scientific reports

OPEN

A novel intuitionistic fuzzy Yager aggregation framework for decision making in green supply chains

Yeshvandhini Kumar[1], Sujatha Ramalingam[1]✉ & Gizachew Bayou Zegeye[2]✉

Aggregation operators constitute the core mathematical mechanism of multi-criteria decision-making by integrating multiple attribute evaluations into a single representative assessment. However, in intuitionistic fuzzy environments characterized by uncertainty, vagueness, and human hesitation, many existing aggregation operators inadequately capture nonlinear interactions among criteria and often weaken the influence of hesitancy information during aggregation. This paper examines the role of Yager-based aggregation mechanisms in preserving the structural integrity of intuitionistic fuzzy information and improving decision reliability under ambiguous conditions. A comprehensive family of Yager-based intuitionistic fuzzy aggregation operators is developed for multi-attribute decision-making problems, including weighted, ordered weighted, hybrid weighted averaging, and corresponding geometric operators. The principal advantage of the proposed operators lies in their flexible control of conjunction and disjunction behaviour through Yager's t-norm and t-conorm, which enables a more balanced treatment of truth, indeterminacy, and falsity degrees compared with existing approaches. The system achieves logical stability through its core axiomatic properties which include idempotency and monotonicity and boundedness and commutativity. The framework shows stable ranking results which produce understandable outputs that identify differences between options during the selection process of eco-friendly companies in green supply chain management. The results show that Yager-based intuitionistic fuzzy aggregation creates a strong decision-support system which helps users make sustainable choices in complicated industrial ecosystems.

Keywords Intuitionistic fuzzy numbers, Yager t-norm and t-conrom, Aggregation operators, Decision making, Green supply chain management

Decision-making is a central part of cognition; it entails virtual experiences of choosing between different options. It is omnipresent in human and organizational behaviour at individual, team, and organizational levels. We make decisions every day, from what to wear in the morning to the architecture of the organization. Options get weighed up or considered, with advantages balanced against disadvantages, before a decision is reached about which action course to follow – one that best fits the goal in question or the choice preferences. The decision process that leads to a choice might be explicit and involve rational estimates, or it might be half-conscious or intuitive. One typical example of control in decision-making is the application of crisp sets exact, binary differentiations, or dichotomies with direct causal relations between exact inputs and exact outputs. Real-life circumstances, however, are often fuzzy, inexact, and uncertain, with information and knowledge of uncertain quality. Therefore, Zadeh introduced fuzzy set theory which excel in these applications by accommodating the inherent variability and imprecision[1]. Although the fuzzy sets serve as the foundation for the uncertain evaluations, they fall short of fully describing the non-membership degree.

To address this shortcoming of fuzzy sets, Atanassov created intuitionistic fuzzy sets (IFSs)[2] which are effective and superior than regular Fuzzy sets due to their dual degrees of satisfaction and discontent. The major problem in MADM systems is the complexity of synthesizing and comparing many attributes or criteria with different units of measurement or scales. This complexity can make it difficult to reach a consensus conclusion

[1]Mathematics, School of Science and Humanities, Shiv Nadar University Chennai, Rajiv Gandhi Salai (OMR), Kalavakkam, Chengalpattu District, Tamil Nadu 603110, India. [2]Department of Mathematics, Bahir Dar University, P.O. Box,79, Bahir Dar, Ethiopia. ✉email: sujatharamalingam@snuchennai.edu.in; gizachew.bayou@bdu.edu.et

that correctly reflects stakeholders' preferences and objectives. To meet this challenge, the concept of operators is established. These operators offer a systematic way to combining and normalizing multiple criteria, allowing decision-makers to properly compare and prioritize qualities using various units of measurement or scales. Pei, Z., and Zheng, L. investigated the use of IFSs in decision-making situations[3]. Xu investigated the concepts of averaging operators based on IF sets[4]. Xu and Yager investigated several weighted, ordered weighted, and hybrid operators on geometricity in an IFSs context[5]. Wei investigated the stochastic Intuitionistic based MADM problems[6]. Liu et al. developed the theory of type-2 hesitant fuzzy sets by integrating hesitant fuzzy information with type-2 fuzzy membership structures, thereby overcoming limitations related to repeated membership values in hesitant fuzzy sets[7]. Naim and Hagras introduced a type-2 hesitation fuzzy logic–based multi-criteria group decision-making system that combines interval type-2 fuzzy sets with intuitionistic fuzzy hesitation indices[8]. Simić et al. proposed a picture fuzzy group MCDM framework for railway infrastructure risk assessment by integrating entropy-based weighting and the MARCOS method, highlighting the suitability of picture fuzzy modelling for complex risk and infrastructure evaluation problems[9]. Özlü & Sezgin introduced the concept of soft covered ideals in semigroups and investigated their structural properties and interrelationships[10]. Özlü developed picture type-2 hesitant fuzzy sets and vector similarity measures for MCDM applications. The study demonstrates improved discrimination capability and robustness through consistency and comparative analyses under multiple fuzzy environments Their work extends soft set theory into algebraic structures, highlighting its applicability in uncertainty modelling and decision-making contexts[11].

Alcantud recently developed the ability to aggregate unlimited chains of IF sets using temporal intuitionistic decision-making procedures[12]. Wei introduced induced geometric aggregation operators based on IF context for group decision-making[13]. Zhao, X., and Wei explored the IF Einstein hybrid aggregation operators[14]. IF Hamacher aggregation operators were discussed by Huang[15]. The concept of "Intuitionistic fuzzy t- norms and t-conorms" was discussed by Deschrijver[16,17]. Li discussed the concept of "generalized ordered weighted averaging operators" for intuitionistic fuzzy sets (IFS)[18]. Radhakrishnan et al. introduced the concept of single valued neutrosophic sets as a generalization of fuzzy and intuitionistic fuzzy sets by incorporating truth, indeterminacy, and falsity memberships[19]. Deli et al. developed single-valued neutrosophic sets and applied them to multi-attribute decision-making problems by proposing aggregation operators and score functions. The study demonstrated that single-valued neutrosophic models outperform traditional fuzzy-based methods in handling uncertainty and hesitation in practical decision scenarios[20]. Karaaslan et al. extended single-valued neutrosophic sets to the Type-2 framework to address higher-order uncertainty. Their work enhanced the expressive capability of neutrosophic models, making them more suitable for complex decision-making problems involving imprecise and unreliable information[21]. Özlü introduced single-valued neutrosophic type-2 hesitant fuzzy sets and generalized Dice similarity measures for MCDM. The framework offers enhanced information capacity and flexibility, particularly for handling hesitation and indeterminacy in expert evaluations[22]. Liu employed the mean operator based on Dombi-Bonferroni on to address group MADM in an IF paradigm[23]. Kaur and Garg[24] discussed MADM approach based on a cubic intuitionistic fuzzy set, whereas Muneeza discussed MCDM on the same fuzzy set[25] Rani and Garg[26] suggested complex intuitionistic power accumulation operators. Liu employed aggregation operations to investigate centroid modifications of intuitionistic fuzzy values[27].

Multi Attribute Decision Making which is aimed at providing a basis of systematic approach mainly used in organizations to analyse choices with several decision-making qualities influencing the selection of an option. It helps to face the challenge of such contexts because it provides a structure and proper line of thinking. There is another important aspect of MADM: the creation of the decision matrix that incorporates systematic comparison of alternatives to the attributes. The scores attributed to the attributes are probabilities and sum to one for each attribute as each attribute in the decision situation is assigned a weight. This structured matrix helps in comparing the differences in between the options with much of accuracy and it gives the decision makers a measurement yard stick that helps them in choosing which option to take on the basis of relative values. Gulfam proposed the use of Yager operators as aggregation tools for fuzzy sets based on Pythagorean values in decision-making[28]. Garg introduced new aggregation operators based on Yager's t-norm and t-conorm to accumulate fuzzy data based on fermatean fuzzy values in decision-making contexts, evaluated using MADM strategy and COVID-19 test application[29]. Shabeer Khan introduced a decision-making technique using aggregation operators based on neutrosophic sets for selecting "solar power plant locations in Bahawalpur, Pakistan"[30]. More recently, Özlü introduced the concept of (M, N, Q)-spherical hesitant fuzzy sets framework was proposed by integrating spherical fuzzy theory with hesitant information, offering independent control over truth, indeterminacy, and falsity degrees. Aczél–Alsina–based aggregation operators further enhanced aggregation flexibility and comparability in complex decision-making scenarios[31].

Liu introduced aggregation operators for MADM problems based on q-rung sets, discussing their applications and impact on decision-making[32]. Özlü & Aktaş proposed r, s, t-spherical hesitant fuzzy sets and developed correlation coefficients to support clustering-based MCDM and TOPSIS analysis. The framework enhances decision flexibility by allowing richer uncertainty representation through generalized spherical fuzzy structures[33]. Kamacı et al. introduced generalized temporal intuitionistic fuzzy sets and corresponding soft-set-based MCDM methods to model decision problems with time dependent uncertainty. Their approach emphasizes dynamic decision environments and integrates temporal evolution into intuitionistic fuzzy reasoning[34]. Özlü developed q-rung orthopair fuzzy Aczel Alsina weighted geometric operators with ordered and hybrid extensions for group-based MCDM. The study demonstrates that parameterized aggregation improves adaptability and comparative decision analysis[35]. Özlü proposed bipolar-valued complex hesitant fuzzy Dombi aggregation operators and applied them to MCDM problems. The work highlights the role of bipolarity and complex-valued information in capturing conflicting expert opinions more effectively[36]. Despite these advances, two critical limitations persist in the existing literature. First, many intuitionistic fuzzy aggregation frameworks rely on fixed or linear aggregation mechanisms, which restrict their ability to model nonlinear interactions and varying compensation effects among criteria. Second, conventional t-norms and t-conorms often impose rigid aggregation behaviour, limiting their adaptability to different decision attitudes and uncertainty intensities. These shortcomings become particularly pronounced in sustainability-driven decision environments, where criteria are interdependent, conflicting, and commonly expressed through hesitant or linguistic assessments.

Motivation

The growing emphasis on sustainability has positioned Green Supply Chain Management as a strategic priority for modern industries seeking to balance environmental responsibility, economic efficiency, and operational performance. The evaluation and selection of sustainable supply chain alternatives involve multiple conflicting criteria, such as green procurement, eco-design, logistics efficiency, and environmental compliance. These criteria are commonly assessed using qualitative judgments, expert opinions, and linguistic information, rather than precise numerical data.

In such decision environments, decision-makers often express not only partial acceptance of an alternative but also varying degrees of rejection and hesitation due to incomplete knowledge, divergent expert opinions, and evolving sustainability regulations. Traditional crisp or fuzzy decision models are often unable to fully capture these characteristics. Consequently, intuitionistic fuzzy aggregation mechanisms, which explicitly model membership, non-membership, and hesitation degrees, provide a more faithful and cognitively realistic representation of human reasoning in sustainability-oriented MADM problems. This makes intuitionistic fuzzy frameworks particularly well suited for complex GRSCM evaluation contexts characterized by uncertainty, ambiguity, and hesitation.

Research gap

Despite significant advances in intuitionistic fuzzy MADM methodologies, a critical examination of the existing literature reveals several unresolved limitations. First, most intuitionistic fuzzy aggregation frameworks rely on fixed or non-parameterized aggregation structures, which restrict their ability to model heterogeneous decision attitudes and nonlinear compensation effects among criteria. Such rigidity becomes problematic in sustainability-driven decision problems, where decision-makers may exhibit varying degrees of optimism or pessimism depending on regulatory pressure, environmental risk, or strategic priorities. Although Yager's parameterized t-norm and t-conorm operators have been successfully employed in other fuzzy environment such as Pythagorean, Fermatean, neutrosophic, and q-rung fuzzy settings their systematic integration within an intuitionistic fuzzy MADM framework remains largely unexplored, particularly in conjunction with sustainability-oriented applications.

Although substantial progress has been made in intuitionistic fuzzy MADM research, the literature lacks an integrated framework that combines Yager's parameterized aggregation operators with entropy-based objective weighting, rigorous axiomatic validation, and explicit sustainability-oriented applications such as Green Supply Chain Management. Addressing this limitation, the proposed framework is designed to provide enhanced aggregation flexibility, strengthened theoretical consistency, and improved decision reliability for complex green supply chain evaluation problems. This research gap motivates the development of the proposed framework, which aims to enhance aggregation flexibility, theoretical soundness, and decision reliability in complex green supply chain evaluation scenarios.

Contributions and Novelty

The principal contributions and novel aspects of this study are summarized as follows:

- A novel and unified family of Yager-based intuitionistic fuzzy aggregation operators including weighted, ordered weighted, hybrid, averaging, and geometric forms is systematically constructed.
- The fundamental axiomatic properties of the proposed operators, such as idempotency, monotonicity, boundedness, and commutativity, are rigorously established to ensure mathematical soundness and logical consistency.
- An entropy-integrated intuitionistic fuzzy MADM algorithm is developed to objectively determine criteria weights and reduce subjective bias in sustainability evaluations.
- The proposed framework is validated through a real-world GRSCM case study, demonstrating enhanced robustness, stability, and discrimination capability in complex decision environments.
- A comparative analysis with existing aggregation approaches confirms the superiority of the proposed method in handling hesitation, uncertainty, and nonlinear criteria interactions.

By combining flexible Yager-based aggregation with the expressive power of intuitionistic fuzzy sets, this study provides a reliable and adaptable decision-support framework for sustainability-oriented MADM problems characterized by high uncertainty and complex interdependencies.The abbreviations used throughout this manuscript are listed in Table 1.

The innovative aspect of the proposed model is the methodical inclusion of Yager's parameterized t-norm and t-conorm in an intuitive fuzzy multiple criteria decision-making context, which allows for the aggregation of criteria with different weights to be done under uncertainty. Besides, the use of entropy-based objective weighting increases the reliability of the importance assessment of the criteria, while the axiomatic validation of the aggregating operators guarantees the mathematical rigor and stability of the proposed method. All these characteristics form a sound and flexible decision-support system for problems relating to the evaluation of sustainability.

The paper's structure is: In Sect. 2 the review for definitions of Intuitionistic Fuzzy Sets and other models. Moving on to Sect. 3 we will delve into the exploration of the operators: Intuitionistic Fuzzy Yager Weighted Averaging Aggregation (IFYWAA), Intuitionistic Fuzzy Yager Ordered Weighted Averaging Aggregation

(IFYOWAA) as well as Intuitionistic Fuzzy Yager Hybrid Weighted Averaging Aggregation (IFYHWAA) along, with presenting key findings related to these operators. When we reach Sect. 4 our focus will shift to discussing the Intuitionistic Fuzzy Yager Weighted Geometric Aggregation (IFYWGA), Intuitionistic Fuzzy Yager Ordered Weighted Geometric Aggregation (IFYOWGA) and lastly Intuitionistic Fuzzy Yager Hybrid Weighted Geometric Aggregation (IFYHWGA) operators. In Sect. 5 we will discuss an algorithm for MADM and a numerical example of IFN-based supply chain management. In Sect. 6, we will compare our model to another existing model. In Sect. 7 we will analyse the findings drawn from our stated theory.

Preliminaries

Definition 1[1] A fuzzy set is A over the domain $\mathcal{H}$ is defined as $A = \{< h, \mu_A(h) >\}$ where $\mu_A : \mathcal{H} \to [0,1]$ is a membership function of an element $h \in \mathcal{H}$.

Definition 2[2] An Intuitionistic Fuzzy set A over the domain $\mathcal{H}$ is defined as $A = < h, \mu_A\left(h\right), \gamma_A\left(h\right) >\}$ where $\mu_A\left(h\right) : \mathcal{H} \to [0,1]\,, \gamma_A\left(h\right) : \mathcal{H} \to [0,1]$ is a membership function and a non-membership function of an element $h \in \mathcal{H}$ respectively with the condition that $\mu_A\left(h\right) + \gamma_A\left(h\right) \leq 1, forallh \in \mathcal{H}$

For each IFS A in $\mathcal{H}$,if.

