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Picture fuzzy soft set TAOV approach for material selection for cryogenic storage tank for liquid nitrogen transportation

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A picture fuzzy soft set (PFSS) is a reliable tool for handling uncertainties in data through three membership degrees: positive, negative, and neutral which is common in real-world applications such as medical diagnoses, financial decisions or risk assessments. Material selection for cryogenic storage tanks used in liquid nitrogen transportation requires accuracy of multiple criteria under uncertainty. This study introduced picture fuzzy soft sets, a specific instance of PFSS, represented by membership and non-membership degrees and refusal. The proposed methodology is applied to rank candidate materials based on essential properties such as thermal conductivity, tensile strength, and cost-efficiency.

Keywords Soft set, Picture fuzzy soft set, TAOV method, MCDM

To address such TAOV systems effectively, a number of theories have been proposed in recent years, including probability theory, fuzzy set theory, soft set theory, fuzzy sets, rough set theory, etc. Numerous real-world issues in the fields of engineering, social and medical sciences, economics, etc., include imprecise data and require the application of mathematical concepts on the basis of uncertainty and imprecision to be solved[1,2]. The fuzzy soft set was first introduced by Zadeh[3], who limited its degree of membership to [0, 1]. Atanassov[4] further explained the fuzzy set concept to encompass the idea of an intuitionistic fuzzy set by introducing a new point known as the degree of membership value in addition to the non-member ship value. The idea of the picture fuzzy set was first suggested by Cuong and Kreinovich[5]. It has three degrees: membership (μ), non-membership (η), and refusal (ν). Both Biswas and Sarkar[6] and Ashraf et al.[7,8] concentrated on creating decision-making methods for group decision-making issues under fuzzy soft set features. As a solution to problems involving ambiguity or uncertainty, Molodtsov[9] created soft set theory. One of the most well-known approaches to solving decision-making issues is multicriteria decision-making (MCDM) based on Pythagorean fuzzy set[10]. The most popular method for handling uncertainty is the picture fuzzy soft set[11], which has three membership degrees: neutral, positive, and negative. The decision-making methods for group decision-making problems under picture fuzzy soft set environments were the focus of[5,12–17].

Picture Fuzzy Soft Sets (PFSS) offer a powerful mathematical framework for handling such complexities. PFSS extends traditional fuzzy sets by incorporating degrees of membership, non-membership, and neutrality, enabling a more nuanced representation of uncertain information. When combined with the Total Area of Orthogonal Vectors (TAOV) method, which quantifies and compares alternatives based on multi-criteria evaluations, this approach provides a comprehensive and reliable decision-making model. This study proposes a PFSS-TAOV-based methodology for selecting materials for cryogenic storage tanks designed for liquid nitrogen transportation. By integrating the strengths of PFSS in uncertainty modeling with the analytical capabilities of TAOV, the approach evaluates materials against critical properties such as thermal resistance, mechanical strength, and economic feasibility.

In an effort to solve decision-making difficulties, several academics and writers have defined numerous MCDM algorithms. These procedures are restricted by difficulties where the decision is made in terms of either yes or no or uncertainty. Saunders et al.[18], the identification of decision makers and stakeholders is the first step

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in decision making. This helps minimize disagreements on problem definition, needs, goals, and criteria. In an effort to solve decision-making difficulties, several academics and writers have defined numerous MCDM algorithms. These procedures are restricted by difficulties where the decision is made in terms of either yes or no or uncertainty. The "total area based on orthogonal vectors" (TAOV) MCDM technique, introduced by Hajiagha et al. in[13], is dependent on the orthogonality of decision criteria. To date, only a few uncertainty-based models have been presented in the literature, including fuzzy sets, intuitionistic fuzzy sets, rough sets, and soft sets. Other models include interval mathematics, probability theory and soft sets. This study enhances material selection methodologies by introducing a more robust, uncertainty-aware, and mathematically sound approach for cryogenic storage tank materials. The PFSS-TAOV method ensures a more accurate, flexible, and practical decision-making framework compared to traditional methods like TOPSIS and AHP.

The Total Area of Orthogonal Vectors (TAOV) method is useful for solving MCDM problems because it:

1. Handles complex uncertainty better, especially in fuzzy environments.
2. Provides clear geometric representation of alternatives for better distinction.
3. Offers robust differentiation between closely ranked options.
4. This method is less sensitive to criteria weight variations.
5. Works well with advanced fuzzy systems like Picture Fuzzy Soft Sets, unlike many traditional methods.

While other methods like TOPSIS, AHP, and VIKOR are effective, TAOV is more suitable for problems with high uncertainty and complex criteria.

The industrial applications of TAOV method:

1. *Quality control* Evaluates product attributes (durability, safety, etc.) for quality assurance.
2. *Supply chain* Compares suppliers on cost, delivery, and reliability to optimize selection.
3. *Risk management* Assesses risks in operations (environment, safety, cost) to choose the best strategy.
4. *Product design* Evaluates design options based on performance, safety, and cost.
5. *Environmental Impact* Compares project impacts on emissions, energy use, and waste.
6. *Process optimization* Compares production parameters to improve efficiency and reduce costs.

Objectives

This Paper has following objectives.

