1 + \sum_{k=3}^{r} n^{-\frac{k}{2}+1} P_k(x)) = 0\left(n^{-\frac{r}{2}+1}\right) \tag{3}$$

where, $P_k(x)$ represents a real polynomial which does not depend on *n* and *r* but only on the moments $\mu_1, \mu_2, \cdots \mu_r$; and $\phi(x)$ represents normally distribution.

The aforementioned theorem demonstrates that the probability density function $f(x)$ can be approximated through the expansion of the higher-order moment, with this expansion represented as a correction coefficient multiplied by the normal distribution. Therefore, $f(x)$ can be defined as a function that expanded into a polynomial with a weight function $\phi(x)$. To estimate $f(x)$ because of its simplicity and ease of operation, Hermite orthogonal polynomial is chosen in this study, which is expressed as follow:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n(e^{-x^2})}{dx^n}, n = 0, 1, \cdots \tag{4}$$

If $Z = g(X) = g(x_1, x_2, \cdots x_n)$, and $f(X)$ is the probability density function of *X*, then the origin moments of each order are expressed as follow:

$$M_k(g) = \int_{-\infty}^{+\infty} (g(X))^k f(X) dX, k = 1, 2, \cdots, N \tag{5}$$

Let $Y = \frac{z-\mu_z}{\sigma_z} = \frac{g(X)-M_1(g)}{\sqrt{M_2(g)-M_1(g)^2}}$ be the new function, then the origin moments of each order are equal to the center moments, namely:

$$M_k(g) = \mu_k(Y) = \int_{-\infty}^{+\infty} Y^k f(X) dX, k = 1, 2, \cdots, N \tag{6}$$

Take $f(X)$ as a weight function when it represents a specific type of probability distribution, and the accuracy and efficiency are higher when the corresponding type of Gaussian integration point is used. For example, in the cases of normal distribution, exponential distribution, and uniform distribution, the type of weight function is respectively e^{-x^2}, e^{-x} and 1. Then the corresponding Gaussian integral points can be used respectively.

Therefore, the process of approximating probability density function using orthogonal polynomials is briefly described as follows: let $\rho(x)$ denote a weight function, the orthogonal polynomial defined on the corresponding interval $[a, b]$ is expressed as follow:

$$\omega_k(x) = \sum_{m=0}^{k} A_{km} x^m, k = 0, 1, 2, \cdots \tag{7}$$

where A_{km} represents a fixed constant. Based on the properties of orthogonal polynomials, it can be stated that there are:

$$\int_a^b \rho(x)\omega_i(x)\omega_j(x) dx = \begin{cases} h_i, & i = j; \\ 0, & i \neq j. \end{cases} \tag{8}$$

Then $f(x)$ can be obtained by:

$$f(x) \approx \rho(x) \sum_{k=0}^{N} a_k \omega_k(x) \tag{9}$$

where a_k denotes an undetermined coefficient, which determined by $a_k = \sum_{m=0}^{k} A_{km}\mu_m(x)/h_k$; the weight function is $\rho(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{(-\frac{(x-\mu)^2}{2\sigma^2})}$, or a standardized function is adopted, and the weight function is $\phi(y) = \frac{1}{\sqrt{2\pi}} e^{(-\frac{y^2}{2})}$, the integration interval is $(-\infty, +\infty)$, and the coefficient of the highest order term of the polynomial is 1.

To sum up, the computational procedures are outlined as follows:

(1) Generate N samples of random parameter using Sobol series. After extensive experimentation, it was determined that the error in both the mean and standard deviation does not exceed 5/10,000 when the sample size exceeds 500.
(2) Establish the calculation model utilizing OpenSees, inputting the sample values of random parameters to obtain the output response $y_k(k = 1, 2, \cdots, N)$ of various seismic requirements Y.
(3) Employ the Hermite orthogonal polynomial estimation method to calculate estimated values $\hat{f}_Y(y)$ of $f_Y(y)$ for different seismic demands Y, followed by an estimation of their original entropy H_Y.
(4) Repeat steps (2) and (3) to estimate the conditional entropies $H_{Y|X_i=x_{ij}}$ of various seismic demands Y.
(5) Calculate $\left|H_Y - H_{Y|X_i=x_i}\right|$ of the difference between the original entropy and the conditional entropy, subsequently deriving the importance index $\overline{\varepsilon_i}$ based on the information entropy as described in Eq. (2).

The flowchart is shown in Fig. 1.

Verification method
In importance analysis, the Monte Carlo-based variance method (abbreviated as VAR) is considered the most classic approach and is typically regarded as an exact solution. To test and validate the proposed method, we employ the variance importance analysis method for comparison. Both the information entropy importance analysis method and the variance importance analysis method are capable of examining changes in output responses when random parameters assume all possible values, thus falling under Global Sensitivity Analysis (GSA). In contrast, traditional sensitivity analysis methods belong to Local Sensitivity Analysis (LSA), which can only assess changes in output responses when random parameters take on specific values. To elucidate the