$\pi_A(h) = 1 - \left(\mu_A\left(h\right) + \gamma_A\left(h\right)\right)$ for all $h \in \mathcal{H}$ then $\pi_A(h)$ is called as hesitation degree of h to A and $0 \leq \pi_A(h) \leq 1$ for all $h \in \mathcal{H}$.

Definition 3[4] Consider a IFN $\widetilde{a} = (\mu_A, \gamma_A)$. The score and accuracy functions of $\widetilde{a}$ is given as.

$S(\widetilde{a}) = \mu_A - \gamma_A$ where $S(\widetilde{a}) \in [-1,1]$
$H(\widetilde{a}) = \mu_A + \gamma_A$ where $H(\widetilde{a}) \in [0,1]$

Definition 4[5] Consider two IFNs $I_1 = \left(\mu_{I1}, \gamma_{I1}\right), I_2 = \left(\mu_{I2}, \gamma_{I2}\right)$.Then.

$$S\left(I_1\right) < S\left(I_2\right), I_1 < I_2;$$
$$S\left(I_1\right) > S\left(I_2\right), I_1 > I_2;$$

$S\left(I_1\right) = S\left(I_2\right)$, then the accuracy functions are compared as

$$H\left(I_1\right) < H\left(I_2\right), I_1 < I_2$$
$$H\left(I_1\right) > H\left(I_2\right), I_1 > I_2$$
$$H\left(I_1\right) = H\left(I_2\right), I_1 \sim I_2$$

Definition 5[37] Given any two real values p and q, Yager t-norms and t- conorms are given as follows:

$$T\left(p, q\right) = 1 - \min(1, ((1-p)^{\gamma} + (1-q)^{\gamma})^{1/\gamma}) \tag{1}$$

$$T\prime\left(p, q\right) = \min(1, (p^{\gamma} + q^{\gamma})^{1/\gamma}), \gamma \in (0, \infty) \tag{2}$$

Intuitionistic fuzzy numbers under Yager operations

We now present the Yager Weighted Averaging, and Yager Ordered Weighted Averaging operators in the Intuitionistic Fuzzy context, to aggregate uncertain and imprecise evaluation information.

Definition 6 Let $I_1 = < \mu_1,\gamma_1 >$ and $I_2 = < \mu_2,\gamma_2 >$ be two IFNs where $\eta > 0,\ \lambda > 0$. The Yager t-norm and t- conorm functions for IFNs have been defined as.

$$I_1 \oplus I_2 = \min\ (1, \left({\mu_1}^{\eta} + {\mu_2}^{\eta}\right)^{\frac{1}{\eta}}\right), 1 - \min(1, (1-\gamma_1)^{\eta} + (1-\gamma_2)^{\eta})^{1/\eta})$$

$$I_1 \otimes I_2 = 1 - \min(1, \left((1-\mu_1)^{\eta} + (1-\mu_2)^{\eta}\right)^{1/\eta}\right), \min\left(1, (\gamma_1^{\eta} + \gamma_2^{\eta})^{1/\eta}\right)$$

$$\lambda I_1 = \min(1, \left(\lambda\mu_1^{\eta}\right)^{\frac{1}{\eta}}\right), 1 - \min\left(1, (\lambda(1-\gamma_1)^{\eta})^{1/\eta}\right)$$

$$I_1^{\eta} = 1 - \min\ \left((1, (\lambda\left(1-\mu_1\right))^{\eta})^{1/\eta}\right), \min\ \left(1, (\lambda\gamma_1^{\eta})^{1/\eta}\right)$$

Example 1 Consider $I_1 = < 0.2,0.3 >, I_2 = < 0.4,0.5 >$ two IFNs, by Yager computations on IFNs using the above definition for $\eta = 3$ and $\lambda = 4$

$$I_1 \oplus I_2 = \min\left(1, \left(\left(0.2^3 + 0.4^3\right)^{\frac{1}{3}}\right)\right),$$
$$1 - \min\left(1, (1-0.3)^3 + (1-0.5)^3\right)^{1/3})$$
$$= \langle 0.42, 0.22\rangle$$

$$I_1 \otimes I_2 = 1 - \min(1, \left((1-0.2)^3 + (1-0.4)^3\right)^{\frac{1}{3}}\right), \min\left(1, \left(0.3^3 + 0.5^3\right)^{\frac{1}{3}}\right) = \langle 0.1, 0.54\rangle$$

$$\lambda I_1 = \min(1, (4\left(0.2^3\right)^{\frac{1}{3}}\right), 1 - \min(1, (4(1-0.3)^3)^{\frac{1}{3}})$$

$$=< 0.32{,}0.11 >$$

$${I_1}^{\eta} = 1 - \min\left(1, (4(1-0.2)^3)^{\frac{1}{3}}\right), \min(1, \ (4\left(0.3^3\right)^{\frac{1}{3}})$$

$$=< 0.27{,}0.48 >$$

Theorem 1 Let $I =< \mu, \gamma >$, $I_1 =< \mu_1, \gamma_1 >$ and $I_2 =< \mu_2{,}\gamma_2 >$ be three IFNs, then.

$$I_1 \oplus I_2 = I_2 \oplus I_1$$
$$I_1 \otimes I_2 = I_2 \otimes I_1$$
$$\lambda(I_1 \oplus I_2) = \lambda I_1 \oplus \lambda I_2, \lambda > 0$$
$$(\lambda_1 + \lambda_2)I = \lambda_1 I + \lambda_2 I, \lambda_1, \lambda_2 > 0$$
$$(I_1 \otimes I_2)^{\lambda} = I_1^{\lambda} \otimes I_2^{\lambda}, \ \lambda > 0$$

Proof For 3 IFNs,$I, I_1, I_2,$ and $\lambda, \lambda_1, \lambda_2 > 0$ by using above definition, we get.

$$I_1 \oplus I_2 = \min\ \left(1, (\mu_1^{\eta} + \mu_2^{\eta})^{1/\eta}\right), 1 - \min\ \left(\left(1, ((1-\gamma_1)^{\eta} + (1-\gamma_2)^{\eta})^{1/\eta}\right)\right)$$
$$= \min\ \left(1, (\mu_2^{\eta} + \mu_2^{\eta})^{1/\eta}\right), 1 - \min\ \left(\left(1, ((1-\gamma_2)^{\eta} + (1-\gamma_1)^{\eta})^{1/\eta}\right)\right) = I_2 \oplus I_1$$

$$I_1 \otimes I_2 = 1 - \min\ \left(1, \left((1-\mu_1)^{\eta} + (1-\mu_2)^{\eta\frac{1}{\eta}}\right)\right), \min\left(1, (\gamma_1^{\eta} + \gamma_2^{\eta})^{1/\eta}\right)$$
$$= 1 - \min\ \left(1, \left((1-\mu_2)^{\eta} + (1-\mu_1)^{\eta\frac{1}{\eta}}\right)\right), \min\ \left(1, \left((\gamma_2^{\eta} + \gamma_1^{\eta})^{1/\eta}\right)\right) = I_2 \otimes I_1$$

$$\lambda\left(I_1 \oplus I_2\right) = \lambda\left(\min\left(1, (\mu_1^{\eta} + \mu_2^{\eta})^{1/\eta}\right)\right), 1 - \min\left(1, ((1-\gamma_1)^{\eta} + (1-\gamma_2)^{\eta})^{1/\eta}\right)$$
$$= \min\left(1, (\lambda\mu_1^{\eta} + \lambda\mu_2^{\eta})^{\frac{1}{\eta}}\right), 1 - \min\left(1, \lambda\left((1-\gamma_1)^{\eta} + \lambda\left(1-\gamma_2\right)^{\eta}\right)^{\frac{1}{\eta}}\right),$$
$$\lambda I_1 \oplus \lambda I_2 = \min\left(1, \ (\lambda\mu_1^{\eta})^{\frac{1}{\eta}}\right), 1 - min\left(1, (\lambda(1-\gamma_1)^{\eta}\right)^{\frac{1}{\eta}}$$
$$\oplus\min\ \left(1, (\lambda\mu_2^{\eta})^{\frac{1}{\eta}}\right) 1 - \min\begin{pmatrix} 1, \left(\lambda\left((1-\gamma_2)^{\eta}\right)^{\frac{1}{\eta}}\right) \\ = \min\left(1, \ (\lambda\mu_1^{\eta} + \lambda\mu_2^{\eta})^{\frac{1}{\eta}}\right) \end{pmatrix},$$
$$1 - \min\left(1, \left(\lambda\left((1-\gamma_1)^{\eta} + (1-\gamma_2)^{\eta}\right)^{\frac{1}{\eta}}\right)\right) = \lambda\left(I_1 \oplus I_2\right)$$

$$\lambda_1 I \oplus_2 I = min\ \left(1, (\lambda_1\mu^{\eta})^{1/\eta}\right), \left(1 - \min\left(1, \lambda_1\left(1-\gamma\right)^{\eta}\right)^{1/\eta}\right)$$
$$= \min\left(1, \left((\lambda_1 + \lambda_2)\,\mu^{\eta}\right)^{\frac{1}{\eta}}\right), 1 - \min\left(1, \left((\lambda_1 + \lambda_2)\left(1-\gamma\right)^{\eta}\right)^{\frac{1}{\eta}}\right) = (\lambda_1 + \lambda_2)I$$

Similarly, other properties can be verified.

Definition 7 Assume that $I_i = (\mu_i, \gamma_i), i = 1{,}2, \ldots, n$ is a set of IFNs. The IFYWAA is a function $I_n \to I$ such that.

$$IFYWAA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \oplus_{i=1}^{n} \alpha_i I_i$$

where $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_n)^T$ is the weight vector of I_i with $\alpha_i > 0 and \sum_{i=1}^{n} \alpha_i = 1$.

Theorem 2 Assume that $I_i = (\mu_i, \gamma_i) i = 1{,}2, \ldots, n$ is a set of IFNs, then the accumulated value of I_i by IFY-WAA is an IFN.

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_n\right)=\oplus_{i=1}^{n}\alpha_i I_i$$
$$=\min\left(1,\left(\left(\sum_{i=1}^{n}\left(\alpha_i\mu_i^{\eta}\right)\right)\right)^{1/\eta}\right)1-\min\left(\left(1,\left(\sum_{i=1}^{n}(\alpha_i(1-\gamma_i)^{\eta}\right)\right)^{1/\eta}\right) \tag{3}$$

where $\alpha=(\alpha_1,\alpha_2,\ldots\alpha_n)^T$ is the weight vector of I_i , $\alpha_i>0 and \sum_{i=1}^{n}\alpha_i=1$.

Proof We apply mathematical induction to establish the theorem.

When $n=2$

$$\alpha_1 I_1=\min\left(1,\alpha_1(\mu_1)^{\eta}\right),1-\min(1,((\alpha_1(1-\gamma_1)^{\eta})^{1/\eta})$$
$$\alpha_2 I_2=\min\left(1,\alpha_2(\mu_2)^{\eta}\right),1-\min(1,(\alpha_2(1-\gamma_2)^{\eta})^{1/\eta})$$

Therefore,

$$\alpha_1 I_1\oplus\alpha_2 I_2=\min\ \left(1,(\alpha_1\mu_1^{\eta})^{\frac{1}{\eta}}\right),\ 1-\min\left(1,(\alpha_1\left(1-\gamma_1\right)^{\eta})^{\frac{1}{\eta}}\right)\ \oplus\ \min\left(1,(\alpha_2\mu_2)^{\eta}\right)^{\frac{1}{\eta}},$$
$$1-\min\left(1,(\alpha_2\left(1-\gamma_2\right)^{\eta})^{1/\eta}\right)1-\min\left(1,(\alpha_2\left(1-\gamma_2\right)^{\eta})^{1/\eta}\right)$$
$$=\min\left(1,(\alpha_1\mu_1^{\eta}+\alpha_2\mu_2^{\eta})^{\frac{1}{\eta}}\right),1-\min\left(1,(\alpha_1\left(1-\gamma_1\right)^{\eta})^{\frac{1}{\eta}}\right)+\left((\alpha_2\left(1-\gamma_2\right)^{\eta})^{1/\eta}\right)$$
$$=\min\left(1,\left(\sum_{i=1}^{2}(\alpha_i\mu_i^{\eta})\right)^{\frac{1}{\eta}}\right),1-\min\left(1,\left(\sum_{i=1}^{2}(\alpha_i(1-\gamma_i)^{\eta})\right)^{1/\eta}\right)$$

Therefore, by Eq. (3) is true for $n=2$

So, by induction hypothesis, by Eq. (3) is true for $n=k$

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_n\right)=\oplus_{i=1}^{k}\alpha_i I_i$$

i.e.
$$=\min\left(1,\left(\sum_{i=1}^{k}(\alpha_i\mu_i^{\eta})\right)\right)^{\frac{1}{\eta}},1-\min\left(1,\left(\sum_{i=1}^{k}(\alpha_i(1-\gamma_i)^{\eta})\right)^{1/\eta}\right)$$

Now we will prove for $n=k+1$

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_{k+1}\right)=\min\left(1,\left(\sum_{i=1}^{k}(\alpha_i\mu_i^{\eta})\right)^{\frac{1}{\eta}}\right),$$
$$1-\min\left(1,\left(\sum_{i=1}^{k}(\alpha_i(1-\gamma_i))^{\eta}\right)^{\frac{1}{\eta}}\right)\min\left(1,\left(\alpha_{k+1}\mu_{k+1}^{\eta}\right)^{\frac{1}{\eta}}\right),$$
$$-\min\left(1,(\alpha_{k+1}(1-\gamma_{k+1})^{\eta})^{\frac{1}{\eta}}\right)$$
$$=\min\left(1,\left(\sum_{i=1}^{k+1}(\alpha_i\mu_i^{\eta})\right)^{\frac{1}{\eta}}\right),1-\min\left(1,\left(\sum_{i=1}^{k+1}(\alpha_i(1-\gamma_i))^{\eta}\right)^{1/\eta}\right)$$

Therefore, by Eq. (3) is true for $n=k+1$.

Therefore, by Eq. (3) is true $\forall n$.

Example 2 Four judges want to examine an athlete's running skills for winning a 100 m race. The Approximated values of 4 judges for athletes are given in intuitionistic fuzzy information such as $I_1=<0.3,0.4>$ $I_2=<0.2,0.6>$ I_3= <0.1,0.3> $I_4=<0.2,0.4>$ with α= $(0.2,0.3,0.2,0.3)^T$ where α represents the importance of running ability for 4 judges and $\eta=2$

By (3), we can write the clamped value for the ability of running of an athlete as shown below.

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_n\right)=\oplus_{i=1}^{n}\alpha_i I_i \min\left(1,\begin{pmatrix}0.2\left(0.3\right)^2+0.3\left(0.2\right)^2\\+0.2\left(0.1\right)^2+0.3\left(0.2\right)^2\end{pmatrix}^{\frac{1}{2}}\right),$$

$$1-\min\left(1,\begin{pmatrix}0.2\left(1-0.4\right)^2+0.3\left(1-0.6\right)^2\\+0.2\left(1-0.3\right)^2+0.3\left(1-0.4\right)^2\end{pmatrix}^{\frac{1}{2}}\right)$$

$$=\langle 0.21,0.43\rangle.$$

Theorem 3 (Idempotency) If all IFNs are similar, i.e., $I_n=I$ then

$$IFYWAA\left(I_1,I_2,\ldots,I_n\right)=I$$

Proof As.$I_i=<\mu_i,\gamma_i>,(i=1,2,\ldots,n)$ Then by Eq. (3).