Multi-criteria evaluation and prioritization
To evaluate and prioritize alternatives using multiple criteria, taking into account the degrees of membership, non-membership, and hesitation inherent in picture fuzzy soft sets.

Decision-making under uncertainty
To balance different factors under uncertainty to make the best possible decision.

Framework development for TAOV in PFSS
To identify the picture fuzzy soft set basis for the TAOV approach.

Preliminaries

In this section, we briefly recall some related basic concepts. These concepts include fuzzy soft sets, intuitionistic fuzzy soft sets, Pythagorean fuzzy soft sets and picture fuzzy soft sets, and some examples are given to illustrate the concepts. The ∂ here represent the membership, $\propto$ shows the non-membership and μ represent the refusal.

Definition 2.1 [19] A fuzzy soft set A over a universe X and a soft parameter E is defined as a set ordered pairs (e, A_e),where $\mathrm{e} \in \mathrm{E}$ and A_e is an fuzzy set in X.

Mathematically, an FSS can be represented as:

$$\mathrm{A_e} = \{\mathrm{e}, \partial_{\mathrm{A_e}}(\mathrm{x}); \mathrm{x} \in \mathrm{X}\} \quad (1)$$

where , $\partial_{\mathrm{A_e}} : \mathrm{X} \to [0,1]$ with the condition

$$0 \leq \partial_{\mathrm{A_e}}(\mathrm{x}) \leq 1; \forall \mathrm{x} \in \mathrm{X}.$$

Definition 2.2 [20] An intuitionistic fuzzy soft set A over a universe X and a soft parameter E is defined as a set of ordered pairs (e, A_e),where $\mathrm{e} \in \mathrm{E}$ and A_e is an intuitionistic fuzzy set in X.

Mathematically, an IFSS can be represented as:

$$\mathrm{A_e} = \{\mathrm{e}, \partial_{\mathrm{A_e}}(\mathrm{x}), \propto_{\mathrm{A_e}}(\mathrm{x}) : \mathrm{x} \in \mathrm{X}\}$$

where , $\partial_{\mathrm{A_e}} : \mathrm{X} \to [0,1]$ and $\propto_{\mathrm{A_e}} : \mathrm{X} \to [0,1]$ satisfy the following condition:

$$0 \leq \partial_{\mathrm{A_e}}(\mathrm{x}) + \propto_{\mathrm{A_e}}(\mathrm{x}) \leq 1;\ \forall \mathrm{x} \in \mathrm{X}$$

The values $\partial_{\mathrm{A}}(\mathrm{x})$ and $\propto_{\mathrm{A}}(\mathrm{x})$ represents the membership degree and non-membership degree respectively of the element X to the set A.

Definition 2.3 [10] An Pythagorean fuzzy soft set A over a universe X and a soft parameter E is defined as a set of ordered pairs (e, A_e),where $\mathrm{e} \in \mathrm{E}$ and A_e is an Pythagorean fuzzy set in X.

Mathematically, an PFSS can be represented as:

$$\mathrm{A_e} = \{\mathrm{e}, \partial_{\mathrm{A_e}}(\mathrm{x}), \propto_{\mathrm{A_e}}(\mathrm{x}), \mu_{\mathrm{A_e}}(\mathrm{x}) : \mathrm{x} \in \mathrm{X}\}$$

where satisfy the following $\partial_{\mathrm{A_e}}(\mathrm{x}), \propto_{\mathrm{A_e}}(\mathrm{x})$ and $\mu_{\mathrm{A_e}}(\mathrm{x})$ condition:

$$0 \leq \partial_{\mathrm{A_e}}(\mathrm{x})^2 + \mu_{\mathrm{A_e}}(\mathrm{x})^2 \leq 1$$

Definition 2.4 [11] An picture fuzzy soft set A over a universe X and a soft parameter E is defined as a set of ordered pairs (e, A_e),where $\mathrm{e} \in \mathrm{E}$ and A_e is an picture fuzzy set in X.

Mathematically, an PFSS can be represented as:

$$\mathrm{A_e} = \{\mathrm{e}, \partial_{\mathrm{A_e}}(\mathrm{x}), \propto_{\mathrm{A_e}}(\mathrm{x}), \mu_{\mathrm{A_e}}(\mathrm{x}) : \mathrm{x} \in \mathrm{X}\} \tag{2}$$

where satisfy the following $\partial_{\mathrm{A_e}}(\mathrm{x}), \propto_{\mathrm{A_e}}(\mathrm{x})$ and $\mu_{\mathrm{A_e}}(\mathrm{x})$ condition:

$$0 \leq \partial_{\mathrm{A_e}}(\mathrm{x}) + (\mathrm{x}) + \mu_{\mathrm{A_e}}(\mathrm{x}) \leq 1$$

Here $\sqrt{1 - \partial_{\mathrm{A_e}}(\mathrm{x}) + (\mathrm{x}) + \mu_{\mathrm{A_e}}(\mathrm{x})}$; is called the degree of refusal membership of X in A.

Theorem [11] Let P_1 and P_2 be the two PFSs; then, $\mathrm{C}_a(P_1, P_2; A)$ is a distance measure.

Proof: It is clear that $\mathrm{C}_a(P_1, P_2; A) \geq 0$.