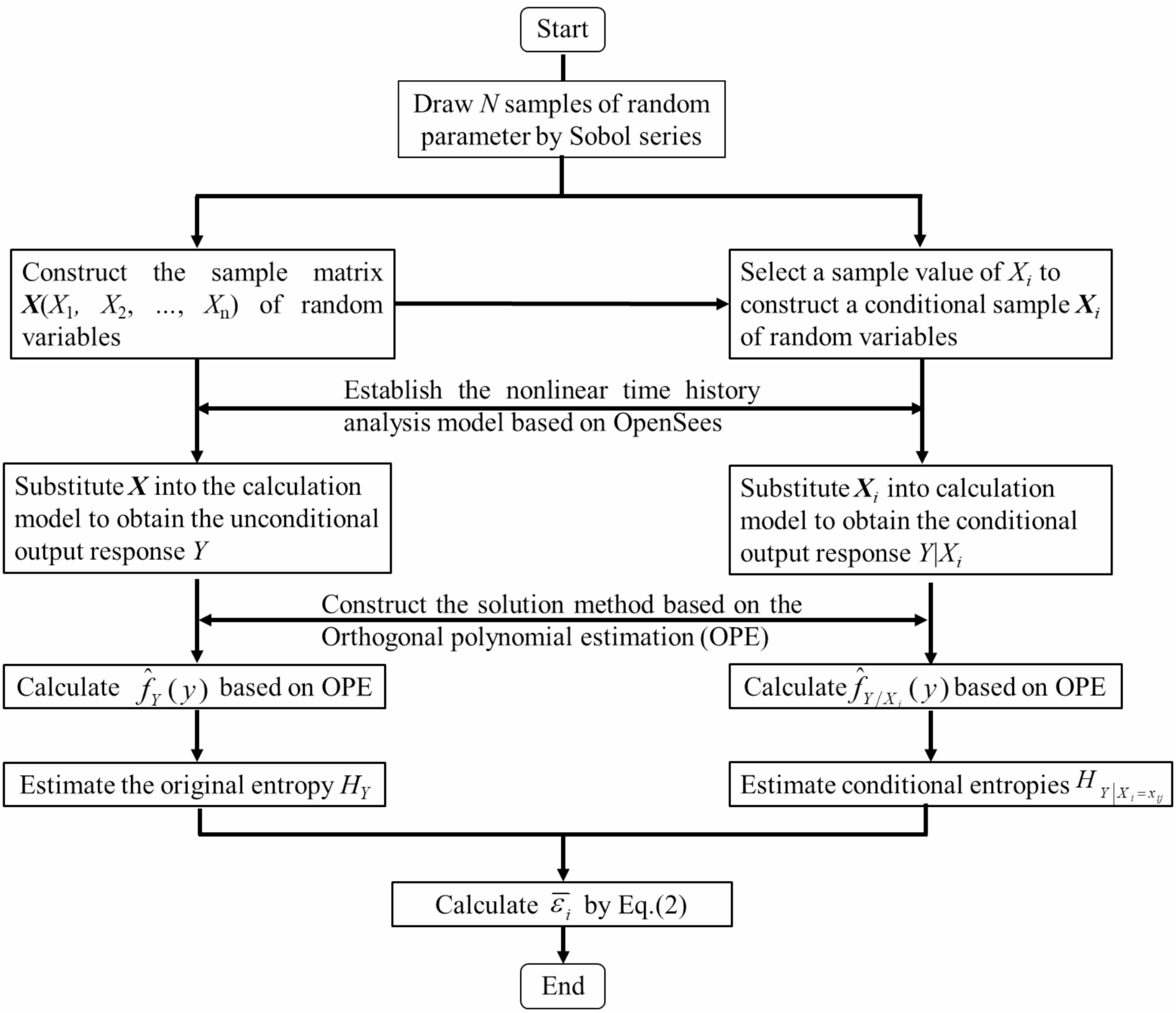

Fig. 1. The flowchart of the proposed method.

differences between GSA and LSA, the Tornado graph method—a technique associated with LSA—also has been used to conduct sensitivity analyses. The results obtained from this approach are then compared with those derived from both the proposed method and the variance-based methodology.

According to Satteli[34], the importance index δ_i^{ν} of the MC-based variance method is expressed as follow:

$$\begin{aligned}\delta_i^{\nu} &= \frac{\mathrm{Var}(\mathrm{E}(Y|\boldsymbol{X}_{\mathrm{i}}))}{\mathrm{Var}(Y)} \\ &= \frac{\mathrm{Var}(Y) - E(\mathrm{Var}(Y|\boldsymbol{X}_{\mathrm{i}}))}{\mathrm{Var}(Y)}\end{aligned} \tag{10}$$

where Var(·) means variance, and $\mathrm{Var}(\mathrm{E}(Y|\boldsymbol{X}_i))$ is the conditional variance of mathematical expectation of the structural seismic demand Y; $\mathrm{E}(\mathrm{Var}(Y|\boldsymbol{X}_i))$ means the expectation of the conditional variance of Y; $\boldsymbol{X}_i$ is a random parameter X_i, or a group of random parameters $[X_{i1}, X_{i2}, \cdots, X_{ir}]$ $(1 \leqslant i_1 \leqslant \cdots \leqslant i_r \leqslant n)$.

Due to space constraints, a detailed introduction to the Tornado graphics method is no longer provided. Please refer to references[26,27,35] for further information.

Case study

A reinforced concrete frame structure

A 3-span, 7-story reinforced concrete frame structure equipped with viscous dampers is presented as an engineering example for importance analysis, as illustrated in Fig. 2. The specifications of this structure are detailed as follows: the standard floor height measures 3600 mm, while the bottom floor height is 4200 mm; the slab thickness is set at 120 mm, and all column spacing are uniformly spaced at 6000 mm. The viscous damper features a damping index of 1, with concrete classified as C40 and reinforcement designated as HRB335. Furthermore, the details regarding input variables are provided in Table 1, and information pertaining to beam and column sections can be found in Table 2.

The finite element analysis model for nonlinear time history analysis in this study is established using OPENSEES. The seismic wave employed is the El Centro seismic wave, and the viscous damper is simulated through a Maxwell element. For the column and beam, a nonlinear fiber beam-column element is utilized, while the concrete material is represented by the Concrete02 element, and the Steel02 material model is adopted for steel reinforcement. Considering that the existing sampling method requires thousands of samples to get good results, which takes a lot of time to perform finite element simulation on the structure. Aiming at this problem, the Sobol sequence is adopted for sampling in this study. With several hundred samples, improved outcomes can be obtained while maintaining high computational efficiency. The process can be summarized as follows:

(1) Drawing N samples according to the joint distribution density of each random parameter; thus, the sample matrix A = n×N (n is the number of variables) is expressed as:

$$A = \begin{bmatrix} X_1^{(1)} & \cdots X_i^{(1)} \cdots & X_n^{(1)} \\ X_1^{(2)} & \cdots X_i^{(2)} \cdots & X_n^{(2)} \\ \vdots & \ddots & \vdots \\ X_1^{(N)} & \cdots X_i^{(N)} \cdots & X_n^{(N)} \end{bmatrix}$$

(2) Replacing the elements in the i-th column of matrix A with the mean of the i-th random parameter, the matrix B is obtained as:

$$B = \begin{bmatrix} X_1^{(1)} & \cdots \overline{X_i^{(1)}} \cdots & X_n^{(1)} \\ X_1^{(2)} & \cdots \overline{X_i^{(2)}} \cdots & X_n^{(2)} \\ \vdots & \ddots & \vdots \\ X_1^{(N)} & \cdots \overline{X_i^{(N)}} \cdots & X_n^{(N)} \end{bmatrix}$$

(3) Substituting the sample matrices A and B into the finite element model for nonlinear time history analysis, respectively, and obtain the corresponding unconditional output responses $Y = (y_1, y_2, \cdots y_N)^T$ and conditional output responses $Y|X_i = (y_1|\bar{x}_i, y_2|\bar{x}_i, \cdots y_N|\bar{x}_i)^T$.

The relationships between the unconditional sample values of the three seismic demands and their corresponding damping ratios are illustrated in Fig. 3. It is evident that these relationships exhibit inconsistencies. For instance, both the maximum interstory displacement angle demand and the roof displacement demand appear to be roughly inversely related to the damping ratio, while the relationship between base shear force demand and damping ratio is less pronounced. Overall, the effects of damping ratio on these three types of seismic demands demonstrate considerable variability. This variability arises from the fact that these seismic demands are influenced not solely by a single random parameter—namely, the damping ratio—but also by eight additional random parameters selected for importance analysis in this study. The relationships among these three seismic demands and other random parameters show similar patterns; therefore, they are not included here due to space constraints.