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_n\right)=\oplus_{i=1}^{n}\left(\alpha_i I_i\right)$$
$$=\min\left(\left(1,\left(\sum_{i=1}^{n}(\alpha_i\mu_i^{\eta})\right)\right)^{\frac{1}{\eta}}\right),1-\min\left(\sum_{i=1}^{n}\left((\alpha_i\left(1-\gamma_i^{\eta}\right))^{\frac{1}{\eta}}\right)\right)$$

Theorem4(Boundedness) Assumethat$I_i=\left(\mu_i,\gamma_i\right),i=1,2,\ldots,n$isasetofIFNsand$I^{-}=\min(I_1,I_2,\ldots,I_n)$ and $I^{+}=\max(I_1,I_2,\ldots,I_n)$

then $I^{-}\leq IFYWAA(I_1,I_2,\ldots,I_n)\leq I^{+}$

Proof Consider that $I^{-}=\min\left(I_1,I_2,\ldots,I_n\right)=\left(\mu^{-},\gamma^{-}\right)$ and $I^{+}=\max\left(I_1,I_2,\ldots,I_n\right)=\left(\mu^{+},\gamma^{+}\right)$ where $\mu^{-}=\min\left(\mu_i\right),\gamma^{-}=\max\left(\gamma_i\right),\mu^{+}=\max\left(\mu_i\right),\gamma^{+}=\min(\mu_i)$

$$\min(1,(\sum_{i=1}^{n}(\alpha_i{\mu^{-}}^{\eta}))^{1/\eta})\leq\min(1,(\sum_{i=1}^{n}(\alpha_i{\mu_i}^{\eta}))^{1/\eta})\leq\min(1,(\sum_{i=1}^{n}(\alpha_i{\mu^{+}}^{\eta}))^{1/\eta})$$

$$1-\min(1,(\sum_{i=1}^{n}(\alpha_i(1-\gamma^{+})^{\eta}))^{1/\eta})\leq 1-\min(1,(\sum_{i=1}^{n}(\alpha_i(1-\gamma_i)^{\eta}))^{1/\eta})\leq 1-\min(1,(\sum_{i=1}^{n}(\alpha_i(1-\gamma^{-})^{\eta}))^{1/\eta})$$

Therefore,$I^{-}\leq IFYWAA(I_1,I_2,\ldots,I_n)\leq I^{+}$.

Theorem 5 (Monotonicity) Assume that $I_i\prime=(I_1\prime,I_2\prime,\ldots,I_n\prime)$ and $I_i=(I_1,I_2,..,I_n)$ are two sets of IFNs, If $\mu_i\prime\leq\mu_i$ and $\gamma_i\prime\leq\gamma_i,\forall i$,then

$$IFYWAA(I_1\prime,I_2\prime,\ldots,I_n\prime)\leq IFYWAA(I_1,I_2,\ldots,I_n)$$

Proof Let $IFYWAA\left(I_1\prime,I_2\prime,\ldots,I_n\prime\right)=(G\prime,K\prime)$ and.

$$IFYWAA\left(I_1,I_2,\ldots,I_n\right)=(G,K)$$

First, we will show that $G\prime\leq G$. As $\mu_i\prime\leq\mu_i$, Moreover,

$$(\sum_{i=1}^{n}(\alpha_i{\mu_i\prime}^{\eta})^{1/\eta})\leq(\sum_{i=1}^{n}(\alpha_i{\mu_i}^{\eta})^{1/\eta})$$

$$\min(1,(\sum_{i=1}^{n}(\alpha_i{\mu_i\prime}^{\eta}))^{1/\eta})\leq\min(1,(\sum_{i=1}^{n}(\alpha_i{\mu_i}^{\eta}))^{1/\eta})$$

Hence $G\prime\leq G$
In similar way, we can prove $K\prime\geq K$
Then $(G\prime,K\prime)\leq(G,K)$
Therefore

$$IFYWAA\left(I_1\prime,I_2\prime,\ldots,I_n\prime\right)\leq IFYWAA\left(I_1,I_2,\ldots,I_n\right).$$

Theorem 6 (Reducibility) Assume that $I_i=<\mu_i,\gamma_i>(i=1,2,\ldots,n)$ is set of IFNs with the associated weight vector $\alpha=\left(\alpha_1,\alpha_2,\ldots,\alpha_n\right)^T=\left(\frac{1}{n},\frac{1}{n},\frac{1}{n},\ldots\frac{1}{n}\right)^T$

The IFYWA operator is

$$IFYWAA_{\alpha}\left(I_1,I_2,\ldots,I_n\right)=\min\left(1,\frac{1}{n}\left(\sum_{i=1}^{n}\left(\mu_i^{\eta}\right)\right)^{\frac{1}{\eta}}\right),1-\min\left(1,\frac{1}{n}\left(\sum_{i=1}^{n}\left(1-\gamma_i^{\eta}\right)\right)^{\frac{1}{\eta}}\right)$$

Theorem 7 (Commutativity) Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is a set of IFNs. If $I_i\prime$ is permutation of I_i, then.

$$IFYWAA_\alpha (I_1, I_2, \ldots, I_n) = IFYWAA_\alpha (I_1\prime, I_2\prime, ..., I_n\prime).$$

In the case of multi-attribute decision-making, different aggregation structures need to be used in order to deal with different decision behaviours. The weighted aggregation structure is used to represent the weight of the criteria or the weight of the expert opinions. The order weighted aggregation operation does not rely on the positions of the criteria. Rather, the operation is dependent on the magnitudes of the criteria in order. This operation is used when the decision-makers intend to demonstrate an optimistic or pessimistic attitude towards the choice information. The operation is used when the extreme choice carries more weight in the final choice. The hybrid aggregation operation is used in combination with the weighted operation and the order weighted operation. The hybrid operation allows the decision-maker to consider the importance of the criteria as well as the order of the criteria. Therefore, by using the three aggregation operations in the proposed framework, the flexibility of the model increases significantly. The model allows the decision-maker to choose the aggregation operation depending on the environment of the decision.

Definition 8 Assume that $I_i =< \mu_i, \gamma_i >, (i = 1,2, ..., n)$ is set of IFNs, with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ with $\alpha_i > 0 and \sum_{i=1}^{n} \alpha_i = 1$. The $IFYOWAA_\alpha (I_1, I_2, \ldots, I_n) = \oplus_{i=1}^{n} \left(\alpha_i I_{\rho(i)}\right)$

Where $\rho(1), \rho(2), \ldots..\rho(n)$ is the permutation of $i = 1,2, \ldots n$ such that $I_{\rho(i-1)} \geq I_{\rho(i)} \forall, i = 1,2, \ldots n$.

Theorem 8 Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is a set of IFNs, with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T with\ \alpha_i > 0 and \sum_{i=1}^{n} \alpha_i = 1$ then the clumped value of I_i by IFYOWAA function is an IFN.

$$\begin{aligned} &IFYOWAA_\alpha (I_1, I_2, \ldots, I_n) = \oplus_{i=1}^{n} \left(\alpha_i I_{\rho(i)}\right) \\ &= \min\left(1, \left(\sum_{i=1}^{n} \left(\alpha_i \mu_{\rho(i)}^{\eta}\right)^{1/\eta}\right)\right), 1 - \min\left(\sum_{i=1}^{n} \left(\alpha_i \left(1 - \gamma_{\rho(i)}^{\eta}\right)^{1/\eta}\right)\right) \end{aligned} \tag{4}$$

Where $\rho(1), \rho(2), \ldots..\rho(n)$ is the permutation of $i = 1,2, \ldots n$ such that $I_{\rho(i-1)} \geq I_{\rho(i)} \forall, i = 1,2, \ldots n$.

Example 3 Four interview panel members assigned various intuitionistic fuzzy values $I_1 =< 0.2,0.5 >, I_2 =< 0.2,0.4 >, I_3 =< 0.3,0.4 > and I_4 =< 0.2,0.6 >$ with $\alpha = (0.2, 0.3, 0.2, 0.3)^T$ to assess the applicant's ability for the HR position in HCL Tech. We use the IFYOWAA function Eq. (4) to determine the applicant's clumped value. The score functions are.

$$S(I_1) = -0.3$$
$$S(I_2) = -0.2$$
$$S(I_3) = -0.1$$
$$S(I_4) = -0.4$$

Since $S(I_1) > S(I_2) > S(I_3) > S(I_4)$, by suitably reordering,

$$I_{\rho(1)} = I_3 =< 0.3,0.4 >$$
$$I_{\rho(2)} = I_2 =< 0.2,0.4 >$$
$$I_{\rho(3)} = I_1 =< 0.2,0.5 >$$
$$I_{\rho(4)} = I_4 =< 0.2.0.6 >$$

Then by applying the IFYOWAA operator, we get

$$\begin{aligned} &IFYOWAA_\alpha (I_1, I_2, \ldots, I_n) = \oplus_{i=1}^{n} \left(\alpha_i I_{\rho(i)}\right) \\ &= \min\left(1, \left(\sum_{i=1}^{n} (\alpha_i \mu_{\rho(i)}^{\eta})\right)^{\frac{1}{\eta}}\right), 1 - \min\left(\sum_{i=1}^{n} \left(\alpha_i \left(1 - \gamma_{\rho(i)}^{\eta}\right)^{\frac{1}{\eta}}\right)\right) \\ &= \min\left(1, \begin{pmatrix} 0.2(0.3)^2 + 0.3(0.2)^2 \\ +0.2(0.2)^2 + 0.3(0.2)^2 \end{pmatrix}^{\frac{1}{2}}\right), \\ &1 - \min\left(\begin{pmatrix} 0.2(1-0.4)^2 + 0.3(1-0.4)^2 \\ +0.2(1-0.5)^2 + 0.3(1-0.6)^2 \end{pmatrix}^{\frac{1}{2}}\right) = \langle 0.22, 0.47 \rangle. \end{aligned}$$

Theorem 9 (Idempotency) If all IFNs are similar, i.e., $I_n = I$ then

$$IFYOWAA\left(I_1, I_2, \ldots, I_n\right) = I.$$

Theorem 10 (Boundedness) Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is set of IFNs. If $I^- = \min(I_1, I_2, \ldots, I_n)$ and $I^+ = \max\left(I_1, I_2, \ldots, I_n\right)$,

$$I^- \leq IFYOWAA(I_1, I_2, \ldots, I_n) \leq I^+.$$

Theorem 11 (Monotonicity) Assume that $I_{i\prime} = (I_{1\prime}, I_{2\prime}, \ldots, I_{n\prime})$ and $I_i = (I_1, I_2, .., I_n)$ are two sets of IFNs, If $\mu_{i\prime} \leq \mu_i$ and $\gamma_{i\prime} \leq \gamma_i, \forall i$,then.

$$IFYOWAA\left(I_{1\prime}, I_{2\prime}, ..., I_{n\prime}\right) \leq IFYOWAA\left(I_1, I_2, \ldots, I_n\right).$$

Theorem 12 (Reducibility) Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is set of IFNs with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T = \alpha = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$

The IFYOWAA operator is $IFYOWAA_\alpha(I_1, I_2, \ldots, I_n)$

$$= \min(1, \frac{1}{n}(\sum\nolimits_{i=1}^{n}(\mu_{\rho(i)}^{\eta}))^{1/\eta}), 1 - \min(1, \frac{1}{n}(\sum\nolimits_{i=1}^{n}(1-\gamma_{\rho(i)}^{\eta}))^{1/\eta}).$$

Theorem 13 (Commutativity) Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is a set of IFNs. If $I_{i\prime}$ is permutation of I_i, then

$$IFYOWAA_\alpha\left(I_1, I_2, \ldots, I_n\right) = IFYOWAA_\alpha\left(I_{1\prime}, I_{2\prime}, ..., I_{n\prime}\right).$$

Definition 9 A IFYHWAA is a function $I^n \to I$ with the correlated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T with$ $\alpha_i > 0$ and $\sum_{i=1}^{n}\alpha_i = 1$ such that.

$$IFYHWAA_\alpha\left(I_1, I_2, \ldots, I_n\right) = \oplus_{i=1}^{n}\left(\alpha_i \dot{I}_{\rho(i)}\right)$$
$$= \min\left(1, \left(\sum_{i=1}^{n}\left(\alpha_i \dot{\mu}_{\rho(i)}^{\eta}\right)\right)^{1/\eta}\right), 1 - \min\left(1, \left(\sum_{i=1}^{n}\left(\alpha_i(1-\dot{\gamma}_{\rho(i)})^{\eta}\right)\right)^{1/\eta}\right) \tag{5}$$

where $\dot{I}_{\rho(i)}$ is i^{th} largest weighted Intuitionistic fuzzy values $\dot{I}_i(\dot{I}_i = n\alpha_i I_i, i = 1,2, ..., n)$ and n is the balancing co-efficient.

Example 4 Suppose four experts are invited to evaluate the overall feasibility of a renewable energy project (such as a solar power installation) under uncertain and imprecise conditions. Due to incomplete technical data, environmental uncertainty, and subjective judgment, each expert expresses their evaluation in the form of an intuitionistic fuzzy number (IFN) as values $I_1 =< 0.5,0.3 >$, $I_2 =< 0.6,0.2 >$, $I_3 =< 0.3,0.4 >$ and $I_4 =< 0.2,0.6 >$ with $\alpha = (0.3, 0.1,0.2, 0.4)^T$. Then by applying the IFYHWAA operator Eq. (5), we get, with a balancing co-efficient $n = 0.5$,

$$IFYHWAA_\alpha\left(I_1, I_2, \ldots, I_n\right) = \oplus_{i=1}^{n}\left(\alpha_i \dot{I}_{\rho(i)}\right)$$
$$= \min\left(1, \begin{pmatrix}0.3\,(0.25)^2 + 0.1\,(0.3)^2 \\ +0.2\,(0.15)^2 + 0.4\,(0.1)^2\end{pmatrix}^{\frac{1}{2}}\right),$$
$$1 - \min\left(\begin{pmatrix}0.3\,(1-0.15)^2 + 0.1\,(1-0.1)^2 \\ +0.2\,(1-0.2)^2 + 0.4\,(1-0.3)^2\end{pmatrix}^{\frac{1}{2}}\right)$$
$$= \langle 0.19, 0.21\rangle.$$

Remark For $\alpha = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$, $IFYWAA$ and $IFOYWAA$ are specific example of $IFYHWA$ operator. Thus, $IFYHWA$ is a generalization of both operators.

Intuitionisticfuzzy Yager geometric operators

We now subsequently propose Yager Weighted Geometric and Yager Ordered Weighted Geometric operators in the Intuitionistic Fuzzy context, to provide a multiplicative aggregation of uncertain decision information.

Definition 10 Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is set of IFNs. The IFYWGA operator is a function $I^n \to I$, such that.

$IFYWGA_{\alpha}(I_1, I_2, \ldots, I_n) = \otimes_{i=1}^{n} {I_i}^{\alpha_i}$ where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ is weight vector of $I_i with\, \alpha_i > 0 and$ $\sum_{i=1}^{n} \alpha_i = 1$.

Theorem 14 Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is set of IFNs, then the accumulated value of I_i by IFYWGA is an IFN.

$$IFYWGA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \otimes_{i=1}^{n} I_i^{\alpha_i}$$
$$= 1 - \min\left(1, \left(\sum_{i=1}^{n} (\alpha_i (1-\mu_i)^{\eta})\right)^{1/\eta}\right), \min\left(1, \left(\sum_{i=1}^{n} (\alpha_i \gamma_i^{\eta})\right)^{1/\eta}\right) \tag{6}$$

Proof: We can demonstrate the result by utilizing analogous explanations as used in theorem 2.

Example 5 Considering Example 2 and by Eq. (6), the aggregated value of an athlete's running efficiency is.

$IFYWGA_{\alpha}(I_1, I_2, \ldots, I_n) = \otimes_{i=1}^{4} (I_i)^{\alpha_i} = 1 - \min\left(1, \left(\sum_{i=1}^{4} (\alpha_i (1-\mu_i)^{\eta})^{1/\eta}\right)\right),$

$\min\left(1, \left(\sum_{i=1}^{4} (\alpha_i (\gamma)^{\eta}))1/\eta\right)\right) = 1 - \min\left(1, \left(\begin{pmatrix} 0.2 (1-0.3)^2 + 0.3 (1-0.2)^2 \\ +0.2 (1-0.1)^2 + 0.3 (1-0.2)^2 \end{pmatrix}^{\frac{1}{2}}\right)\right) \min\left(1, \begin{pmatrix} 0.2 (0.4)^2 + 0.3 (0.6)^2 \\ +0.2 (0.3)^2 + 0.3 (0.4)^2 \end{pmatrix}^{\frac{1}{2}}\right),$

$=< 0.20, 0.45 >$

Theorem 15 (Idempotency) If all IFNs are similar, i.e., $I_n = I$ then

$$IFYWGA\left(I_1, I_2, \ldots, I_n\right) = I$$

Proof: We can demonstrate the result by utilizing analogous explanations as used in theorem 3.