Furthermore, we have

$$0 \leq \mathrm{A}(\partial_1(\mathrm{x_i}), \partial_1(\mathrm{x_j})) \leq 1$$

and

$$0 \leq \mathrm{A}(\partial_2(\mathrm{x_i}), \partial_2(\mathrm{x_j})) \leq 1$$

From the Perron-Frobenius theorem, it follows that the

$$\|{}^\circ\mathrm{F}(\partial_1(x)) - {}^\circ\mathrm{F}(\partial_2(x))\| \leq \sqrt{n}$$

Similarly, we have

$$\|{}^\circ\mathrm{F}(\propto_1(x)) - {}^\circ\mathrm{F}(\propto_2(x))\| \leq \sqrt{n}$$
$$\|{}^\circ\mathrm{F}(\mu_1(x)) - {}^\circ\mathrm{F}(\mu_2(x))\| \leq \sqrt{n}$$

and

$$\|{}^\circ\mathrm{F}(\gamma_1(x)) - {}^\circ\mathrm{F}(\gamma_2(x))\| \leq \sqrt{n}$$

Thus,

$$0 \leq \mathrm{C}_a \leq 1$$

If $\mathrm{C}_a(P_1, P_2; A) = 0$, then we have

$$\|{}^\circ\mathrm{F}(\partial_1(x)) - {}^\circ\mathrm{F}(\partial_2(x))\| + \|{}^\circ\mathrm{F}(\propto_1(x)) - {}^\circ\mathrm{F}(\propto_2(x))\| + \|{}^\circ\mathrm{F}(\mu_1(x)) - {}^\circ\mathrm{F}(\mu_2(x))\| + \|{}^\circ\mathrm{F}(\gamma_1(x)) - {}^\circ\mathrm{F}(\gamma_2(x))\| = 0$$

By definition of norm, we have

$$^\circ\mathrm{F}(\partial_1(x)) = {}^\circ\mathrm{F}(\partial_2(x))$$
$$^\circ\mathrm{F}(\propto_1(x)) - {}^\circ\mathrm{F}(\propto_2(x))$$
$$^\circ\mathrm{F}(\mu_1(x)) - {}^\circ\mathrm{F}(\mu_2(x))$$

$$^{\circ}\mathrm{F}\left(\gamma_1\left(x\right)\right)-{}^{\circ}\mathrm{F}\left(\gamma_2\left(x\right)\right)$$

Then, we obtain

$$\mathrm{A}\left(\partial_1\left(\mathrm{x_i}\right),\partial_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\partial_2\left(\mathrm{x_i}\right),\partial_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\propto_1\left(\mathrm{x_i}\right),\propto_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\propto_2\left(\mathrm{x_i}\right),\propto_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\mu_1\left(\mathrm{x_i}\right),\mu_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\mu_2\left(\mathrm{x_i}\right),\mu_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\gamma_1\left(\mathrm{x_i}\right),\gamma_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\gamma_2\left(\mathrm{x_i}\right),\gamma_2\left(\mathrm{x_j}\right)\right)$$

As A is a strictly increasing (or decreasing) function, we obtain

$$P_1=P_2$$

Conversely, if $P_1=P_2$, we obtain

$$\partial_1\left(\mathrm{x_i}\right)=\partial_2\left(\mathrm{x_i}\right)$$
$$\propto_1\left(\mathrm{x_i}\right)=\propto_2\left(\mathrm{x_i}\right)$$
$$\mu_1\left(\mathrm{x_i}\right)=\mu_2\left(\mathrm{x_i}\right)$$
$$\gamma_1\left(\mathrm{x_i}\right)=\gamma_2\left(\mathrm{x_i}\right)$$

Because A is a strictly increasing (or decreasing) function, we have

$$\mathrm{A}\left(\partial_1\left(\mathrm{x_i}\right),\partial_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\partial_2\left(\mathrm{x_i}\right),\partial_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\propto_1\left(\mathrm{x_i}\right),\propto_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\propto_2\left(\mathrm{x_i}\right),\propto_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\mu_1\left(\mathrm{x_i}\right),\mu_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\mu_2\left(\mathrm{x_i}\right),\mu_2\left(\mathrm{x_j}\right)\right)$$
$$\mathrm{A}\left(\gamma_1\left(\mathrm{x_i}\right),\gamma_1\left(\mathrm{x_j}\right)\right)=\mathrm{A}\left(\gamma_2\left(\mathrm{x_i}\right),\gamma_2\left(\mathrm{x_j}\right)\right)$$

Then,

$$\left\|^{\circ}\mathrm{F}\left(\partial_1\left(x\right)\right)-{}^{\circ}\mathrm{F}\left(\partial_2\left(x\right)\right)\right\|=0$$
$$-\left\|^{\circ}\mathrm{F}\left(\propto_1\left(x\right)\right)-{}^{\circ}\mathrm{F}\left(\propto_2\left(x\right)\right)\right\|=0$$
$$\left\|^{\circ}\mathrm{F}\left(\mu_1\left(x\right)\right)-{}^{\circ}\mathrm{F}\left(\mu_2\left(x\right)\right)\right\|=0$$
$$\left\|^{\circ}\mathrm{F}\left(\gamma_1\left(x\right)\right)-{}^{\circ}\mathrm{F}\left(\gamma_2\left(x\right)\right)\right\|=0$$

Thus, $\mathrm{C}_a\left(P_1,P_2;A\right)=0$.