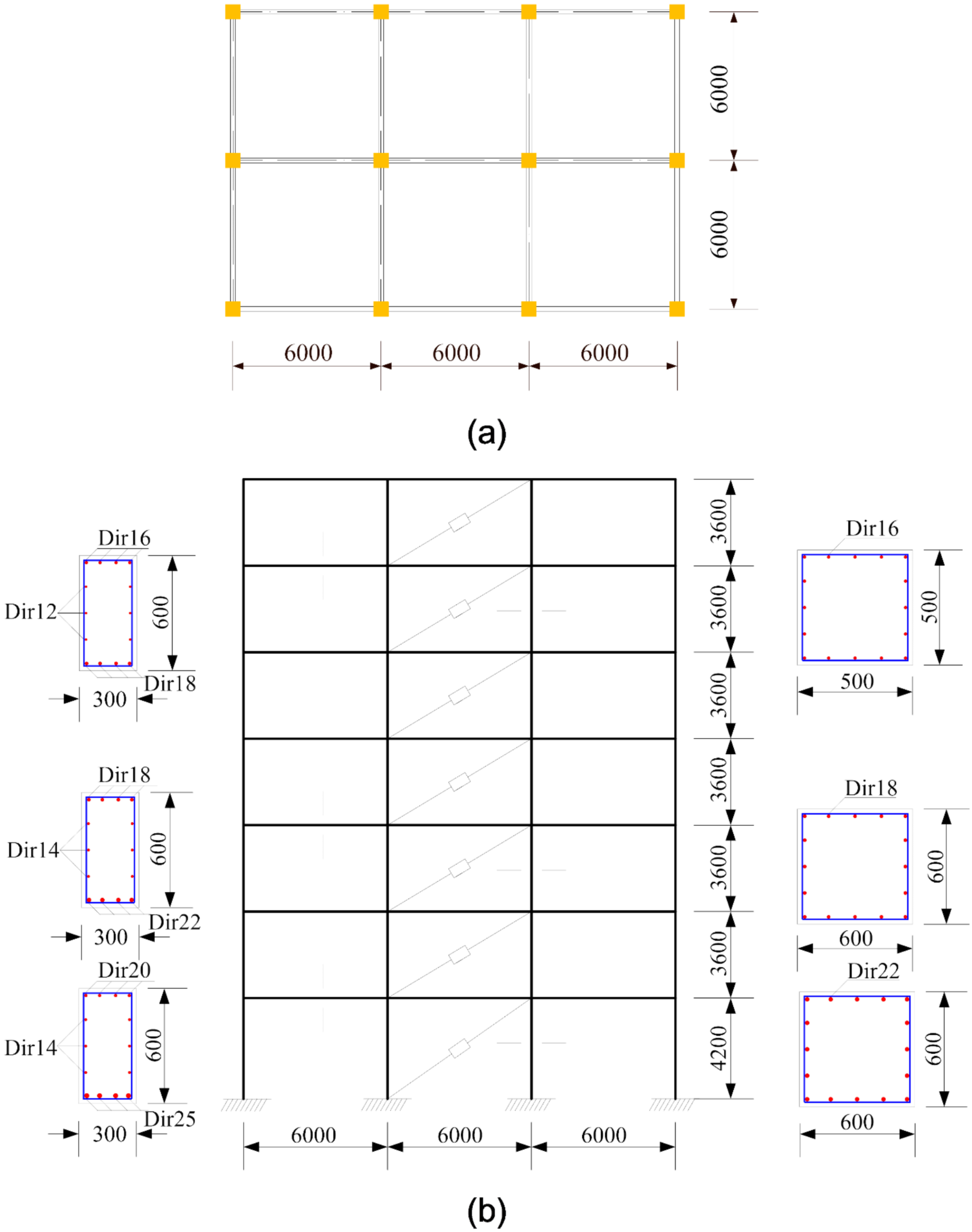

Fig. 2. Structure diagram: (**a**) Building plans; (**b**) Building elevation.

The effects of 8 random parameters on three kinds of seismic demands are illustrated in Fig. 4. As observed from Fig. 4, the results calculated by the proposed method (IE) closely align with those derived from the variance-based method (VAR), indicating that the proposed method yields accurate results. The analysis presented in Fig. 4 reveals that the eight random parameters exert varying degrees of influence on the three seismic demands. For example, Fig. 4a demonstrates that the representative value of gravity load and damping ratio significantly impact the roof displacement demand of the structure, followed by yield strength of reinforcement and compressive strength of concrete; other factors exhibit minimal influence. In Fig. 4b, it is evident that base shear

Random parameters	Units	Symbol	Distributions	Means	Variation coefficients	References
Compressive strength of concrete	MPa	f_c	Normal	34.82	0.14	[36]
Modulus of elasticity of concrete	MPa	E_c	Normal	33,904	0.08	[37]
Modulus of elasticity of steel	MPa	E_s	Normal	228,559	0.033	[38]
Yield strength of steel rebars	MPa	f_y	Lognormal	384	0.078	[36]
Structural damping ratio		ζ	Normal	0.055	0.2	[25]
Stiffness of viscous damper	$kN\cdot mm^{-1}$	k	Normal	100	0.1	
Damping coefficient of viscous damper	$kN\cdot s\cdot mm^{-1}$	c	Normal	3	0.1	
Representative value of gravity load	kN/m^2	M_s	Normal	6	0.1	[36]

Table 1. Statistical information of random parameters.

Floor	Column section/(mm×mm)	Area of reinforcement/mm²		Beam section/(mm×mm)	Area of reinforcement/mm²	
		Middle position of section	Edge position of section		Top of section	Bottom of section
1	600 × 600	2281	3800	300 × 600	1256	1964
2 ~ 4	500 × 500	1570	2544		1017	1520
5 ~ 7	500 × 500	1206	2010		804	1017

Table 2. Section information.

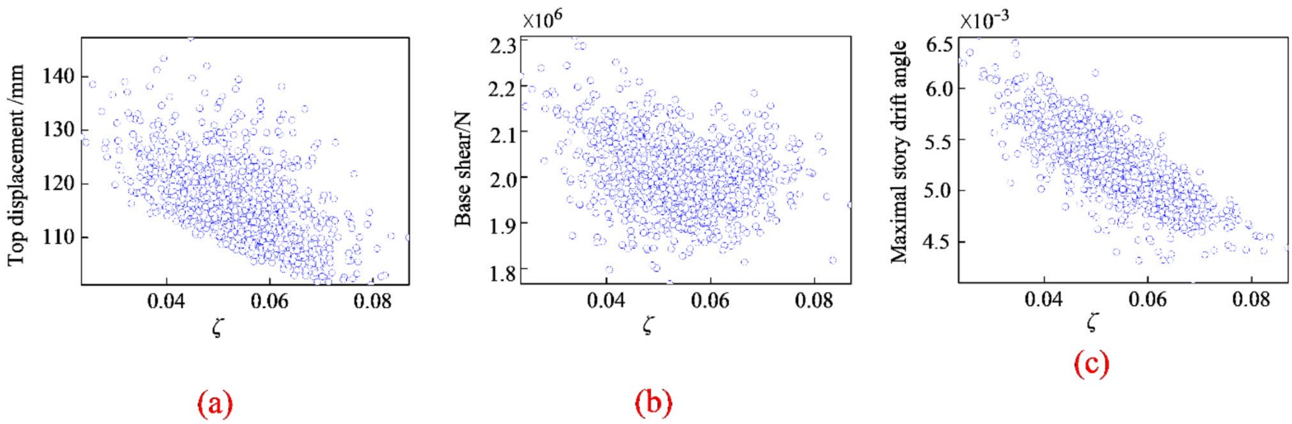

Fig. 3. The relation of seismic demands and damping ratio, (**a**) (top displacement demand), (**b**) base shear demand, (**c**) maximal story drift angle demand.

demand is predominantly affected by three random parameters: yield strength of reinforcement, representative value of gravity load, and compressive strength of concrete, while other random parameters have little effects. Furthermore, as shown in Fig. 4c, damping ratio has a substantial effect on maximum inter-story displacement angle demand for the structure, followed by yield strength of steel bars; however, elastic modulus of steel bars, compressive strength of concrete, elastic modulus of concrete, and stiffness characteristics associated with viscous dampers show little to no influence on this parameter.