Theorem 16 (Boundedness) Assume that $I_i =< \mu_i, \gamma_i > (i = 1,2, ..., n)$ is set of IFNs and $I^- = \min(I_1, I_2, \ldots, I_n)$ and $I^+ = \max(I_1, I_2, \ldots, I_n)$

then $I^- \leq IFYWGA(I_1, I_2, \ldots, I_n) \leq I^+$
Proof: We can demonstrate the result by utilizing analogous explanations as used in theorem 4.

Theorem 17 (Monotonicity) Assume that $I_i\prime = (I_1\prime, I_2\prime, ..., I_n\prime)$ and $I_i = (I_1, I_2, \ldots, I_n)$ are two sets of IFNs, if $\mu_i\prime \leq \mu_i$ and $\gamma_i\prime \leq \gamma_i, \forall i$.

$$IFYWGA(I_1\prime, I_2\prime, ..., I_n\prime) \leq IFYWAA(I_1, I_2, \ldots, I_n)$$

Proof: We can demonstrate the result by utilizing analogous explanations as used in theorem 5.

Theorem 18 (Reducibility) Assume that $I_i =< \mu_i, \gamma_i >, (i = 1,2, \ldots, n)$ is a set of IFNs with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$

The IFYWGA operator is

$$IFYWGA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \min\left(1, \frac{1}{n}\left(\sum_{i=1}^{n} (\mu_i^{\eta})\right)^{\frac{1}{\eta}}\right),$$
$$1 - \min\left(1, \frac{1}{n}\left(\left(\sum_{i=1}^{n} ((1-\gamma_i^{\eta})\right)^{\frac{1}{\eta}}\right)\right).$$

Theorem 19 (Commutativity) Assume that $I_i =< \mu_i, \gamma_i >, (i = 1,2, \ldots, n)$ is a set of IFNs. If $I_i\prime$ is permutation of I_i, then

$$IFYWGA_{\alpha}(I_1, I_2, \ldots, I_n) = IFYWGA_{\alpha}(I_1\prime, I_2\prime, ..., I_n\prime)$$

Definition 11 Assume that $I_i =< \mu_i, \gamma_i >, (i = 1,2, \ldots, n)$ is a set of IFNs with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ with $\alpha_i > 0 and$ $\sum_{i=1}^{n} \alpha_i = 1$.The IFYOWGA operator is a function $I^n \to I$ such that.

$$IFYOWGA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \otimes_{i=1}^{n} (I_{\rho(i)})^{\alpha_i}$$

where $(\rho\left(1\right), \rho\left(2\right), \ldots..\rho(n))$ is the permutation of $i = 1,2, \ldots, n$ such that $I_{\rho(i-1)} \geq I_{\rho(i)} \forall, i = 1,2, \ldots n$

Theorem 20 Assume that $I_i = < \mu_i, \gamma_i >, (i = 1,2, \ldots, n)$ is a set of IFNs with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ with $\alpha_i > 0 and \sum_{i=1}^{n} \alpha_i = 1$, then the accumulated value I_i of by IFYOWGA is a IFN.

$$IFYOWGA_\alpha (I_1, I_2, \ldots, I_n) = \otimes_{i=1}^{n} (I_{\rho(i)})^{\alpha_i}$$

$$IFYOWAA_\alpha (I_1, I_2, \ldots, I_n) = \oplus_{i=1}^{n} \left(\alpha_i I_{\rho(i)}\right)$$

$$= 1 - \min \left(1, \left(\sum_{i=1}^{n} (\alpha_i \left(1 - \mu_{\rho(i)}\right)^\eta)\right)^{\frac{1}{\eta}}\right),$$

$$\min \left(\sum_{i=1}^{n} \left(\alpha_i \left(\gamma_{\rho(i)}\right)^\eta\right)^{1/\eta}\right). \tag{7}$$

where $(\rho(1), \rho(2), \ldots ..\rho(n))$ is the permutation of $(i = 1{,}2, ..., n)$ such that $I_{\rho(i-1)} \geq I_{\rho(i)} \forall, i = 1{,}2, \ldots n$.
Proof: We can demonstrate the result by utilizing analogous explanations as used in theorem 2.

Example 6 Considering Example 3 and by Eq. (7), the aggregated value of an athlete's running efficiency is.

$$IFYOWGA_\alpha (I_1, I_2, \ldots, I_n) = \otimes_{i=1}^{4} (I_{\rho(i)})^{\alpha_i}$$

$$1 - \min \left(1, \left(\sum_{i=1}^{4} (\alpha_i 1 - \mu_{\rho(i)})^\eta\right) 1/\eta\right), \min \left(1, \left(\sum_{i=1}^{4} \alpha_i \gamma_{\rho(i)}^\eta\right)^{\frac{1}{\eta}}\right)$$

$$= 1 - \min \left(1, \left(\begin{pmatrix} 0.1(1-0.3)^2 + 0.2(1-0.2)^2 \\ +0.3(1-0.2)^2 + 0.3(1-0.2)^2 \end{pmatrix}^{\frac{1}{2}}\right)\right),$$

$$\min \left(1, ((0.1(0.4)^2 + 0.2(0.4)^2 \right.$$
$$\left. +0.3(0.5)^2 + 0.3(0.6)^2)^{1/2}\right) = (0.22{,}0.49)$$

Theorem 21 (Idempotency) If all IFNs are similar, i.e., $I_n = I$,

$$IFYOWGA (I_1, I_2, \ldots, I_n) = I$$

Theorem 22 (Boundedness) Assume that $I_i = < \mu_i, \gamma_i > (i = 1{,}2, \ldots, n)$ is set of IFNs and $I^- = \min(I_1, I_2, \ldots, I_n)$ and $I^+ = \max(I_1, I_2, \ldots, I_n)$

Then $I^- \leq IFYOWGA(I_1, I_2, \ldots, I_n) \leq I^+$

Theorem 23 (Monotonicity) Assume that $I_i\prime = (I_1\prime, I_2\prime, ..., I_n\prime)$ and $I_i = (I_1, I_2, \ldots., I_n)$ are two sets of IFNs, $\mu_i\prime \leq \mu_i$ and $\gamma_i\prime \leq \gamma_i, \forall i$,

$$IFYOWGA(I_1\prime, I_2\prime, ..., I_n\prime) \leq IFYWOGA(I_1, I_2, \ldots., I_n)$$

Theorem 24 (Reducibility) Assume that $I_i = < \mu_i, \gamma_i >, (i = 1{,}2, \ldots, n)$ is set with the associated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T = \left(\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$.

The IFYOWGA operator is $IFYOWGA_\alpha(I_1, I_2, \ldots, I_n)$

$$= \min \left(1, \frac{1}{n}\left(\sum_{i=1}^{n} (\mu_i{}^\eta)\right)^{\frac{1}{\eta}}, 1 - \min \left(1, \frac{1}{n}\left(\sum_{i=1}^{n} ((1-\gamma_i)^\eta)\right)\right)^{\frac{1}{\eta}}\right).$$

Theorem 25 (Commutativity) Assume that $< \mu_i, \gamma_i > (i = 1{,}2, ..., n)$ is set of IFNs. If $I_i\prime$ is permutation of I_i, then

$$IFYOWGA_\alpha (I_1, I_2, \ldots, I_n) = IFYOWGA_\alpha (I_1\prime, I_2\prime, ..., I_n\prime).$$

Definition 12 A IFYHWGA is a function: $I^n \to I$ with the correlated weight vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ with $\alpha_i > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$ such that.

$$IFYHWGA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \otimes_{i=1}^{n}\left(\dot{I}_{\rho(i)}\right)^{\alpha_i}$$
$$= 1 - \min\left(1, \left(\sum_{i=1}^{n}\left(\alpha_i(1-\dot{\mu}_{\rho(i)})^{\eta}\right)\right)^{1/\eta}\right), \min\left(1, \left(\sum_{i=1}^{n}\left(\alpha_i(\dot{\gamma}_{\rho(i)})^{\eta}\right)^{1/\eta}\right)\right). \quad (8)$$

where $\dot{I}_{\rho(i)}$ is the largest weighted intuitionistic fuzzy values $\dot{I}_{(i)}\left(\dot{I}_{(i)} = {I_i}^{n\alpha_i}\right), i = 1,2,\ldots,n$, n=balancing co-efficient.

Example 7 Considering Example 4 and by Eq. 8, the aggregated value is given as.

$$IFYHWGA_{\alpha}\left(I_1, I_2, \ldots, I_n\right) = \otimes_{i=1}^{n}\left(\dot{I}_{\rho(i)}\right)^{\alpha_i} = 1 - \min\left(1, \left(\sum_{i=1}^{n}\left((\alpha_i(1-\dot{\mu}_{\rho(i)})^{\eta})\right)^{\frac{1}{\eta}}\right)\right),$$
$$\min\left(1, \left(\sum_{i=1}^{n}\left(\alpha_i\left(\dot{\gamma}_{\rho(i)}^{\eta}\right)^{\frac{1}{\eta}}\right)\right)\right) = 1 - \min\left(1, \begin{pmatrix}0.3(1-0.25)^2 + 0.1(1-0.3)^2 \\ +0.2(1-0.15)^2 + 0.4\left(1-0.1)^2\right)^{\frac{1}{2}}\end{pmatrix}\right),$$
$$\min\left(1, \left(\begin{pmatrix}0.1\,(0.15)^2 + 0.2\,(0.1)^2 \\ +0.3\,(0.2)^2 + 0.3\,(0.3)^2\end{pmatrix}^{1/2}\right)\right) = \langle 0.17, 0.21\rangle.$$

In this section, the formulation and methodological framework of the Multi-Attribute Decision-Making (MADM) problem are presented in detail.

Application in MADM

GRSCM could be described as an inventory control strategy that involves advocating for green practices at each link of the chain. In many ways, GRSCM's main objective is to improve supply chain processes with the overall purpose of achieving social and economic outcomes. Another important element of GRSCM is sustaining procurement; By this, the suppliers are selected for their socially responsible policies in labour, environmentally sustainable methods and green material sourcing. Fortes discussed an extensive analysis of the GRSCM literature over last two decades[38]. Fahimnia conducted a thorough bibliometric and network analysis on GRSCM literature, identifying influential works, and providing a roadmap for future investigations[39]. Kumari & Mishra introduced the IF-COPRAS method, which is applied in green supplier selection[40]. Govindan highlighted about the growing importance of environmentally conscious procedures in commercial settings and introduced a DEMATEL strategy for promoting green practices and supply network efficiency[41]. Rouyendegh proposed a hybrid methodology that combines the Intuitionistic Fuzzy Set and TOPSIS method to reduce ambiguity and instability in green supplier selection decision-making[42]. Sarkis presented a strategic decision framework to aid managerial decision-making for GRSCM[43]. Srivastava discussed GRSCM literature, categorizing it based on problem context, methodology, and approach, emphasizing key research concerns and prospects in the field[44]. To assess companies' ecologically friendly performance in GRSCM, Uygun developed a MCDM technique that employs fuzzy DEMATEL, ANP, TOPSIS methodologies[45]. Jalil explored the dynamics of customer trust in e-commerce, emphasizing the essential role of GRSCM in making online purchasing more empirically viable[46]. Letunovska addressed the impact of long-term viability on reverse logistics and also suggests that the firms can collaborate with environmental preservation authorities to manufacture goods that are ecologically friendly[47]. Ahmad presented the study which focuses on examining the influence of GRSCM methods on the sustainable achievement of the fabric and automobile sectors[48].Bhatia evaluated the research methodology of 216 empirical papers in GRSCM and proposed future research paths for empirical study in GRSCM[49]. Fahimnia presented a pragmatic supply chain strategy model that may be used to explore cost-environmental trade-offs while also integrating other features of practical supply chains[50].

Lee analysed 133 data sets from Malaysian manufacturing enterprises to identify a link between GRSCM procedures and technological progression[51]. Mathiyazhagan observed 26 hurdles to GRSCM deployment and aims to determine the most dominant one.[52]. Iddik discovered that non-bureaucratic culture, familial social structure, and structured culture had a favourable impact on the execution of GRSCM activities and also demonstrated how ecological preservation and culture are interconnected.[53]. Abdallah assessed the impact of GRSCM on circular economy performance by collecting data from 278 firms using an elementary stochastic approach, revealing that GRSCM had an immediate beneficial effect on circular economy performance[54]. Xing's presentation on product-service innovation and GRSCM sheds light on how it contributes to ecological resilience[55]. Saini developed a theoretical model that comprises an effective theory-building method for the essential aspects of global logistics management practices[56]. Tiep Le studied how business social accountability activities impact diverse stakeholders, including GRSCM, in fostering equitable patterns of consumption and production.[57]. Roh investigated the effect of ethical managerial creativity and rights of intellectual property on GRSCM and environmental efficacy, emphasizing the key role of supply chain administration[58]. Selepe proposed better inventory management techniques, supplier evaluation, and ERP system customization as potential solutions aimed to improve supply chain efficiency and reduce costs. The study aims to understand the basis for poor supply chain quality at a steel product manufacturing setup and finds key issues such as inventory stock-out, deviations in process, and unreliable ERP systems[59].

GRSCM aims to transform authentic supply chain processes into more environmentally friendly and sustainable practices. GRSCM focuses on conserving natural resources and minimizing pollution, while simultaneously reducing the negative impact of climate change. Furthermore, GRSCM adheres to regulatory requirements and industry standards, ensuring compliance and limiting legal risks associated with environmental violations. The key objective of this application is to determine the Suitable green Supplier (Companies which implements GRSCM) in GRSCM by applying IFYWAA and IFYWGA operators. Assume that $L = \{L_1, L_2, L_3, L_4\}$ is a set of alternatives (Companies). Assume that $K = \{K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}\}$ is a set of attributes for the evaluation GRSCM performance of alternative companies where K_1 represents green design,K_2 represents green procurement K_3 represents green collaboration K_4 represents green manufacturing K_5 represents green logistics K_6 represents green inventory,K_7 represents green reverse logistics K_4 represents green readiness K_5 represents green procurement K_6 represents green inventory,K_7 represents green reverse logistics K_8 represents green technology,K_9 represents green information systems ,K_{10} represents green readiness,K_{11} represents green human resource management K_{12} represents sustainable performance. Development of a sustainable and eco-friendly supply chain is dependent on three main factors—eco-friendliness, a delightful product experience, and savings in time and energy. The conceptual framework shown in Fig. 1 illustrates the structural and functional interrelationships of the 12 key dimensions of Green Supply Chain Management (GRSCM) as a whole. Figure 2 presents the overall methodological framework of the proposed intuitionistic fuzzy entropy-based MADM approach. Figure 3 illustrates the detailed algorithmic workflow and stage-wise execution of the proposed decision-making process.

The framework ultimately represents the transformation from environmental intent to sustainable performance outcomes through a systematic layering of four interdependent layers—Strategic, Operational, Enabling, and Outcome.

The Strategic Layer consists of Green Design (GD), Green Procurement (GP), and Green Collaboration (GC) which define the long-term environmental strategy and external relationships of the organization by indicating that sustainability issues are less about each organization but more about the group formed from collaboration with different members of the supply chain.

The Operational Layer consists of Green Manufacturing (GM), Green Logistics (GL), Green Inventory (GI), and Green Reverse Logistics (GRL) which turn strategic commitments into carbon output actions; Simply put, they operationalize sustainability from the production process to distribution. The focus is on cleaner production technology, better use of materials and energy, and sustainable logistics.