Hence, $\mathrm{C}_a\left(P_1,P_2;A\right)=0$ if $P_1=P_2$.

Summary of the TAOV approach

Picture fuzzy soft set relation with the TAOV

In this section, we present the new MCDM methodology with the TAOV method. First, we state some basic MCDM problems with completely unknown weights. This proposed a new weight-determination method to compute the criteria weight by using proposed distance measures. Later, we stated a novel MCDM-TAOV method to solve the DMPs in this section. As shown in Fig. 1, the TAOV method involves a series of interconnected steps, which are summarized in the flowchart. Equations (3) and (4) are used to find the normalized decision matrix,Utilizing Eq. (5), determine that the principal component analysis by finding the weight criteria. The total area of each alternative is determined via Eq. (6) and find the ranking of each alternative using Eq. (7).

We execute the steps of the proposed weight determination method to determine the criteria weight, as described in this section.

Phase 1. Initialization

Step 1 Identify decision alternatives and decision criteria.

Step 2 Construct decision matrix $\mathrm{X}=[x_{ij}]$.

Step 3 Determine the criteria weight vector $\mathrm{w}=(\mathrm{w}_{1,}\ \mathrm{w}_{2,}\ldots\mathrm{w}_{n})$ via the pairwise comparison method.

Step 4 Normalize the decision matrix.

For benefit criteria:

$$\mathrm{r_{ij}}=\frac{x_{ij}}{max_i x_{ij}} \tag{3}$$

For cost criteria:

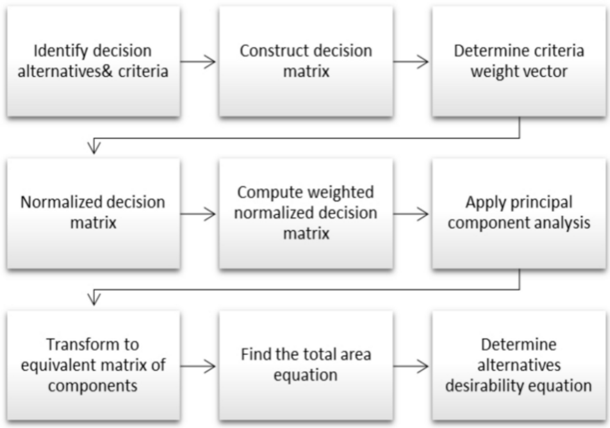

Fig. 1. Flow Chart of the TAOV Method.

$$\mathrm{r_{ij}} = \frac{min_i x_{ij}}{x_{ij}} \quad (4)$$

Step 5 Compute the weighted normalized decision matrix WN.

By using the normalized decision matrix and multiply with weight which is obtained from pairwise comparison method.

Phase 2: Orthogonalization

Step 6 Principal component analysis is applied, and the WN matrix is transformed to the equivalent matrix of components.

$$\mathrm{Z} = \mathrm{value} - \mathrm{mean} \backslash \mathrm{S.D} \quad (5)$$

Phase 3: Comparison

Step 7 Applying the ranking order of alternatives.

$$\mathrm{TA} = \sum_{j=1}^{n} \sqrt{(\boldsymbol{k_j})^2 + (\boldsymbol{k_{j+1}})^2} + \sqrt{(\boldsymbol{k_{j+1}})^2 + (\boldsymbol{k_{j+2}})^2} \quad (6)$$

Step 8 Find the total area of each alternative. The score function is then applied for ranking.

$$\mathrm{S}\,(\mathrm{o}) = \mu - \eta - \ln(1 + \nu) \quad (7)$$

$$\mathrm{S}\,(\mathrm{o}) = \mu - \eta \quad (8)$$

Decision making method based on the picture fuzzy soft set

Numerical example

We explore a problem involving the application of our recommended approach to assess and rank the best materials for liquid nitrogen transportation in cryogenic storage tanks[10].

To ensure transparency in the decision-making process, the specific values for membership, non-membership, and refusal in the Picture Fuzzy Soft Set (PFSS) framework were determined based on expert evaluations and standardized criteria.

Membership

Membership values represent the degree to which an alternative satisfies a particular criterion.

Non-membership

Non-membership values reflect the degree to which an alternative fails to meet a criterion. These were calculated using the complement of membership values, adjusted for any uncertainties expressed by experts.

Refusal

Refusal values account for hesitancy or neutrality in expert opinions, capturing uncertainty in evaluations. These values were determined through a consensus among experts when precise agreement on membership or non-membership was not achievable.

Description

The transport of – 1960C is common in cryogenic storage tanks. Because cryogenic storage tanks must endure low temperatures and pressure changes during shipping, material selection is essential. To choose the right material for the cryogenic storage tank, a thorough assessment of each material's mechanical characteristics, thermal conductivity, and resilience to low-temperature embrittlement was conducted. 304 stainless steel was ultimately chosen after a number of materials, including carbon steels, stainless steels, and aluminum alloys, were weighed for their advantages and disadvantages.