Additionally, when sample size $N \leq 256$ is considered, there exists considerable variation in importance ranking among each random parameter concerning output response; conversely, when $N \geq 384$ is applied, this ranking remains relatively stable with only minor changes observed in importance index values for each parameter. This suggests that larger sample sizes enhance result accuracy significantly. Notably depicted in Fig. 4 is that at $N = 1024$—results generated through the proposed method remain consistent—satisfying established accuracy requirements.

In addition, When the sample size $N \leq 256$, the importance ranking of each random parameter on the output response is quite different, while when $N \geq 384$, the importance ranking basically remains unchanged, and the value of the importance index of each random parameter also change very little. It indicates that the larger the sample size N, the more accuracy the results. As shown in Fig. 4, when $N = 1024$, the results obtained by the proposed method remains unchanged, which can meet the accuracy requirements. Compared to the variance method that relies on Monte Carlo, which requires thousands of samples, it is obvious that the proposed method in this study significantly improves the computing efficiency.

The rankings of the importance effect of each random parameter are shown in Fig. 5, where $N = 1024$. It can be seen from Fig. 5 that the random parameters exert varying degrees of influence on the three seismic demands.

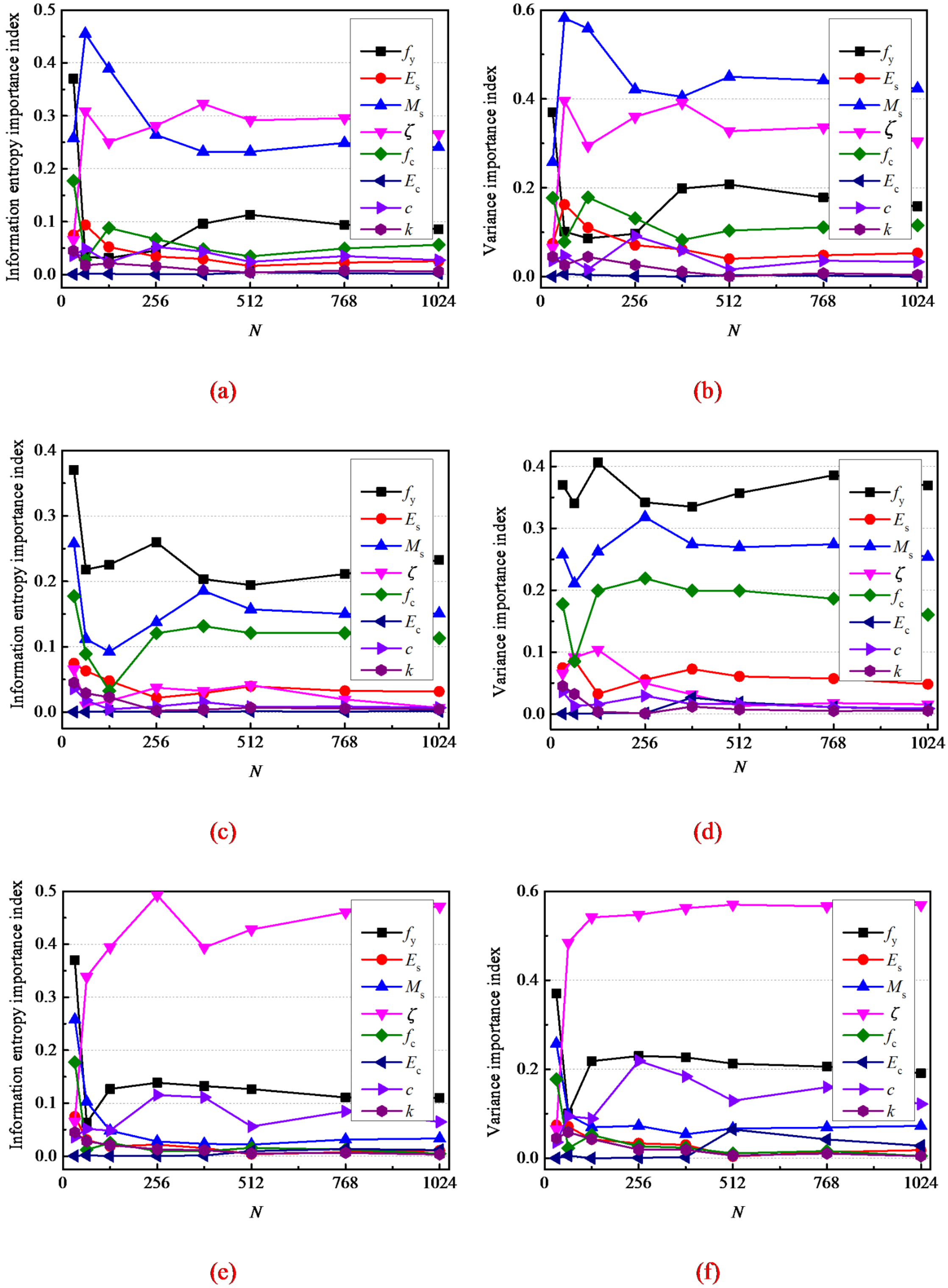

Fig. 4. Importance index for case study 1, (**a**) Top displacement demand (IE), (**b**) Top displacement demand (VAR), (**c**) Base shear demand (IE), (**d**) Base shear demand (VAR), (**e**) Maximal story drift angle demand (IE), (**f**) Maximal story drift angle demand (VAR).

For example, the representative value of gravity load (M_s) and the structural damping ratio (ζ) significantly impact the top displacement demand; conversely, ζ has a lesser effect on the base shear demand. The steel strength (f_y) and the representative value of gravity load (M_s) have a pronounced influence on base shear demand but exhibit a reduced impact on maximal story drift angle demand. In addition, there is a notable discrepancy between the values of importance indices derived from information entropy (IE) and those obtained through variance