The Enabling Layer consists of Green Technology for Decision Making (GTDM), Green Information Systems (GIS), Green Readiness (GR), and Green Human Resource Management (GHRM) as the support for GRSCM practices, to create an implementation and continuous improvement of GRSCM practices. These enablers support the digital integration, make the organizations ready and, with the help of the data-informed insights and a focused environmental culture, the humans' capacity to uphold the green initiatives is being developed.

In the last instance, with regard to the Outcome Layer, Sustainable Performance (SP) indicates the primary objective of GRSCM—discovering the balance among the environment, the economy, and the society domains that are often referred to as the "triple bottom line." The close connections among the different layers of the framework demonstrate their interdependent character. A sustainable performance is not an isolated event; it

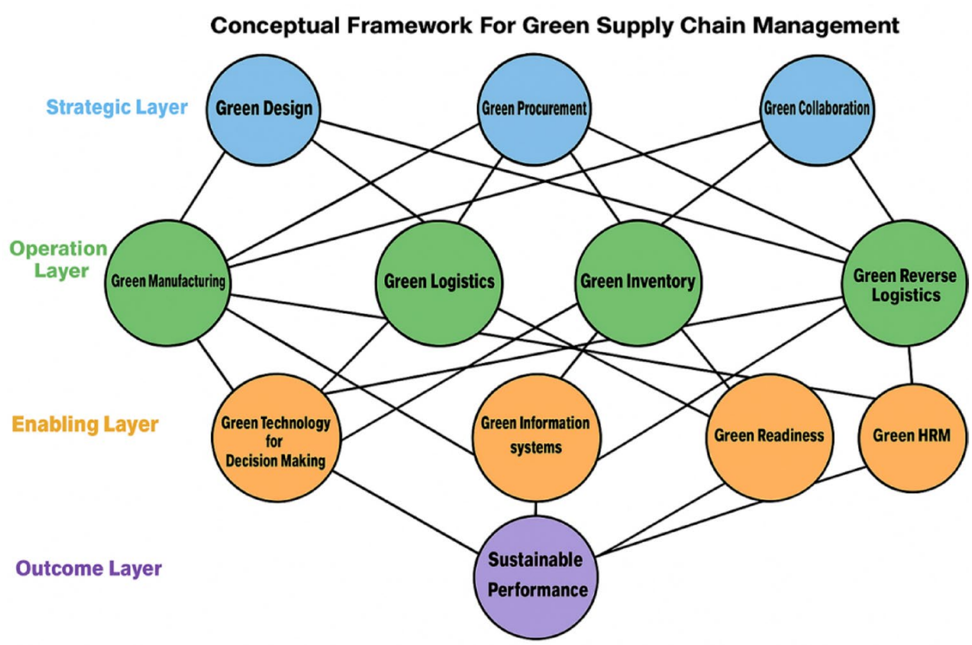

Fig. 1. Conceptual Framework for Green Supply Chain Management.

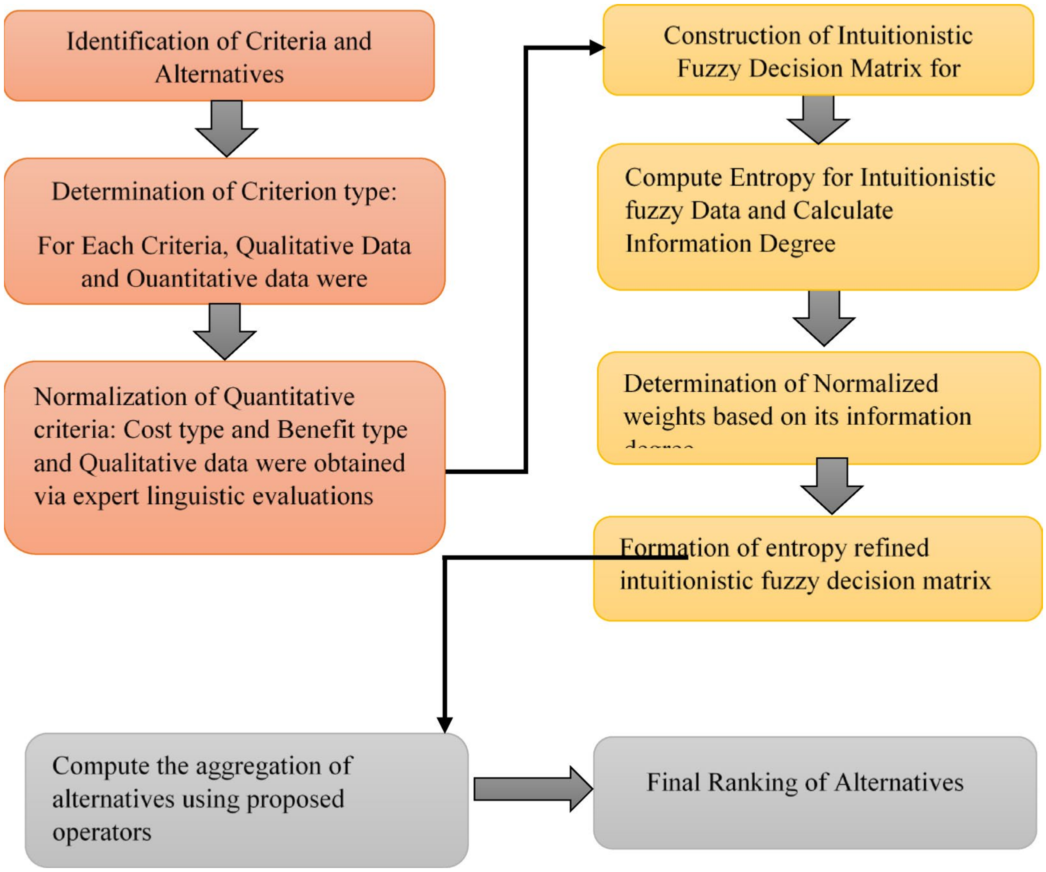

Fig. 2. Flowchart of the proposed intuitionistic fuzzy entropy-based MADM framework.

is the result of a strategic intent, operational delivery and enablers playing their respective roles in an integrated manner and are thus relying on their mutually supporting relationships.

Thus, as a whole the conceptual framework offers a comprehensive outlook and emphasizes how multiple green attributes interact with dynamic processes within a supply chain system. Sustainability is not a single function; it is an emergent outcome of the integrated environmental behaviours of everyone in every aspect of the organization.

Green design
Green design promotes the deployment of eco-products, optimizes the consumption of natural resources, and reduces products that could be reused by 100% of building materials. Thus, this approach reduces waste and the consumption of energy.

Green procurement
Green procurement is about identifying vendors who fulfil their criteria by their environmental consciousness as well as ensuring the necessary feedback of the environment and employees. Also, it is about obtaining the green procurement of materials by green suppliers who are fair and honest in selling sustainably.

Green collaboration
Green collaboration is the sum of the joint efforts of the supply chain partners, such as suppliers, manufacturers, distributors, and customers, in achieving common environmental goals. When collaborating companies share data and ideas, they perform innovative projects together and coordinate their strategies, they can each lower their carbon emissions and make better use of resources, which in turn results in higher sustainability performance. Additionally, green collaboration deepens the trust that the partners share and brings them closer to the final goal of eco-efficiency as a result of the creation of long-term partnerships.

Green manufacturing
The focus of green manufacturing is on the process of minimizing environmental problems like air pollution, energy usage, and saving energy; hence, developing models so that end-users also benefit from the savings produced during the manufacturing process. Accomplishing this can happen through the use of wind or solar power, the transformation of conventional power plants to natural gas, the application of energy-efficient assembly methods, and the implementation of lean logistics solutions.

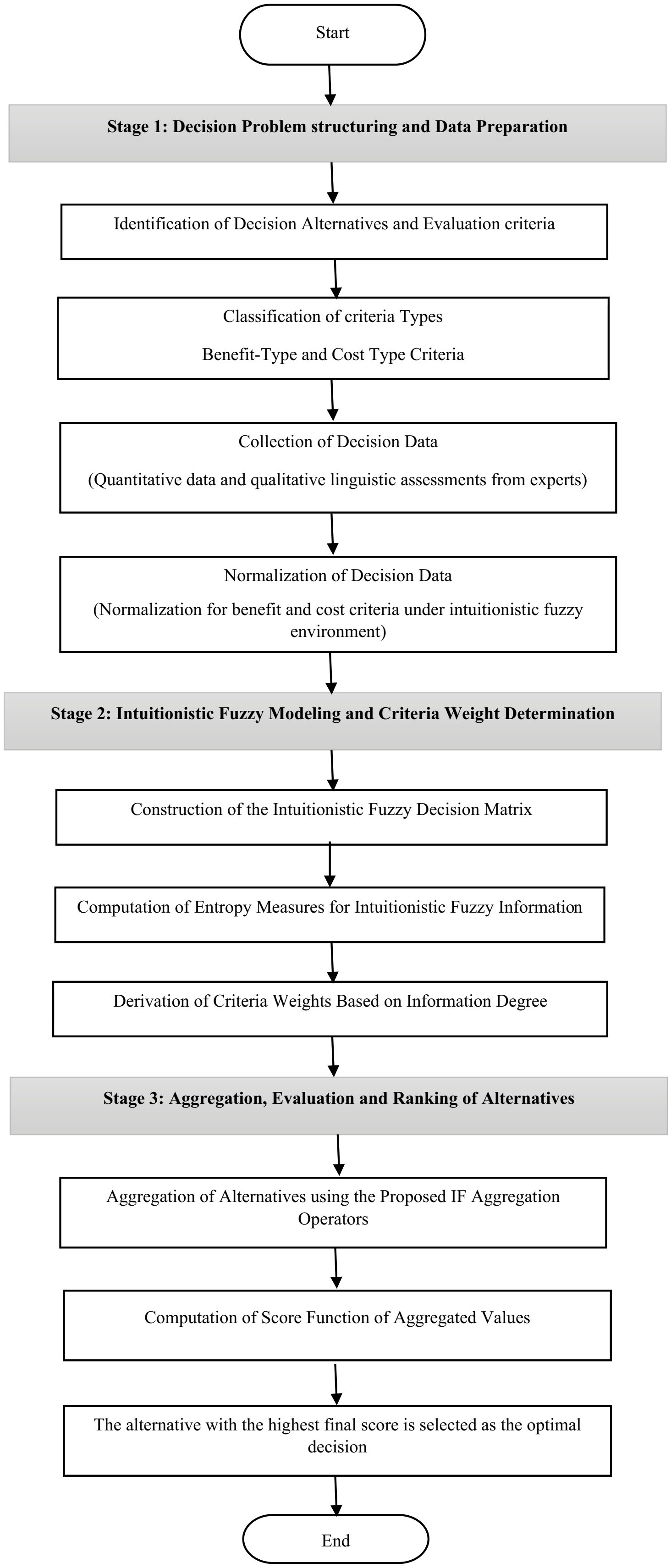

Fig. 3. Algorithmic workflow of the proposed intuitionistic fuzzy MADM method.

Green logistics
The main objective of green logistics is to change ways of doing things so that the degradation of the environment is lessened. Approaches such as this can be accelerated by adopting sustainable energy apart from permitting the access of broad traffic that can conduct lane-based mobility for all societies. Such solutions must also include renewable energy sources, thus decreasing the environmental footprint as well as transportation cost.

Green inventory
Green inventory management is a way to make sure that the inventory levels are kept at an optimal level so that there is neither wastage nor overproduction while at the same time market demand is met. It supports activities such as just-in-time inventory, recycling of the unused stock, and the use of eco-friendly storage solutions. These changes in the company's material flow led to the decrease of material obsolescence, waste disposal costs, and environmental pollution.

Green reverse logistics
Green reverse logistics refers to the processes that deal with the proper handling of returns, defective products, and those which have reached the end of their life cycle. Such processes are the collection, disassembly, refurbishment, and recycling of products which results in the promotion of material recovery and reuse. Applying circular economy principles to GRL not only prolongs product life cycles but also lessens waste generation and generates economic value from the recovered resources, thereby reducing environmental pollution simultaneously.

Green technology
Green technology emerges from the applications of solar power, and use of the latest tech in environmental monitoring, for instance, IoT (Internet of things). Data analytics that help predict potential environmental challenges and thereby prevent the creation of waste and the release of greenhouse gases in the supply chain are more on the focus of technology and green waste less delivery systems.

Green information systems
Green information systems are a well-organized digital framework that helps in capturing, processing, and sharing the data related to the environment coming from the supply chain. These systems encourage the real-time availability of the environmental metrics such as the carbon footprints, resource use, and compliance status. GIS supports environmental auditing, sustainability reporting, and decision-making in organizations; thus, it is a tool that keeps all levels of the supply chain informed and oriented towards green objectives.

Green readiness
Green readiness refers to the mindset of life that is always ready for the environmental challenges such as notable disasters that have occur natural disasters, and global warming that is changing the world regulatory changes. The priority that these companies have besides green and social responsibility as mentioned to be the investment in measures which foster on the growth of supply chain partners. These tasks include the supply of multivendor solutions, the evaluation of business risks, and the adjustment of the emergency response plans.

Green human resource management (GHRM)
Green Human Resource Management aims at building a workforce that not only supports but also is committed to the environmental goals. It comprises environmentally friendly recruitment, various programs, performance appraisal systems, and incentive mechanisms that attract and retain employees who behave responsibly towards the environment. GHRM helps the organization to become green and hence the employees' culture aligns with the sustainability values of the company making them the key drivers for achieving environmental excellence.

Sustainable performance (SP)
Sustainable performance is the end result of the effective and integrated use of GRSCM practices. It is the triple bottom line which are economic prosperity, environmental protection, and social well-being. The right implementation of GRSCM leads to the improvement of the corporate reputation, cost reduction of the operations, and increasing of the resource efficiency thus enabling firms to achieve a long-term competitive advantage while at the same time contributing to global sustainability goals.

Construction of the intuitionistic fuzzy decision matrix
An intuitionistic fuzzy decision matrix (IFDM) was developed in this research for dealing with qualitative and quantitative attributes under uncertainty. The method facilitates a systematic approach to transform raw data and linguistic judgements into a general intuitionistic fuzzy environment, which retains the intrinsic ambiguity associated with human reasoning. The detailed procedure for construction is described in the subsequent text.

Step 1: Identification of Criteria and Alternatives

Let there be m alternatives $A_1, A_2 \ldots, A_m$ and n evaluation criteria $C_{1,}C_2, \ldots, C_m$ Each criterion can be of benefit-type (where higher values are preferred) or cost-type (where lower values are desired). For example, C_4 (Green Manufacturing) and C_5 (Green Logistics) are treated as cost-type criteria since higher energy usage or fuel consumption indicates lower environmental efficiency. All other criteria are considered benefit-type, as their improvement directly contributes to sustainable performance.

Step 2: Determination of Criterion Type and Data Collection

For each criterion, corresponding data were collected.

a) Qualitative data were obtained through expert linguistic evaluations using terms such as Very Good, Good, and Moderate.
b) Quantitative data were collected from numerical measurements such as cost, time and performance scores.

Each qualitative linguistic term was converted into an Intuitionistic Fuzzy Number (IFN) of the form $A = (\mu, \gamma)$,where μ and γ denote the membership and non-membership degrees, respectively with $\mu + \gamma \leq 1$. For instance:

$$\text{Very Good} = (0.82,0.10)\,, \text{Good} = (0.70,0.17)\,,\ \text{Moderate}\ = (0.58,0.31)$$

Step 3: Normalization of Quantitative Criteria

To bring all numeric values to comparable scale, normalization was applied using a linear scale transformation. For benefit-type criteria:

$$x_{ij'} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)}$$

For cost -type criteria:

$$x_{ij'} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)}$$

where x_{ij} is the raw data value for the ith alternative under the jth criterion. The normalized values $x_{ij'}$ were further mapped to corresponding IFNs by defining suitable membership and suitable non-membership degrees proportional to normalized scale.