Toughness index
Designing and building cryogenic storage tanks for the transfer of liquid nitrogen requires careful consideration of the toughness index. The toughness index quantifies the resistance to brittle fracture at low temperatures, which is an essential property for maintaining the tank's structural integrity while in operation. Materials may become brittle and more prone to breaking or cracking when exposed to temperatures as low as – 1960C (3210F), which is the temperature of liquid nitrogen used in cryogenic applications. High toughness indices are required for materials used in cryogenic tanks to endure the low temperatures and possible mechanical stresses that may arise during handling and transit.

Yield strength
To maintain the structural integrity and safety of cryogenic storage tanks used to transport liquid nitrogen, the yield strength of the materials used is crucial. Under stress, materials undergo deformation, and their ability to withstand stress before permanent deformation occurs is termed the yield strength. In cryogenic settings, where temperatures are exceptionally low, materials tend to become more brittle, thereby reducing their yield strength and increasing their susceptibility to cracking or failure. Hence, the selection of materials for cryogenic storage tanks necessitates prioritizing those with high yield strength, even under low-temperature conditions.

Young's modulus
The Young's modulus is crucial in cryogenic storage tank design, as it measures a material's stiffness and ability to withstand stress, as brittle materials may cause irreversible plastic deformation. Because of their high Young's moduli, materials such as nickel alloys, aluminum, and stainless steel are preferred for use in cryogenic tanks. Notably, at cryogenic temperatures, a material's Young's modulus may change. This highlights the necessity of extensive testing and analysis to determine the material's resilience at the targeted working temperature.

Application of alternatives This section provides a comparative analysis of four firms that were assessed on the basis of their ability to manufacture cryogenic liquid storage tanks. The evaluation was conducted utilizing essential material qualities such as yield strength, toughness index, and Young's modulus. Pak Oxygen Limited, Techno Chemicals, Linde Limited Pakistan, and Dynamic Engineering & Automation (DEA) are the companies that were examined.

Literature review
Various Multi-Criteria Decision-Making (MCDM) methods have been employed to tackle this complex problem, including widely recognized techniques like TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), AHP (Analytic Hierarchy Process), and VIKOR. These methods have demonstrated effectiveness in addressing material selection challenges but also exhibit certain limitations when applied to problems characterized by high levels of uncertainty and hesitancy.

TOPSIS is a popular MCDM approach that evaluates alternatives based on their distance from an ideal solution and an anti-ideal solution. AHP, on the other hand, focuses on pairwise comparisons of criteria and alternatives, facilitating a hierarchical structuring of decision problems. Its strength lies in its ability to handle qualitative and quantitative criteria simultaneously. VIKOR emphasizes ranking and selecting alternatives based on a compromise solution that minimizes regret. It is particularly useful for solving decision problems involving conflicting criteria.

Decision model
The company expert gives the information of the data in the form of membership, non-membership and refusal membership. In accordance with the rules stated in Eq. (2), we assume that the decision matrix of the picture fuzzy soft set is in Table 1.
Step I Eqs. (3) and (4) are used to find the normalized decision matrix, which is written as.

Table 2.
Step II The weight vector criteria are W = (0.31, 0.16, 0.25), (0.31, 0.13, 0.28), (0.2, 0.33, 0.21), and (0.17, 0.36, 0.25).

By using the pairwise comparison shown in Tables 3 and 4.
Step III Utilizing Eq. (5), determine that the principal component analysis is written in Table 5, and Table 6 shows the orthogonalization decision matrix.

	K_1	K_2	K_3
O_1	(0.3, 0.2, 0.4)	(0.5, 0.2, 0.1)	(0.6, 0.1, 0.2)
O_2	(0.5, 0.1, 0.3)	(0.8, 0.1, 0.0)	(0.1, 0.2, 0.5)
O_3	(0.1, 0.4, 0.2)	(0.3, 0.4, 0.2)	(0.5, 0.2, 0.2)
O_4	(0.4, 0.0, 0.5)	(0.2, 0.6, 0.1)	(0.2, 0.5, 0.1)

Table 1. Expert information fuzzy decision matrix.

	K_1	K_2	K_3
O_1	(0.5, 0.33, 0.660)	(0.833, 0.33, 0.16)	(0.16, 1, 0.5)
O_2	(0.62, 0.12, 0.37)	(1, 0.12, 0)	(1, 0.5, 0.2)
O_3	(0.2, 0.8, 0.4)	(0.6, 0.8, 0.25)	(0.2, 0.5, 0.5)
O_4	(0.66, 0, 0.83)	(0.33, 1, 0.16)	(0.5, 0.2, 1)

Table 2. The normalized decision matrix R is constituted:

	K_1	K_2	K_3	Sum	Weight
O_1	(0.3, 0.2, 0.4)	(0.5, 0.2, 0.1)	(0.6, 0.1, 0.2)	(1.4, 0.5, 0.7)	(0.311, 0.16, 0.25)
O_2	(0.5, 0.1, 0.3)	(0.8, 0.1, 0.0)	(0.1, 0.2, 0.5)	(1.4, 0.4, 0.8)	(0.311, 0.13, 0.28)
O_3	(0.1, 0.4, 0.2)	(0.3, 0.4, 0.2)	(0.5, 0.2, 0.2)	(0.9, 1, 0.6)	(0.2, 0.33, 0.21)
O_4	(0.4, 0.0, 0.5)	(0.2, 0.6, 0.1)	(0.2, 0.5, 0.1)	(0.8, 1.1, 0.7)	(0.17, 0.36, 0.25)
				(4.5, 3, 2.8)	(1, 1, 1)

Table 3. The weight vectors of the criteria are determined via pairwise comparison. Weight = 1.4\4.5 = 0.311. The weight vectors of the criteria are W = (0.31, 0.16, 0.25), (0.31, 0.13, 0.28), (0.2, 0.33, 0.21), and (0.17, 0.36, 0.25).