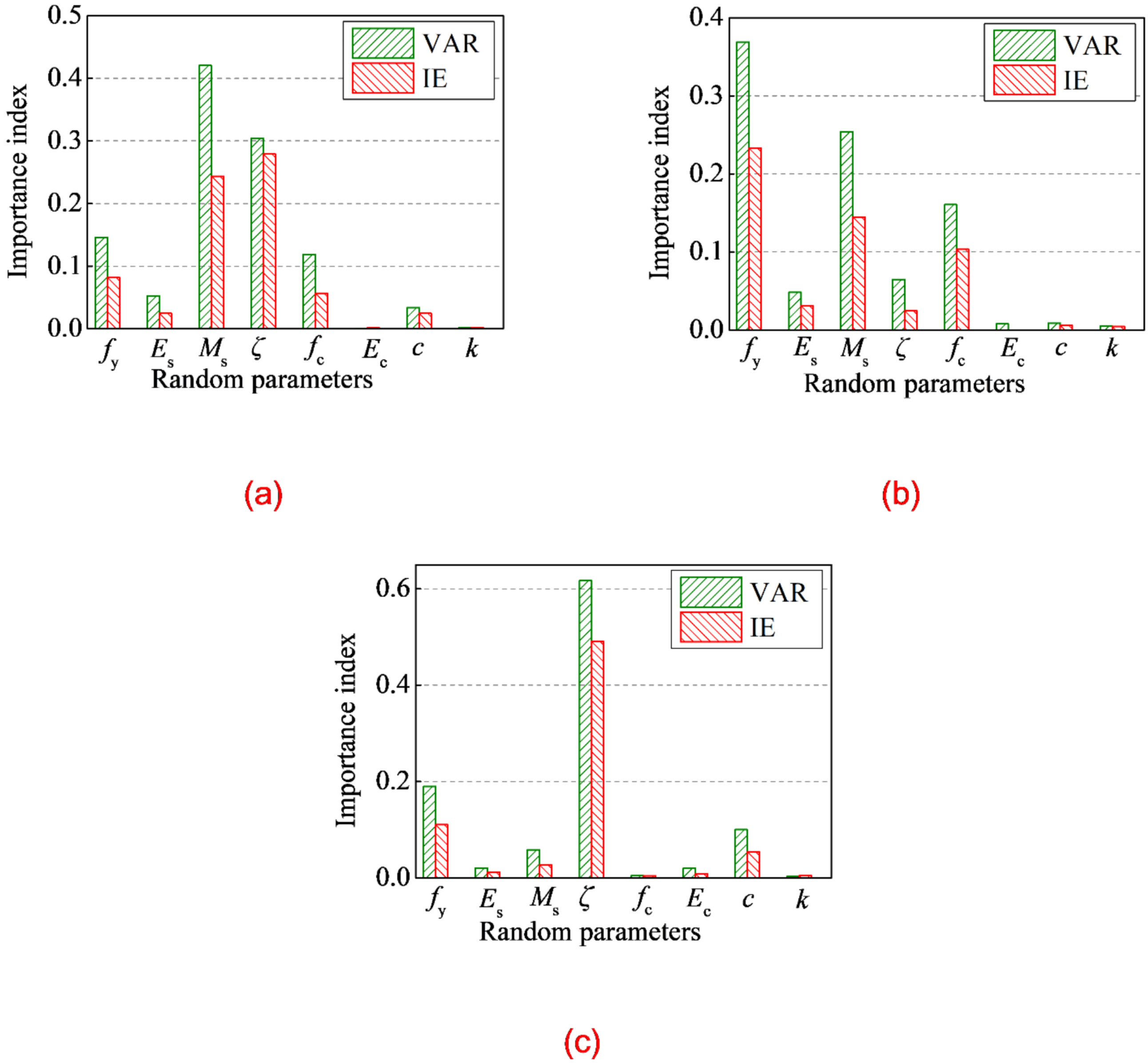

Fig. 5. Comparison of importance index, (**a**) Top displacement demand, (**b**) Base shear demand, (**c**) Maximal story drift angle demand.

analysis (VAR). Generally, values calculated using VAR tend to be higher than those determined by IE. This difference arises from the distinct interpretations associated with each method for assessing importance indices.

The results obtained by Tornado graphic method are shown in Fig. 6. It is obviously that the importance effect of each parameter on the three seismic demands are different. For example, Fig. 6a shows that the representative value of gravity load M_s, damping ratio ζ and compressive strength of concrete f_c exert a more substantial influence on the top displacement demand, whereas the stiffness of the viscous damper k and the elastic modulus of concrete E_c have less influence. Figure 6b indicates that the yield strength of steel bar f_y, structural quality M_s and compressive strength of concrete f_c have a greater influence on the base shear demand of the structural, while the elastic modulus of concrete E_c has the least influence. From Fig. 6c, the damping ratio ζ and compressive strength of concrete f_c have a greater impact on the maximal story drift angle demand, while the elastic modulus of concrete E_c has the least influence. Overall, the ranking of the importance effect of the eight random parameters on the three structural seismic demands are as follows, respectively: M_s>ζ>f_c>c>E_s>f_y>k>E_c, f_y>M_s>f_c>ζ>E_s>k>c>E_c and ζ>f_y>c>M_s>E_s>f_c>k>E_c.

A steel-concrete frame structure

A three-span, seven-story steel-concrete frame structure, the steel strength grade is Q345, and the welded H-section is used. The section steel in the beam is H140 × 440 × 10 × 16, and the section steel in the columns of floors 1 ~ 4 and 5 ~ 7 are respectively H400 × 400 × 11 × 18 and H300 × 300 × 10 × 15. The statistical characteristics

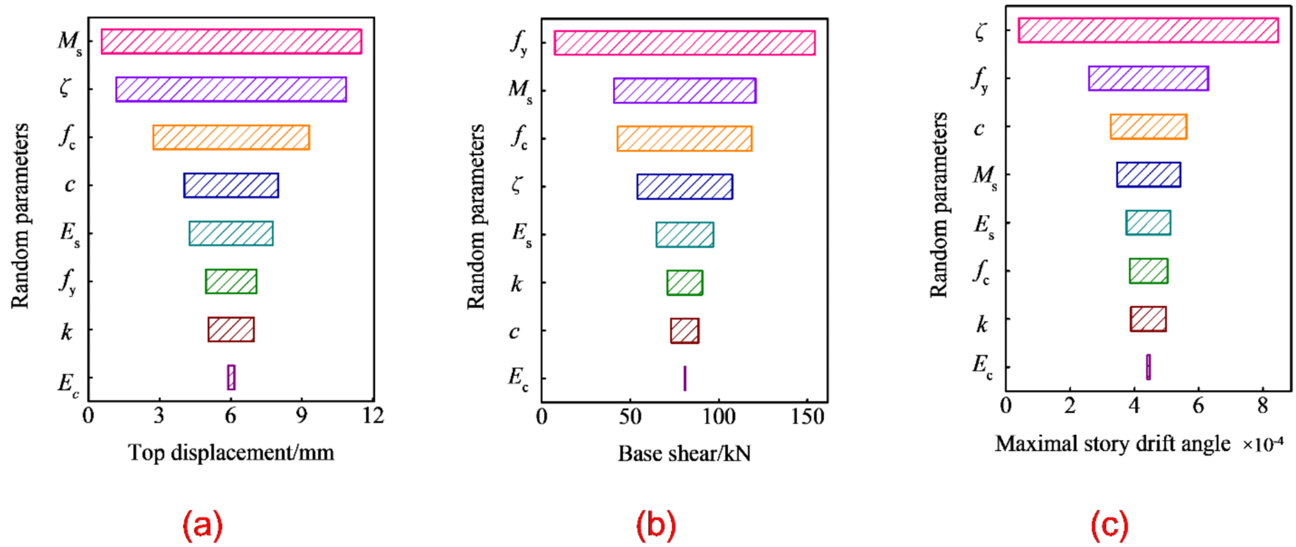

Fig. 6. Sensitivity ordering of random parameters, (**a**) top displacement demand, (**b**) base shear demand, (**c**) maximal story drift angle demand.