Step 4: Construction of Combined IF Decision matrix

After transforming both qualitative and quantitative data into IFNs, the combined intuitionistic fuzzy decision matrix $D = [A_{ij}]_{m \times n}$ was constructed as:

$$D = \begin{bmatrix} (\mu_{11}, \gamma_{11}) & \cdots & (\mu_{1n}, \gamma_{1n}) \\ \vdots & \ddots & \vdots \\ (\mu_{m1}, \gamma_{m1}) & \cdots & (\mu_{mn}, \gamma_{mn}) \end{bmatrix}$$

where each element $A_{ij} = (\mu_{ij}, \gamma_{ij})$ denotes the intuitionistic fuzzy evaluation of alternative A_i under the criterion C_j

Step 5: Compute Entropy for Intuitionistic Fuzzy Numbers

The intuitionistic fuzzy entropy quantifies the degree of uncertainty or fuzziness in each criterion.

For intuitionistic fuzzy data, the entropy [60] can be calculated as:

$$E_j = -\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{1}{2}\left[-4{\mu_{ij}}^2 + 4\mu_{ij} - 4{\gamma_{ij}}^2 + 4\gamma_{ij}\right] + 4\gamma_{ij} + log_2(1 + \pi_{ij}^3)\right\}$$

Higher entropy indicates greater uncertainty in that criterion, while lower entropy signifies higher decisiveness in expert opinions.

Step 6: Calculation of Information Degree

The information degree reflects the level of useful information provided by each criterion and is obtained as $d_j = 1 - E_j$

A criterion with a higher information degree contributes more decisively to the decision process.

Step 7: Determination of Normalized weights

Finally, the normalized weight w_j is derived based on its information degree

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}$$

Subject to $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j \leq 1$

These weights ensure that each criterion's importance is reflected proportionally in the decision-making process while maintaining the normalization condition.

Step 8: Formation of Weighted IF Decision Matrix

Since the decision problem involves both measurable (quantitative) and linguistic (qualitative) attributes, a mixed data approach was employed to construct the Intuitionistic Fuzzy Decision Matrix (IFDM). Quantitative indicators were first normalized based on their attribute nature (benefit or cost type) and subsequently transformed into Intuitionistic Fuzzy Numbers (IFNs) using a small degree of hesitation. The qualitative factors were evaluated using linguistic judgments provided by domain experts, which were converted into corresponding IFNs via a predefined linguistic scale. The resulting matrix seamlessly integrates both data types, allowing uniform processing under the intuitionistic fuzzy environment for subsequent weight determination and aggregation analysis.

The weighted IF decision matrix is then obtained by multiplying each element A_{ij} with its corresponding weight w_j:

$D^* = [A_{ij}\prime]_{m \times n}$ $\mu_{ij}\prime = \mu_{ij} * (E_j)$ and $\gamma_{ij}\prime = \gamma_{ij} * (1 - E_j)$

$$D^* = \begin{bmatrix} (\mu_{11}\prime, \gamma_{11}\prime) & \cdots & (\mu_{1n}\prime, \gamma_{1n}\prime) \\ \vdots & \ddots & \vdots \\ (\mu_{m1}\prime, \gamma_{m1}\prime) & \cdots & (\mu_{mn}\prime, \gamma_{mn}\prime) \end{bmatrix}$$

This matrix serves as the input for subsequent aggregation and ranking procedures, such as intuitionistic fuzzy Yager aggregation operators. To integrate objective information into the fuzzy environment, the intuitionistic fuzzy entropy method was applied to refine the decision matrix. The entropy E_j of each criterion quantifies its discriminative information content. The membership and non-membership degrees were then adjusted proportionally to E_j and $1 - E_j$ respectively. This yields an entropy-refined intuitionistic fuzzy decision matrix that simultaneously accounts for uncertainty and objective variability among criteria.

The algorithm that is proposed guarantees that the uncertainties, doubts, and ambiguities that are embedded in expert evaluations are properly represented by the use of intuitionistic fuzzy modelling, while the measures of information based on entropy improve the overall objectivity and strength of the final assessment. The entire procedure, which is shown in the following Algorithm, yields a clear and replicable framework for the multi-criteria assessment of uncertain decision-making environments.

Input:

$A = A_1, A_2 \ldots, A_m$: Set of alternatives (companies), $C = C_1, C_2, \ldots, C_m$: Set of evaluation criteria

Qualitative criteria expressed in linguistic terms {VG, G, M}, mapped to intuitionistic fuzzy numbers (IFNs): $\text{Very Good} = (0.82{,}0.10), \text{Good} = (0.70{,}0.17)\,,\ \text{Moderate}\ = (0.58{,}0.31)$

Quantitative criteria expressed as numerical data which are Benefit-type criteria and Cost-type (Table 1).

Output:

Final ranking of alternatives based on Score Function Ranking.

1. // Stage 1: Problem Initialization
2. Define $m = 4$ (number of alternatives), $n = 12$ (number of criteria)
3. Identify each criterion as benefit-type or cost-type as per GRSCM domain
4. // Stage 2: Data Transformation
5. For each criterion C_j and alternative A_i
6. if C_j is qualitative:
7. Convert linguistic term to IFN (μ_{ij}, γ_{ij})
8. else if C_j is quantitative:
9. if $C_j \in$ benefit-type:
10. Scale:$x'_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)}$
11. else if $C_j \in$ cost-type
12. Scale: $x'_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)}$
13. Set membership $\mu_{ij} = x'_{ij}$ and non-membership $\gamma_{ij} = 1 - \mu_{ij} - \pi_{ij}$
14. Set hesitation degree $\pi_{ij} = 0.1$ (if uncertainty exists)
15. Construct intuitionistic fuzzy decision matrix:

Notations	Abbreviations
IFS	Intuitionistic Fuzzy Set
IFN	Intuitionistic Fuzzy Number
$S(\widetilde{a})$	Score Function of Intuitionistic Fuzzy Number
$H(\widetilde{a})$	Accuracy Function of Intuitionistic Fuzzy Number $\widetilde{a}$
IFYWAA	Intuitionistic Fuzzy Yager Weighted Averaging Aggregation
IFYOWAA	Intuitionistic Fuzzy Yager Ordered Weighted Averaging Aggregation
IFYHWAA	Intuitionistic Fuzzy Yager Hybrid Weighted Averaging Aggregation
IFYWGA	Intuitionistic Fuzzy Yager Weighted Geometric Aggregation
IFYOWGA	Intuitionistic Fuzzy Yager Ordered Weighted Geometric Aggregation
IFYOWAA	Intuitionistic Fuzzy Yager Hybrid Weighted Geometric Aggregation
MADM	Multi Attribute Decision Making
MCDM	Multi Criteria Decision Making
GSCM	Green Supply Chain Management
IFDWAA	Intuitionistic Fuzzy Dombi Weighted Averaging Aggregation
IFDWGA	Intuitionistic Fuzzy Dombi Weighted Geometric Aggregation

Table 1. Presents the abbreviations used throughout this paper.

17. // Stage 3: Entropy Computation for Each Criterion
18. For each criterion $C_j (j = 1ton)$
19. Compute Entropy
21. End For
22. // Stage 4: Determine Information Degree and Normalized Weights
23. For each criterion C_j:
25. Compute normalized weight:
27. Verify $\sum_{j=1}^{n} w_j \leq 1$
28. // Stage 5: Construct Weighted IF Decision Matrix
29. For each element (μ_{ij}, γ_{ij}) in D:
32. Form weighted matrix $D^* = [(\mu'_{ij}, \gamma_{ij})]_{m \times n}$
33. // Stage 6: Compute Aggregation values Using IFYWAA or IFYWGA Operator
34. for each alternative A_i:
35. Implement Averaging and Geometric aggregation:
36. end for
37. // Stage 7: Ranking of Alternatives
38. for each A_i:
39. Compute Score Function
40. End for
41. Rank alternatives in descending order of Score Function
42. Return final ranking of Alternatives

The above process effectively integrates objective quantitative data and subjective qualitative judgments under the intuitionistic fuzzy environment, while the entropy and information degree methods ensure data-driven weight allocation**.** Thus, the constructed IFDM provides a robust, unbiased, and comprehensive representation of the decision environment. The respective weight vectors of the twelve attributes across the four alternatives are illustrated in the following Figs. 4, 5, 6 and 7.

Table 2 shows the decision matrix of aforementioned alternative according to the different attributes. Then we decide upon to choose the appropriate alternative by the IFYWA by performing the procedures indicated in Fig. 2 and 3.

The Evaluation Performance values ξ_i of alternatives (Companies) by IFYWAA operator for $\eta = 3$ are

$$\xi_1 = (0.445246, 0.091357)$$

$$\xi_2 = (0.43432, 0.141732)$$

$$\xi_3 = (0.358778, 0.042241)$$

$$\xi_4 = (0.444372, 0.108052)$$

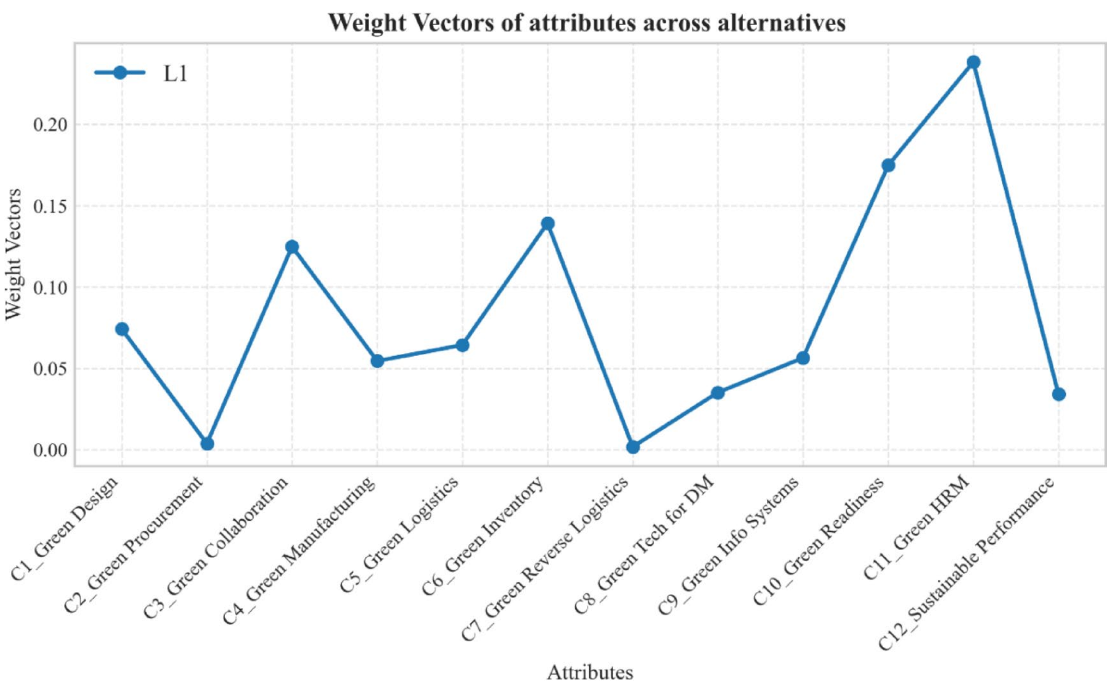

Fig. 4. Weight Vector of Alternative L_1 across 12 attributes.

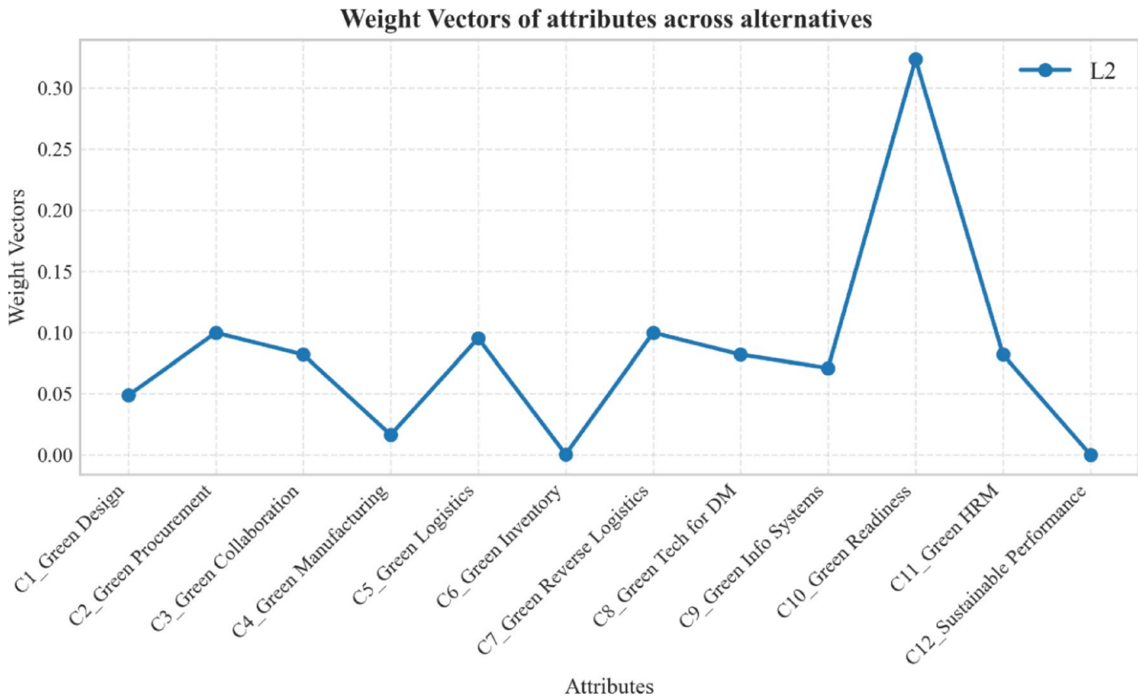

Fig. 5. Weight Vector of Alternative L_2 across 12 attributes.

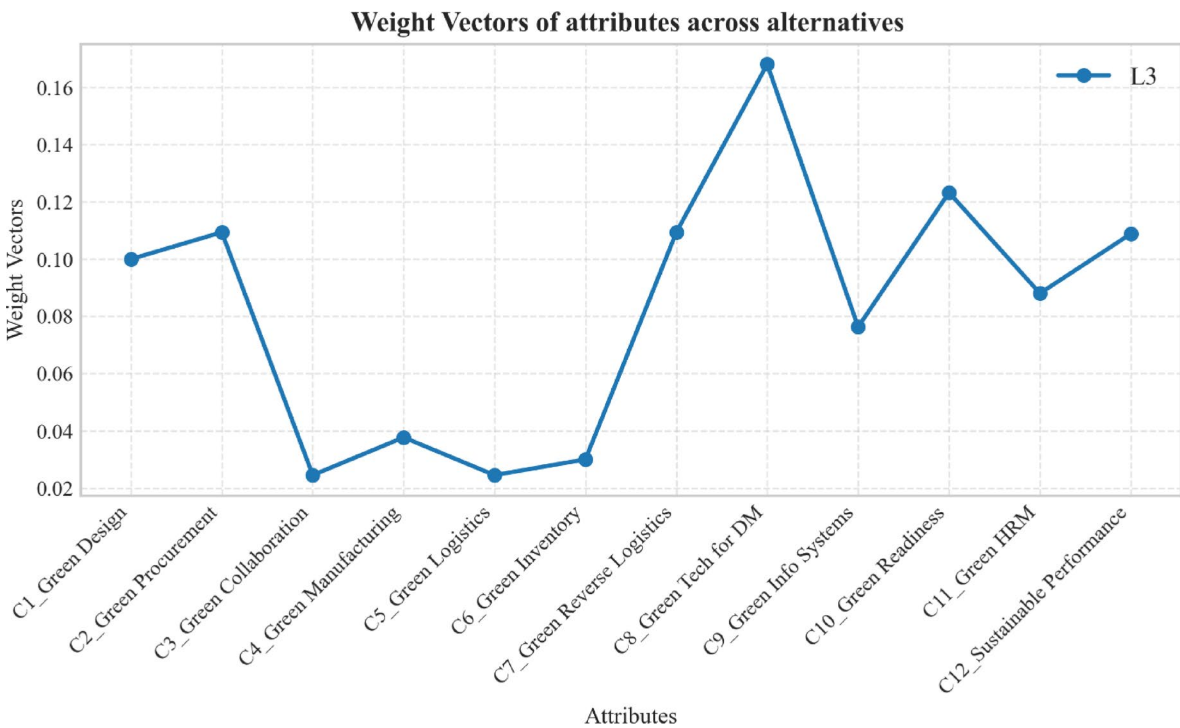

Fig. 6. Weight Vector of Alternative L_3 across 12 attributes.