	K_1	K_2	K_3
O_1	(0.15, 0.05, 0.16)	(0.25, 0.05, 0.04)	(0.04, 0.16, 0.12)
O_2	(0.19, 0.01, 0.10)	(0.31, 0.01, 0)	(0.31, 0.06, 0.05)
O_3	(0.4, 0.26, 0.08)	(0.12, 0.26, 0.05)	(0.04, 0.16, 0.10)
O_4	(0.11, 0.0, 0.20)	(0.05, 0.36, 0.04)	(0.08, 0.07, 0.25)

Table 4. Weighted normalized decision matrix.

	K_1	K_2	K_3
Mean	(0.12, 0.08, 0.13)	(0.18, 0.17, 0.13)	(0.11, 0.11, 0.13)
S.D	(0.08, 0.1, 0.09)	(0.06, 0.09, 0.05)	(0.07, 0.01, 0.04)

Table 5. Principal component analysis.

	K_1	K_2	K_3
O_1	(0.37, − 0.3, 0.33)	(1.16, − 1.33, − 1.8)	(− 1, 5, − 0.25)
O_2	(0.87, − 0.7, − 0.33)	(2.16, − 1.77, -2.6)	(2.85, -5, -2)
O_3	(-1, 1.8, -0.55)	(-1, 1, -1.6)	(-1, 5, -0.75)
O_4	(-0.12, -0.18, 0.77)	(-2.16, 2.11, -1.8)	(-0.42, -4, 3)

Table 6. Orthogonalization decision matrix.

Step IV The total area of each alternative is determined via Eq. (6), which is based on Table 7.
Step V Last, on the basis of the general values, the ranking order of alternatives is determined as follows:

$$O_2 > O_4 > O_1 > O_3$$

Apparently, O_2 is the most desirable alternative. Table 8 shows the ranking of each alternative via Eq. (7).

Figure 2 shows the ranking order of all the alternatives. The likely second alternative is our best choice after finding the values of alternatives.

$$O_2 > O_4 > O_1 > O_3$$

	TA
O_1	(2.73, 6.52, 3.63)
O_2	(5.88, 7.2, 5.88)
O_3	(2.82, 7.14, 3.45)
O_4	(4.36, 6.62, 5.44)

Table 7. Total area of alternatives.

		Ranking
O_1	− 4.45	3
O_2	− 2.15	1
O_3	− 4.96	4
O_4	− 3.06	2

Table 8. Ranking orders of alternatives.

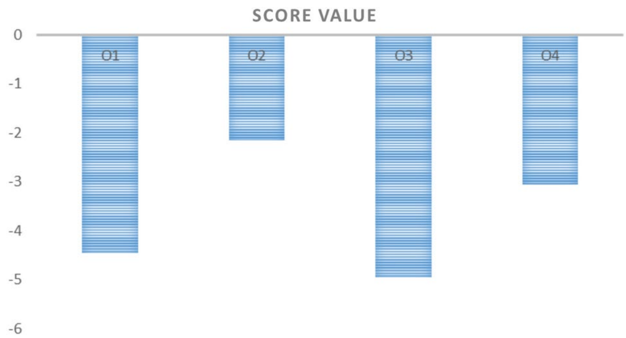

Fig. 2. Ranking alternatives in picture fuzzy soft set value.

	K_1	K_2	K_3
O_1	0.3	0.5	0.6
O_2	0.5	0.8	0.1
O_3	0.1	0.3	0.5
O_4	0.4	0.2	0.2

Table 9. Expert information fuzzy decision matrix.

	K_1	K_2	K_3
O_1	0.5	0.833	0.5
O_2	0.625	1	1
O_3	0.2	0.6	0.2
O_4	1	0.5	1

Table 10. The normalized decision matrix R is constituted:

Comparison analysis

This section provides a comparative analysis of four firms that were assessed on the basis of their ability to manufacture cryogenic liquid storage tanks. The evaluation was conducted utilizing essential material qualities such as yield strength, toughness index, and Young's modulus. Pak Oxygen Limited, Techno Chemicals, Linde Limited Pakistan, and Dynamic Engineering & Automation (DEA) are the companies that were examined.

The company expert gives the information of data in the form of membership that was given in Table 9. (3) and (4) are used to find the normalized decision matrix, which is written in Table 10.

	K_1	K_2	K_3
O_1	0.15	0.25	0.15
O_2	0.19	0.31	0.31
O_3	0.4	0.12	0.4
O_4	0.17	0.08	0.17

Table 11. Weighted normalized decision matrix:

	K_1	K_2	K_3
Mean	0.22	0.19	0.25
S.D	0.11	0.10	0.11

Table 12. Principal component analysis:

	K_1	K_2	K_3
O_1	− 0.63	0.6	− 0.90
O_2	− 0.27	1.2	0.54
O_3	1.63	− 0.7	1.36
O_4	− 0.45	− 1.1	− 0.72

Table 13. Orthogonalization decision matrix.