Random parameters	Units	Symbol	Distributions	Means	Variation coefficients
Structural damping ratio		ζ	Normal	0.05	0.2[39]
Modulus of elasticity of section steel	MPa	E_{ss}	Normal	228,559	0.033
Yield strength of section steel	MPa	f_{ys}	Normal	396	0.078

Table 3. Statistical characteristics of random parameters.

Storey	Column section/(mm×mm)	Area of reinforcement/mm^2	Beam section/(mm×mm)	Area of reinforcement/mm^2
1	600 × 600	6082	300 × 600	2280
2 ~ 7		4072		1526

Table 4. Section information.

Serial number	Earthquake	Magnitude	Occurrence time
RSN902	Big Bear-01	6.5	1992
RSN3747	Cape Mendocino	7.0	1992
RSN130	Friuli_ Italy-02	5.9	1976
RSN6	Imperial Valley-02	7.0	1940
RSN1083	Northridge-01	6.7	1994
RSN947	Northridge-01	6.7	1994
RSN578	TaiwanSMART1(45)	7.3	1986

Table 5. Ground motion records.

of the random parameters are detailed in Table 3; all other parameters remain consistent with those presented in Case study 1. Due to the limitation of space, the structure diagram is not provided herein. The configuration of the cross-section reinforcement is shown in Table 4, and the ground motion records are shown in Table 5. The PGA is uniformly adjusted to 0.6 g, acting on the longitudinal direction of the structure.

The important effects of 8 random parameters on the three structural seismic demands under different ground motion records are shown in Fig. 7. It can be seen from Fig. 7a–f that the results obtained by the proposed method (IE) and the variance method (VAR) exhibit a high degree of consistency, thereby validating the accuracy of the proposed method. It shows that the effect of random parameters on structural seismic demands varies across different ground motion records. For example, Fig. 7a shows that the importance orderings of the 8 parameters on the top displacement demand obtained by IE under the 7 ground motion records are as follows, respectively:

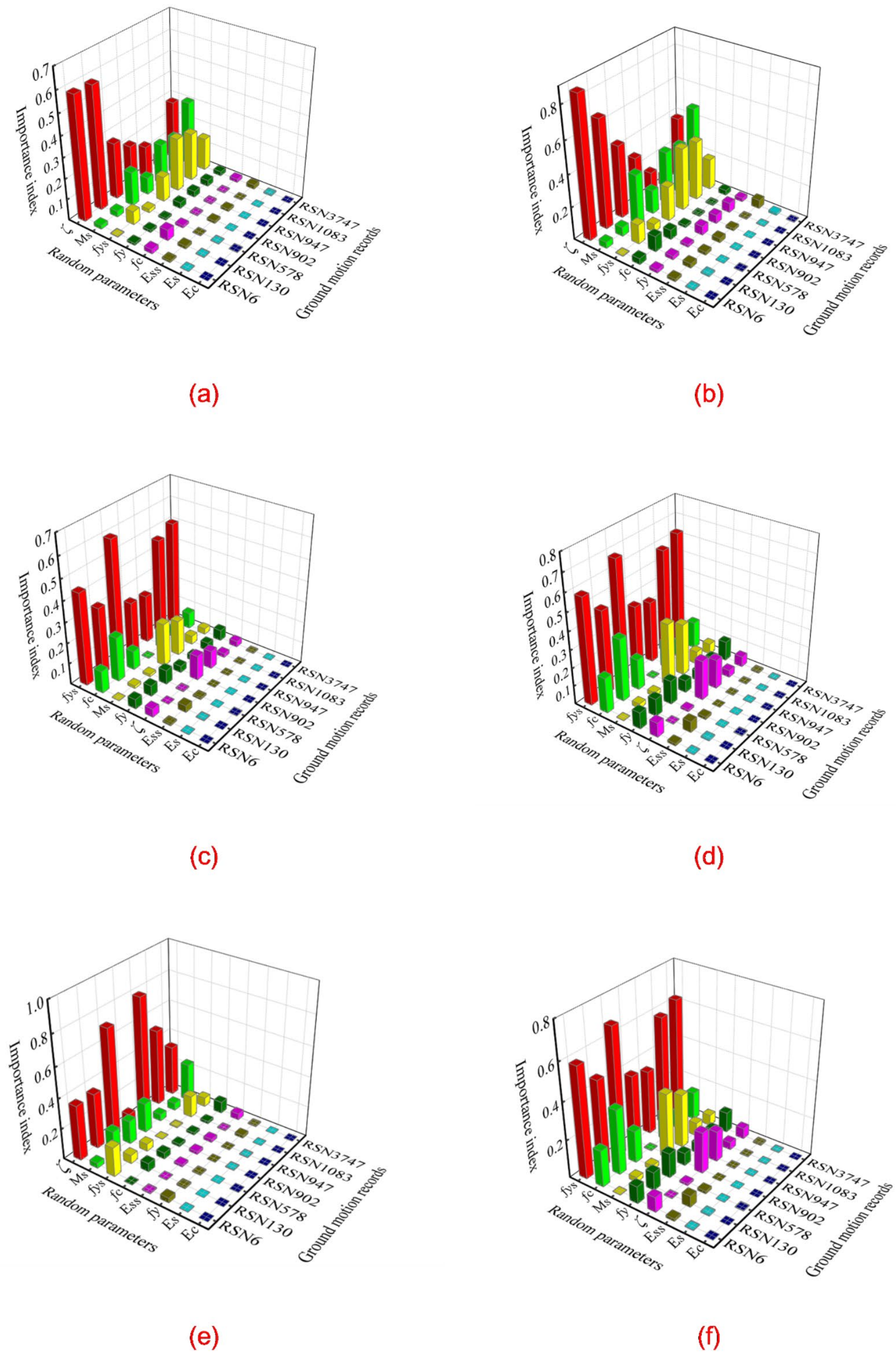

Fig. 7. Comparison of importance index, (**a**) Top displacement demand (IE), (**b**) Top displacement demand (VAR), (**c**) Base shear demand (IE), (**d**) Base shear demand (VAR), (**e**) Maximal story drift angle demand (IE), (**f**) Maximal story drift angle demand (VAR).

$\zeta>M_s>f_c>f_y>E_{ss}>f_{ys}>E_s>E_c$, $\zeta>f_{ys}>f_c>M_s>E_{ss}>f_y>E_s>E_c$, $\zeta>M_s>f_{ys}>f_c>f_y>E_{ss}>E_s>E_c$, $\zeta>f_{ys}>M_s>f_y>E_{ss}>f_c>E_s>E_c$, $f_{ys}>M_s>\zeta>f_y>E_{ss}>f_c>E_s>E_c$, $f_{ys}>M_s>\zeta>f_y>E_{ss}>f_c>E_s>E_c$ and $M_s>\zeta>f_{ys}>f_y>E_{ss}>f_c>E_s>E_c$. Obviously, the orderings are not consistent, and similar discrepancies are observed for the other two seismic demands.