We obtain the score metrics ξ_i, which are given as,

$$S(\xi_1) = 0.35388$$

$$S(\xi_2) = 0.292588$$

$$S(\xi_3) = 0.316537$$

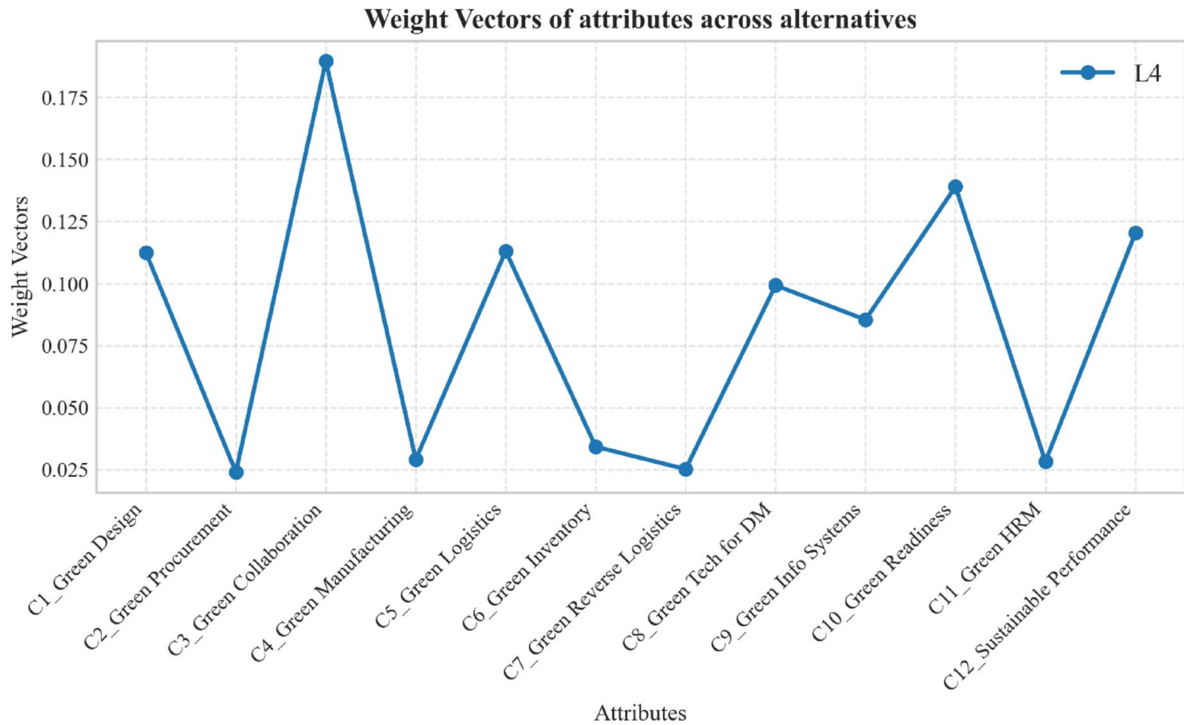

Fig. 7. Weight Vector of Alternative L_4 across 12 attributes.

L_i/k_j	K_1	K_2	K_3	K_4	K_5	K_6
L_1	[0.494,0.05]	[0.558,0.01]	[0.494,0.05]	[0,0]	[0.5735,0.0172]	[0,0.9704]
L_2	[0.494,0.05]	[0,0.9704]	[0.494,0.05]	[0.087,0.375]	[0,0]	[0.46,0.002]
L_3	[0.37,0.05]	[0,0]	[0.53,0.02]	[0,0.97]	[0.19,0.17]	[0.54,0.06]
L_4	[0.37,0.05]	[0.577,0.05]	[0.494,0.52]	[0.37,0.06]	[0,0.97]	[0.54,0.06]
L_i/k_j	K_7	K_8	K_9	K_{10}	K_{11}	K_{12}
L_1	[0.439,0.0057]	[0.5318,0.026]	[0.494,0.05]	[0.494,0.05]	[0.37,0.056]	[0.568,0.0577]
L_2	[0,0.9704]	[0.494,0.05]	[0.37,0.056]	[0.5318,0.026]	[0.494,0.05]	[0.5.0.00009]
L_3	[0,0]	[0.37,0.056]	[0.494,0.05]	[0.494,0.05]	[0,0]	[0,0]
L_4	[0.573,0.553]	[0.494,0.05]	[0.37,0.06]	[0.493,0.05]	[0.532,0.026]	[0,0.97]

Table 2. Intuitionistic Fuzzy Decision Matrix.

$$S(\xi_4) = 0.33632$$

As $S(\xi_1) > S(\xi_4) > S(\xi_3) > S(\xi_2)$. Hence $L_1 > L_4 > L_3 > L_2$. Thus,L_1 is the appropriate green supplier in the GRSCM among other three Green suppliers.

Now applying IFYWGA operator to find the evaluation performance values ξ_i.

$$\xi_1 = (0.173442,0.325889)$$

$$\xi_2 = (0.291438,0.4601285)$$

$$\xi_3 = (0.265487,0.567712)$$

$$\xi_4 = (0.315538,0.502919)$$

We obtain the score metrics ξ_i, which are given as,

$$S(\xi_1) = -0.15245$$

$$S(\xi_2) = -0.30985$$

Operators	$S(\xi_1)$	$S(\xi_2)$	$S(\xi_3)$	$S(\xi_4)$	Order of Rankings
IFYWAA	0.35388	0.292588	0.316537	0.33632	$L_1 > L_4 > L_3 > L_2$.
IFYWGA	-0.15245	-0.30985	-0.30223	-0.18738	$L_1 > L_4 > L_3 > L_2$.

Table 3. Comparison between IFYWAA and IFYWGA operators.

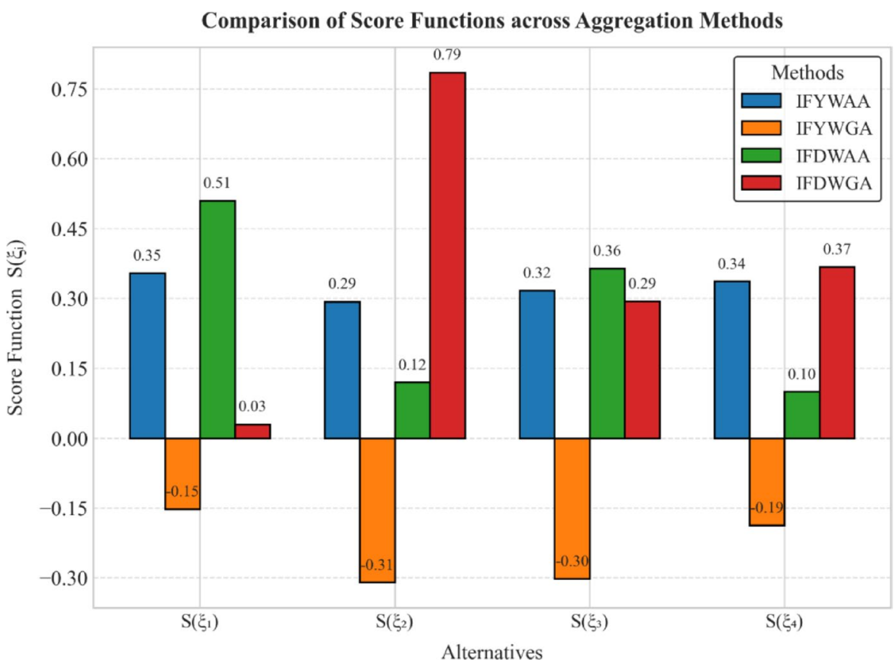

Fig. 8. Comparison of Four Techniques.

$$S(\xi_3) = -0.30223$$

$$S(\xi_4) = -0.18738$$

As $S(\xi_1) > S(\xi_4) > S(\xi_3) > S(\xi_2)$. Hence $L_1 > L_4 > L_3 > L_2$. Thus, L_1 is the appropriate green supplier in the GRSCM among other three Green suppliers. Table 3 shows that using both operators,L_1 is the appropriate green provider. As a result, we are able to employ any operator that fits between the aforementioned operators.

Comparison analysis

The section compares the newly designed Yager-based Intuitionistic Fuzzy Aggregation Operators with the already available Dombi-based operators [61] to see their relative effectiveness in solving Multi-Attribute Decision-Making (MADM) problems. The comparison is mainly between the Intuitionistic Fuzzy Yager Weighted Averaging Aggregation (IFYWAA) and Intuitionistic Fuzzy Yager Weighted Geometric Aggregation (IFYWGA) operators with their Dombi Operators Intuitionistic Fuzzy Dombi Weighted Averaging Aggregation (IFDWAA) and Intuitionistic Fuzzy Dombi Weighted Geometric Aggregation (IFDWGA), respectively. In contrast, the IFDWAA and IFDWGA operators exhibit variability in their outcomes, implying sensitivity to minor changes in attribute weights or membership degrees. The findings from IFYWAA and IFYWGA, as the results show, are both the same and consistent in ranking all alternatives, thus, pointing to a stable aggregation mechanism. On the other hand, the results of IFDWAA and IFDWGA are inconsistent, and hence, different variations of the outcomes have been obtained, thus, indicating a sensitivity to minor changes in attribute weights or membership degrees.

The difference identified here is indicative of the proposed Yager-based operators' superior robustness, flexibility, and computational coherence to handle the intuitionistic fuzzy MADM problems. The comparative performance of the four methods is displayed graphically in Fig. 8, which is, therefore, an additional confirmation of the proposed framework's stability and interpretability. Table 4 Comparison Analysis for Sect. 5.1 with IFDWAA and IFDWGA.

This study systematically altered the parameter η to analyse its impact on the performance and aggregation behaviour of proposed operator. The values of η were chosen as 5, 8, 13, 29, 52,73 and 89 to represent varying levels of sensitivity in the weighting structure of the aggregation. Varying η allows for effective observation of each operator's response to fluctuations in membership, non-membership, and hesitation degrees. Tables 5 and

Methods	$S(\xi_1)$	$S(\xi_2)$	$S(\xi_3)$	$S(\xi_4)$	Order of Rankings
IFYWAA	0.35388	0.292588	0.316537	0.33632	$L_1 > L_4 > L_3 > L_2$
IFYWGA	-0.15245				
	-0.30985	-0.30223	-0.18738	$L_1 > L_4 > L_3 > L_2$	
IFDWAA [61]	0.517	0.27	0.364	0.10	$L_1 > L_3 > L_2 > L_4$
IFDWGA [61]	0.03	0.785	0.293	0.367	$L_2 > L_4 > L_3 > L_1$

Table 4. Comparison Analysis between proposed operators and IFDWAA and IFDWGA operators.

η	Scores of alternatives	Order of Rankings
5	0.393, 0.349, 0.363, 0.390	$L_1 > L_4 > L_3 > L_2$
8	0.424, 0.388, 0.398, 0.422	$L_1 > L_4 > L_3 > L_2$
13	0.448, 0.418, 0.429, 0.447	$L_1 > L_4 > L_3 > L_2$
29	0.485, 0.461, 0.469, 0.475	$L_1 > L_4 > L_3 > L_2$
52	0.510, 0.487, 0.490, 0.493	$L_1 > L_4 > L_3 > L_2$
73	0.522, 0.498, 0.499, 0.505	$L_1 > L_4 > L_3 > L_2$
89	0.529, 0.502, 0.504, 0.510	$L_1 > L_4 > L_3 > L_2$

Table 5. order of rankings of alternatives for different values of η for IFYWAA.

η	Scores of alternatives	Order of Rankings
5	−0.37286, −0.5035, −0.49688, −0.40179	$L_1 > L_4 > L_3 > L_2$
8	−0.55278, −0.65552, −0.64588, −0.57686	$L_1 > L_4 > L_3 > L_2$
13	−0.69515, −0.76841, −0.76158, −0.71532	$L_1 > L_4 > L_3 > L_2$
29	0.83943, −0.87643, −0.87359, −0.85153	$L_1 > L_4 > L_3 > L_2$
52	−0.89565, −0.91718, −0.91564, −0.90319	$L_1 > L_4 > L_3 > L_2$
73	−0.91662, −0.93222, −0.93112, −0.92228	$L_1 > L_4 > L_3 > L_2$
89	−0.92608, −0.93895, −0.93805, −0.93084	$L_1 > L_4 > L_3 > L_2$

Table 6. Order of rankings of alternatives for different values of η for IFYWGA.

6 shows that order of rankings of alternatives for different values of η.The following graphs, Figs. 9 and 10 show the variance of score function values corresponding to each alternative, emphasizing how varied parameter settings influence.

Discussion

Sensitivity analysis

To evaluate the robustness and reliability of the proposed Intuitionistic Fuzzy Multi-Attribute Decision-Making (IF-MADM) model in evaluating sustainable performance among companies implementing Green Supply Chain Management (GRSCM) practices, a sensitivity analysis was conducted. The primary objective of this analysis was to examine the influence of variations in attribute weights on the final ranking of alternatives and to validate the stability of the proposed decision-making framework.

The model considers twelve attributes: Green Design, Green Procurement, Green Collaboration, Green Manufacturing, Green Logistics, Green Inventory, Green Reverse Logistics, Green Technology for Decision-Making, Green Information Systems, Green Readiness, Green Human Resource Management and Sustainable Performance. After entropy-based weight determination, it becomes necessary to analyse whether slight changes in these attribute weights significantly alter the final decision ranking.

Two distinct scenarios were designed for this purpose:

Case 1: Equal Weights Assigned to All Attributes

In the first scenario, each of the twelve attributes was assigned an equal importance weight**,** i.e.,

$$w_j = \frac{1}{12} = 0.0833, j = 1,2,\ldots,12.$$

This case represents a neutral decision environment, where all attributes contribute uniformly to the overall assessment without emphasizing any specific dimension of GRSCM. The weighted Intuitionistic Fuzzy Decision

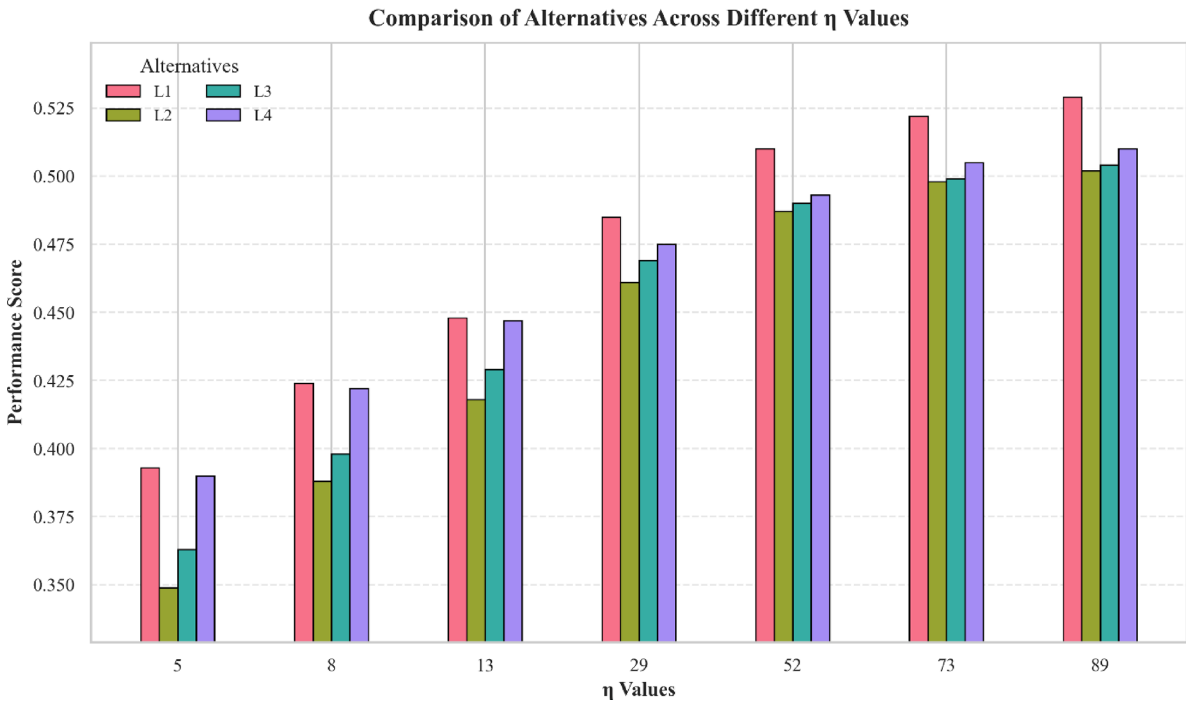

Fig. 9. Comparison of alternatives across different η values for IFYWAA.