	TA	Ranking
O_1	1.94	4
O_2	2.54	3
O_3	4.66	1
O_4	3.48	2

Table 14. Ranking orders of alternatives. $O_3 > O_4 > O_2 > O_1$

A weighted normalized decision matrix is given in Table 11. By using Eq. (5), the principal component analysis results are written in Tables 12, and Table 13 shows the orthogonalization decision matrix. The total area of each alternative is determined via Eq. (6), which is based on Table 14. Figure 3 shows the ranking order of all the alternatives.

Fuzzy soft set

Intuitionistic fuzzy soft set

The company expert gives the information of the data in the form of membership and non-membership, which is given in Table 15. (3) and (4) are used to find the normalized decision matrix, which is written in Table 16. A weighted normalized decision matrix is given in Table 17. By using Eq. (5), the principal component analysis results are written in Table 18, and Table 19 shows the orthogonalization decision matrix. The total area of each alternative is determined via Eq. (6) on the basis of Table 20. The next step is to determine the ranking order of alternatives by using (8) on the basis of Table 21. The ranking order of all the alternatives is shown in Fig. 4.

$$O_2 > O_1 > O_3 > O_4$$

Analyzing the results

Table 22 shows that Linde Limited Pakistan had the best performance across all important material attributes, according to the data that were supplied. Linde Limited Pakistan is the greatest option for manufacturing cryogenic liquid storage tanks because it performs better than other companies do. My thorough study of yield strength, toughness index, and Young's modulus indicates that Linde Limited Pakistan is the top manufacturer of cryogenic liquid storage tanks. They are the best option for this special equipment because of their outstanding material qualities, which guarantee excellent performance and dependability. Figure 5 shows the ranking order of all the alternatives of the picture fuzzy soft set, fuzzy soft set and iterative fuzzy soft set. While the focus on Linde Limited Pakistan is comprehensive, a clearer comparison with the second-ranked alternative would enhance the discussion. Key differentiators such as material durability, cost-effectiveness, and operational

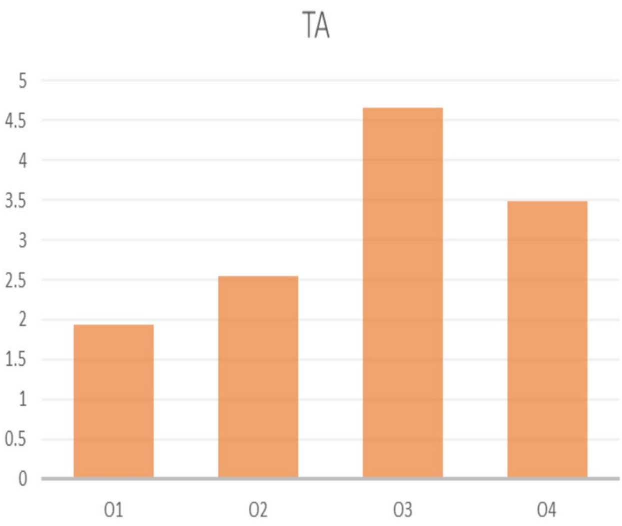

Fig. 3. Ranking alternatives in fuzzy soft set value.

	K_1	K_2	K_3
O_1	(0.3, 0.2)	(0.5, 0.2)	(0.6, 0.1)
O_2	(0.5, 0.1)	(0.8, 0.1)	(0.1, 0.2)
O_3	(0.1, 0.4)	(0.3, 0.4)	(0.5, 0.2)
O_4	(0.4, 0.0)	(0.2, 0.6)	(0.2, 0.5)

Table 15. Expert information fuzzy decision matrix.

	K_1	K_2	K_3
O_1	(0.5, 0.33)	(0.833, 0.33)	(0.16, 1)
O_2	(0.62, 0.12)	(1, 0.12)	(1, 0.5)
O_3	(0.2, 0.8)	(0.6, 0.8)	(0.2, 0.5)
O_4	(0.66, 0)	(0.33, 1)	(1, 0.4)

Table 16. The normalized decision matrix R is constituted:

	K_1	K_2	K_3
O_1	(0.15, 0.05)	(0.25, 0.05)	(0.04, 0.16)
O_2	(0.19, 0.01)	(0.31, 0.01)	(0.31, 0.15)
O_3	(0.04, 0.26)	(0.12, 0.26)	(0.04, 0.16)
O_4	(0.11, 0.0)	(0.05, 0.36)	(0.17, 0.14)

Table 17. Weighted normalized decision matrix:

	K_1	K_2	K_3
Mean	(0.12, 0.08)	(0.18, 0.17)	(0.14, 0.15)
S.D	(0.02, 0.03)	(0.21, 0.35)	(0.06, 0.01)

Table 18. Principal component analysis:

	K_1	K_2	K_3
O_1	(1.5, − 1)	(0.33, − 0.34)	(− 1.66, 1)
O_2	(3.5, − 2.3)	(0.61, − 0.45)	(2.83, 0)
O_3	(− 4, 6)	(− 0.28, 0.25)	(− 1.66, 1)
O_4	(− 0.5, − 2.66)	(− 0.61, 0.54)	(0.5, − 1)

Table 19. Orthogonalization decision matrix.