Considering the characteristics of the results under most ground motion records, for the top displacement demand, the information entropy importance indices of ζ, M_s and f_{ys} tend to be larger compared to those of E_{ss},

E_s and E_c which remain smaller. For the base shear demand, the information entropy importance indices of f_{ys}, f_c and M_s are larger, while those of E_{ss}, E_s and E_c are smaller under most ground motion records. For the maximal story drift angle demand, the information entropy importance indices of M_s, f_{ys} and ζ are larger, while those of E_{ss}, E_s and E_c remain comparatively smaller under most ground motion records.

Taking the ground motion record RSN902 as an example, the results obtained by IE and VAR are shown in Fig. 8. Notable differences exist between the results of the two methods, which can be attributed to the different meanings of the index expressed by the two analysis methods. The importance rankings of 8 random parameters for the three seismic demands derived from IE are respectively $\zeta>f_{ys}>M_s>f_y>E_{ss}>f_c>E_s>E_c$, $f_{ys}>\zeta>M_s>f_y>f_c>E_{ss}>E_s>E_c$, $M_s>\zeta>f_c>E_{ss}>E_s>f_y>f_{ys}>E_c$, which are nearly consistent with those obtained by VAR. It can be seen that the random parameter has different influence degrees on different seismic demands. For example, M_s has a greater impact on the base shear demand and the maximal story drift angle demand, but has a small influence on the top displacement demand. f_{ys} has a greater impact on the structural top displacement demand and the base shear demand, but has a small influence on the maximal story drift angle demand.

The results obtained by the Tornado graphic method under the ground motion record RSN902 are shown in Fig. 9. it can be seen form Fig. 8a,b that ζ, f_{ys} and M_s have greater influence on the top displacement demand, while E_s and E_c have comparatively smaller influence. In Fig. 8c, M_s, ζ and f_c have greater influence on the

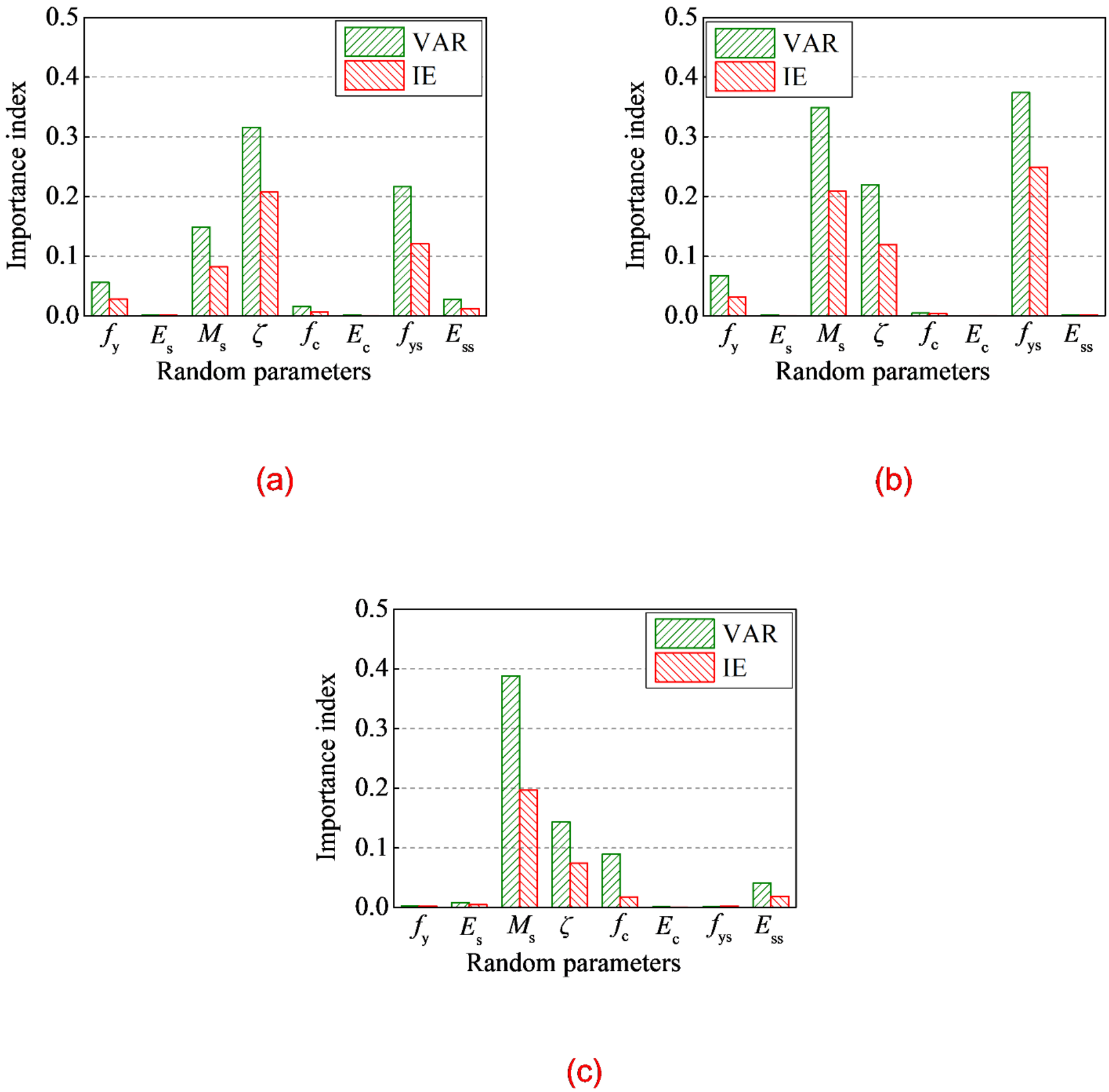

Fig. 8. Comparison of importance index, (**a**) Top displacement demand, (**b**) Base shear demand, (**c**) Maximal story drift angle demand.

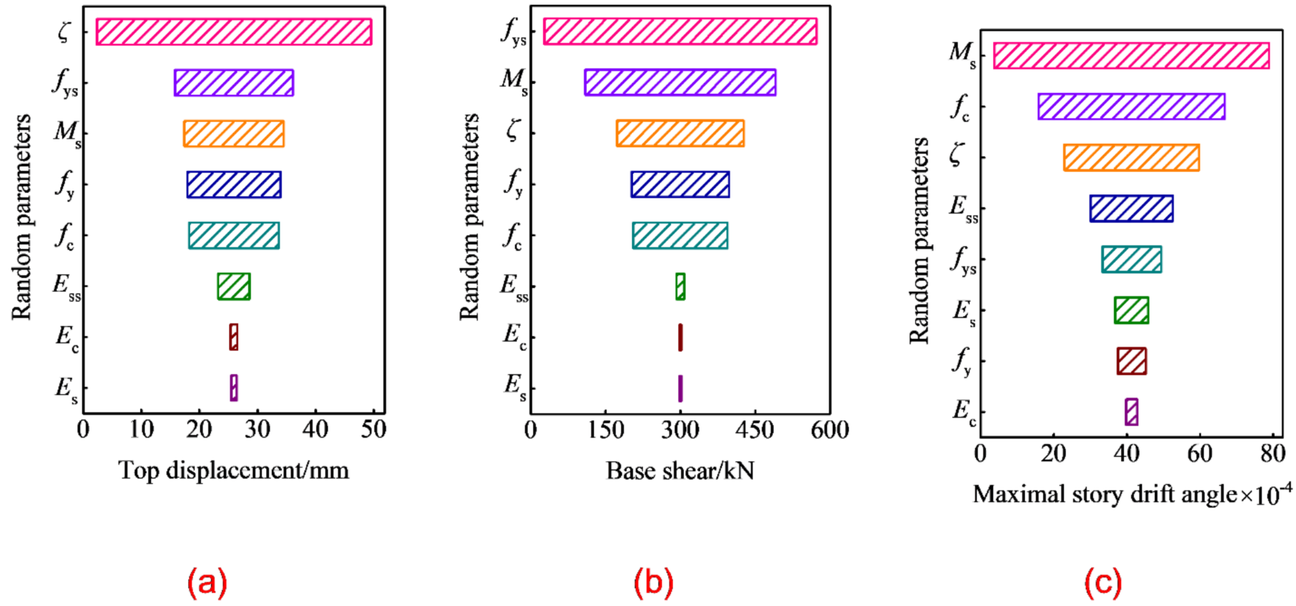

Fig. 9. Sensitivity ordering of random parameters, (**a**) top displacement demand, (**b**) base shear demand, (**c**) maximal story drift angle demand.

Random parameters	Top displacement	Base shear	Maximal story drift angle
f_y	3-3-6*	1-1-1	2-2-2
E_s	5-6-5	5-4-5	5-5-5
M_s	1-2-1	2-2-2	4-4-4
ζ	2-1-2	4-5-4	1-1-1
f_c	4-4-3	3-3-3	7-7-6
E_c	8-8-8	7-8-8	6-6-8
c	6-5-4	6-6-7	3-3-3
k	7-7-7	8-7-6	8-8-7

Table 6. Importance ordering of random parameters for case study 1. 3-3-6*means that, the first number 3 represent the importance ordering obtained by information entropy, the second number 3 represent the importance ordering obtained by MC and the third number 6 represent the importance ordering obtained by Tornado graphic method, respectively. The same as others.

maximal story drift angle demand, while f_y and E_c have smaller influence. In general, the importance rankings of 8 random parameters for the three seismic demands are $\zeta>f_{ys}>M_s>f_y>f_c>E_{ss}>E_s>E_c$, $f_{ys}>M_s>\zeta>f_y>f_c>E_{ss}>E_c>E_s$, $M_s>f_c>\zeta>E_{ss}>f_{ys}>E_s>f_y>E_c$, respectively. These findings indicate that the relative significance of identical random parameters varies across different seismic demands.

Discussions

In order to compare the influence of random variables obtained by the three methods on different seismic demands, the importance orderings of the random parameters obtained by the three methods to the three structures seismic demands are listed in Tables 6 and 7, respectively.

As shown in Table 6, it is evident that the representative value of gravity load M_s and damping ratio ζ significantly impact the top displacement demand. Conversely, the stiffness of the viscous damper k and the elastic modulus of the concrete E_c exert a lesser influence. The yield strength of steel bar f_y and representative value of gravity load M_s have great influence on the base shear demand of structure, while the elastic modulus of concrete E_c has little influence. The damping ratio ζ and the yield strength of steel bar f_y have a great influence on the maximal story drift angle of the structure, while the elastic modulus of concrete E_c has little influence. Taken the top displacement demand of structure as an example, the importance orderings of the random parameters obtained by the three methods respectively are $M_s>\zeta>f_y>f_c>E_s>c>k>E_c$, $\zeta>M_s>f_y>f_c>c>E_s>k>E_c$ and $M_s>\zeta>f_c>c>E_s>f_y>k>E_c$. It is clear that the results obtained by IE and VAR closely aligned, whereas those derived from Tornado graphic method differ notably from both approaches.

As shown in Table 7, there are some differences in the orderings produced by various methods for the same set of random parameters. Nevertheless, these rankings generally exhibit a close correspondence with one

Random parameters	Top displacement	Base shear	Maximal story drift angle
f_y	4-4-4*	4-4-4	6-7-7
E_s	8-7-8	6-7-8	5-5-6
M_s	3-3-3	2-2-2	1-1-1
ζ	1-1-1	3-3-3	2-2-3
f_c	6-6-5	5-5-5	3-4-2
E_c	7-8-7	8-8-7	8-8-8
f_{ys}	2-2-2	1-1-1	7-6-5
E_{ss}	5-5-6	7-6-6	4-3-4

Table 7. Importance ordering of random parameters for case study 2. 4-4-4*means that, the first number 4 represent the importance ordering obtained by information entropy, the second number 4 represent the importance ordering obtained by MC and the third number 4 represent the importance ordering obtained by Tornado graphic method, respectively. The same as others.

another. The random parameters ζ, f_{ys} and M_s have greater influence on the top displacement demand and base shear demand, while E_c and E_s have lesser influence. The random parameters M_s, ζ, and f_c have greater influence on the maximal story drift angle, while E_c and f_y have lesser influence.

Conclusion

The influence of random parameters on the three seismic demands are investigated in this study using the importance analysis method based on information entropy. The results from two case studies align well with those obtained through the variance method, indicating that the proposed approach is both accurate and effective. Additionally, the Tornado graphic method is employed for comparative purposes. The conclusions drawn are as follows:

(1) The importance orderings regarding the influence of random parameters on structural seismic demands, as determined by the proposed method, is consistent with the findings derived from the variance-based importance analysis method. This consistency verifies the accuracy of the proposed method.
(2) Improved results can be achieved when utilizing a sample size of $N = 1024$ based on the proposed method; conversely, the variance-based importance analysis requires thousands of samples to yield comparable outcomes. Thus, the required sample size for the proposed method is significantly smaller than that needed for variance-based methods, leading to a marked increase in efficiency.
(3) For the frame structure, identical random parameters exhibit varying degrees of influence on the three structural seismic demands. In case study 1, M_s and ζ exert considerable effects the top displacement demand, f_y and M_s significantly impact the base shear demand, and ζ and f_y greatly affect the maximal story drift angle, while E_c has little influence on all the three structural seismic demands. In case study 2, ζ, f_{ys} and M_s have greater influence on the top displacement demand and base shear demand, M_s, ζ, and f_c have greater influence on the maximal story drift angle, while E_c has lesser influence on all the three structural seismic demands.
(4) According to Tables 6 and 7, the influence of M_s on the seismic demands is relatively large, whereas E_c demonstrates minimal impact.
(5) When the importance of the frame structure is analyzed with a single ground motion record and multiple ground motion records, the importance orderings of each parameter on seismic demands remain relatively consistent.

In summary, this study proposed an efficient importance analysis method based on information entropy to evaluate the influence of parameter uncertainty on structural seismic demands and identify parameters with higher impact. By controlling the uncertainty associated with these parameters, it is possible to enhance structural safety effectively and provide valuable insights for optimization in structural design.

Data availability

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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Xiuzhen Wang and Zhaoxia Xu wrote the main manuscript text and all figures; Chuanzhi Sun prepared all tables. All authors reviewed the manuscript.

Declarations

Competing interests
The authors declare no competing interests.

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