	TA
O_1	(3.22, 2.1)
O_2	(6.43, 2.78)
O_3	(5.68, 7.02)
O_4	(1.56, 3.84)

Table 20. Total area of alternatives.

		Ranking
O_1	1.12	2
O_2	3.65	1
O_3	− 1.34	3
O_4	− 2.28	4

Table 21. Ranking orders of alternatives.

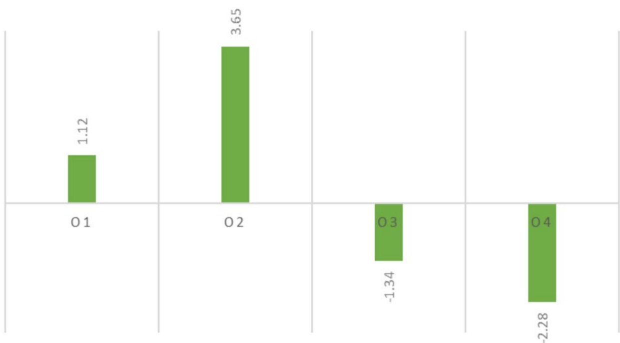

Fig. 4. Ranking of the alternatives of the iterative fuzzy soft set value.

	FSS		IFSS		PFSS	
O_1	1.94	4	1.12	2	− 4.45	3
O_2	2.54	3	3.65	1	− 2.15	1
O_3	4.66	1	− 1.34	3	− 4.96	4
O_4	3.48	2	− 2.28	4	− 3.06	2
	$O_3 > O_4 > O_2 > O_1$		$O_2 > O_1 > O_3 > O_4$		$O_2 > O_4 > O_1 > O_3$	

Table 22. Analyzing the results.

efficiency should be highlighted to justify the ranking. A pairwise comparison or percentage difference analysis can further strengthen the credibility of the results.

Conclusion

This study on picture fuzzy soft sets (PFSS) has resulted in a standard matrix based distance measure that combines a strictly monotonic function with its properties. The proposed measure satisfies axiomatic properties

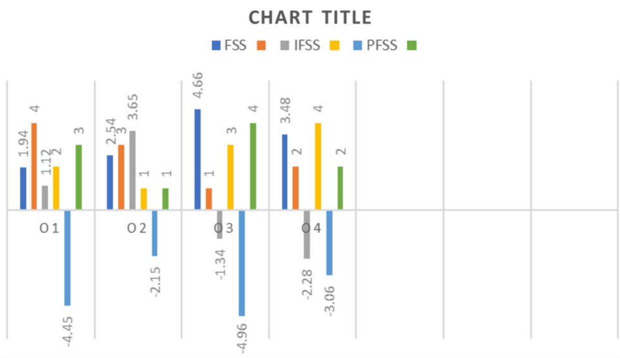

Fig. 5. Comparison between existing and proposed values.

and ranks PFSS pairs effectively. Unlike prevalent methods, the MCDM-TAOV approach developed for the PFSS environment demonstrates a formal procedure ensuring criterion independence. Sensitivity analysis and statistical validation confirm the method's significant impact. By applying this methodology, Linde Limited Pakistan emerged as the top manufacturer of cryogenic liquid storage tanks, scoring highest on key material properties and overall performance. The study illustrates the effectiveness of the new measures and the robustness of the MCDM-TAOV approach in decision-making scenarios. In future cryogenic storage, studies in Picture Fuzzy Soft Sets (PFSS) for use in supply chain management, healthcare, and finance, among other industries. PFSS can be used in medical decision-making, especially in diagnostic procedures where imprecision and ambiguity are frequent. By offering a more accurate depiction of ambiguous data, it can improve decision-making in supply chain management and aid in the optimization of logistics and inventory control. PFSS provides more accurate forecasting models and can be applied to risk analysis and investment strategies in the financial industry.

Discussion

According to the study, the higher material qualities of Linde Limited Pakistan's cryogenic liquid storage tanks such as yield strength, toughness, and Young's modulus allow them to perform better than those of other manufacturers. The MCDM-TAOV approach, which employs a standard matrix based distance measure to effectively rank alternatives and guarantee independence in decision criteria, supports this finding. Recent advancements have significantly improved decision-making in uncertain contexts with extensions such as spherical fuzzy sets[1] and interval-valued picture fuzzy sets. These developments strengthen the MCDM-TAOV strategy for use in the future by enabling more accurate and adaptable handling of data uncertainty, which benefits domains like supply chain management and green supplier evaluation.

Data availability

The datasets used or analyzed during the current study are available from the corresponding author upon reasonable request.

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Author contributions
S.M., A.A., Z.M., M.N., and S.M. contributed to the conceptual framework and methodology. Z.M. conducted the main computational experiments, developed the MCDM-TAOV algorithm, and led the statistical validation. S.M., A.A., and M.N. prepared and processed the data for the case studies. S.M. and S.M. wrote the main manuscript text. A.A. and M.N. prepared Figs. 1–5. Z.M. reviewed and revised the manuscript, ensuring technical accuracy and integrity. All authors reviewed and approved the final manuscript.

Declarations

Competing interests
The authors declare no competing interests.

Additional information
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