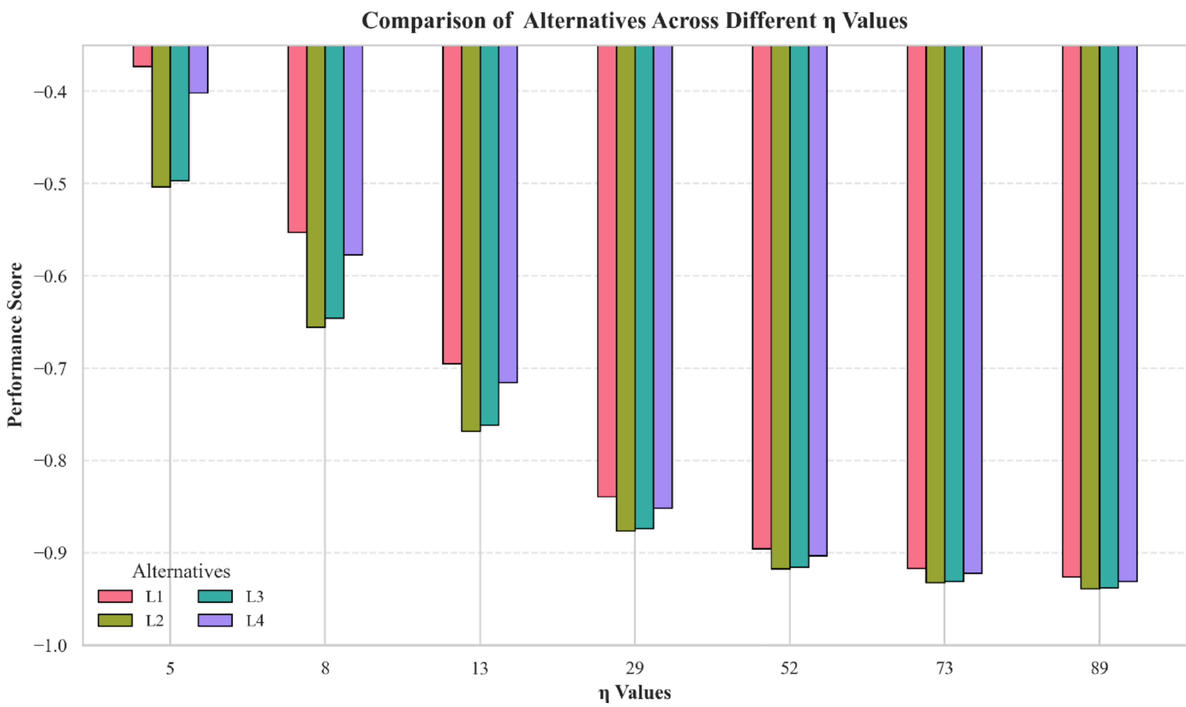

Fig. 10. Comparison of alternatives across different η values for IFYWGA.

Matrix (IFDM) was reconstructed using these uniform weights, and the ranking of alternatives was recomputed using the IFWA–OWAWAD aggregation mechanism.

The results showed that the top-performing company (L_1) remained unchanged, while minor positional variations occurred among lower-ranked alternatives (L_2, L_3, L_4) This indicates that the decision structure remains stable under uniform weighting conditions.

Case 2: Emphasis on Strategic and Operational Attributes.

Criterion	Description	w_j
K_1	Green Design	0.11
K_2	Green Procurement	0.11
K_3	Green Collaboration	0.11
K_4	Green Manufacturing (cost-type)	0.10
K_5	Green Logistics (cost-type)	0.10
K_6	Green Inventory	0.10
K_7	Green Reverse Logistics	0.10
K_8	Green Technology for Decision Making	0.054
K_9	Green Information Systems	0.054
K_{10}	Green Readiness	0.054
K_{11}	Green HRM	0.054
K_{12}	Sustainable Performance	0.054

Table 7. Weight vectors of the attributes for Case 2.

Criterion	Description	w_j
K_1	Green Design	0.06
K_2	Green Procurement	0.06
K_3	Green Collaboration	0.06
K_4	Green Manufacturing (cost-type)	0.06
K_5	Green Logistics (cost-type)	0.06
K_6	Green Inventory	0.06
K_7	Green Reverse Logistics	0.06
K_8	Green Technology for Decision Making	0.10
K_9	Green Information Systems	0.10
K_{10}	Green Readiness	0.05
K_{11}	Green HRM	0.05
K_{12}	Sustainable Performance	0.28

Table 8. Weight vectors of the attributes for Case 3.

In this scenario decision-makers place greater emphasis on strategic $(K_1 - K_3)$ and operational $(K_4 - K_7)$ attributes because these dimensions directly influence process efficiency, environmental performance, whereas enabling and outcome attributes were assigned lower weights. To test this perspective, we specify the following normalized weight vector (sum = 1).The weight vectors of the attributes for case 2 is shown in Table 7.

The weights above were chosen so that (a) strategic criteria each receive a noticeably higher weight 0.11 reflecting managerial priority, (b) operational criteria are collectively dominant (four criteria at 0.10 each), and (c) the remaining enabling/outcome criteria share the residual importance (each ≈ 0.054). All weights were normalized so that $\sum_{j=1}^{12} w_j = 1$. The normalized weights are then applied to the combined IF decision matrix and the aggregation and ranking process is repeated.

This represents a practical managerial perspective where decision-makers prioritize process-based environmental initiatives over supportive systems.

The resulting ranking again placed Company L_1 as the top performer, followed by L_4, L_3, L_2.The consistency of rankings under both cases confirms that the proposed IF-MADM model is robust**,** resilient to weight perturbations, and capable of producing stable decisions even under varying evaluation priorities.

Case 3: Outcome & Enabling Emphasis (policy-driven).

As an alternative plausible viewpoint, one could imagine decision-makers or regulators prioritizing long-term sustainability outcomes and enabling infrastructures. For this case we assign larger weight to sustainable performance (K_{12}) and to enabling criteria (K_8, K_9) while reducing the emphasis on strategic/operational process measures. The weight vectors of the attributes for case 2 is shown in Table 8.

This configuration models stakeholders focused on long-term outcomes: K_{12}, (Sustainable Performance) receives a large share (0.28) reflecting outcome-prioritized policy goals; enabling technologies and information systems receive elevated weights (0.10 each); process-related and strategic items still matter but are down-weighted (0.06 each). Again, weights are normalized to sum to 1. And now recompute the weighted Intuitionistic Fuzzy Decision Matrix, aggregate with IFYWAA and IFYWGA and compute the new scores and ranking.

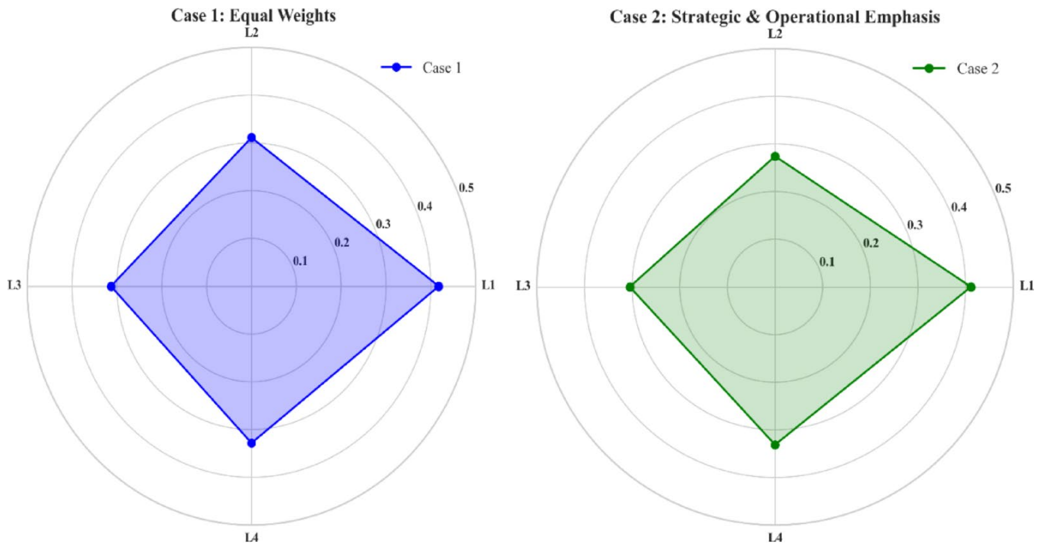

Fig. 11. Sensitivity analysis results for case 1 and case 2.

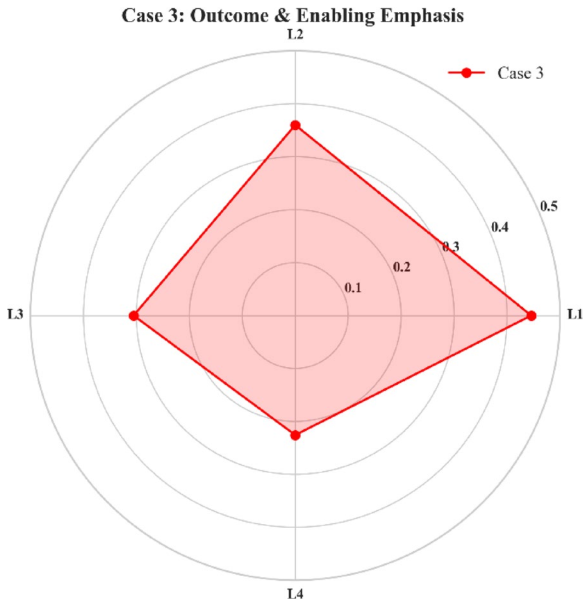

Fig. 12. Sensitivity analysis results for case 3.

Interpretation

The findings of the three cases indicate that small variations in the weighting scheme, whether by allocating equal weights or selectively emphasizing criteria, did not have any substantial effect on the overall ranking of the alternatives. The findings imply that the proposed Hybrid Entropy Intuitionistic Fuzzy Yager Aggregation decision-model is very robust, consistent, and reliable. Furthermore, by introducing weight normalization, we combined both aspects: providing factual basis for all situations under comparison and potential use of our decision model independent of which managerial assumptions or expert preferences are taken into account. The sensitivity analysis results for three cases are depicted in Figs. 11 and 12 and explained in Table 9.

To systematically evaluate the proposed framework from a professional perspective, the key evaluation indicators considered in the application are summarized in Table 10.

Model characteristics, limitations, and extensions

Practical implementations of the proposed intuitionistic fuzzy MADM framework

The Intuitionistic Fuzzy Multi-Attribute Decision-Making (IF-MADM) model operates through a combined system that merges Entropy with Intuitionistic Fuzzy Weighted Yager Averaging and Geometric aggregation operators to deliver a flexible decision analysis framework for sustainability applications. The model enables

	Ranking Values	Ranking order
Case 1	0.417126,0.311196,0.31324,0.328268	$L_1 > L_4 > L_3 > L_2$.
Case 2	0.410976,0.273961,0.30463,0.331284	$L_1 > L_4 > L_3 > L_2$.
Case 3	0.445729,0.359955,0.305276,0.225176	$L_1 > L_2 > L_3 > L_4$.

Table 9. Sensitivity analysis Results.

Indicator	Description
Uncertainty modelling capability	Ability of the proposed framework to represent the uncertainty of expert evaluations based on intuitionistic fuzzy information for GRSCM criteria such as green procurement, eco-design and environmental compliance, etc
Hesitation preservation	Extent to which the aggregation process retains and propagates the degree of hesitation present in the expert judgment(s) when conducting a sustainable assessment
Aggregation flexibility	Capability of Yager's Parameterized T-Norm and T-Conorm to adapt its aggregation behaviour based on the different attitudes towards decision-making found in sustainable evaluations
Criteria weighting objectivity	Effectiveness of the Entropy-Based Weighting Scheme in determining the relative importance of the sustainability criteria objectively rather than only based on expert's subjective preferences
Theoretical consistency	Assurance of mathematical soundness through satisfaction of axiomatic properties such as idempotency, monotonicity, boundedness, and commutativity within the aggregation process
Ranking stability	The degree to which the proposed method produces ranking(s) of GRSCM alternatives that are consistent and reliable with respect to the intuitionistic fuzzy aggregation
Result Interpretation	Clarity with which decision-makers can understand and explain the aggregated results and final rankings in practical sustainability decision scenarios
Practical applicability	Suitability of the proposed framework for real-world GRSCM decision-making problems involving multiple sustainability criteria and uncertain information

Table 10. Description of Professional evaluation indicators.

decision-makers to evaluate and rank multiple options under conditions of uncertainty through its combination of expert intuitive assessments and numerical information within a single computational framework.

The section demonstrates the practical application of the proposed method for Green Supply Chain Management (GRSCM) operations.

Green supplier selection and evaluation

Supplier selection operates as the core GRSCM element which determines the entire supply chain environmental performance results. The proposed IF-MADM framework enables systematic supplier ranking by evaluating environmental and operational sustainability through Green Procurement (K_2), Green Logistics (K_5), and Green Reverse Logistics (K_7)attributes.

Expert assessments of supplier performance begin with linguistic judgments such as Moderate, Good, and Very Good which experts then convert into Intuitionistic Fuzzy Numbers (IFNs) to represent their uncertain and hesitant subjective views. The entropy mechanism performs dual operations to normalize and integrate quantitative indicators.

Sustainable manufacturing process optimization

The drive for low-carbon, resource-efficient production is a must for modern manufacturing to be able to compete in the long run. The presented model makes it possible to easily compare the different production strategies or process designs with respect to the selected criteria like Green Manufacturing (K_4), Green Inventory (K_6), and Green Collaboration (K_3).

By entropy-based weight calculation, the system points to the criteria that most strongly lead to sustainable results, and the IF-based aggregation shows the uncertainty of the operational data (e.g., energy consumption, material recyclability, or process waste). The decision results obtained guide the managers in finding the right moments to make their processes more efficient, in using their resources in the best way, and in bringing their production systems in line with the principles of the circular economy.

Digital transformation and green technology adoption

Industry 4.0 brought digital innovation which is increasingly considered as a sustainability enabler. The suggested IF-MADM model serves as a well-organized tool for assessing and understanding the preparatory work and the technological changes' effects, specifically under criteria such as Green Technology for Decision-Making (K_8), Green Information Systems (K_9), and Green Readiness (K_{10}).

The model combines expert opinions related to the technological feasibility with the quantitative measures like system efficiency, data accuracy, and carbon tracking capability. By uncertainty captured through intuitionistic fuzzification, the model assists the executives in choosing technology portfolios that bring both ecological and strategic benefits to a maximum helping digital transformation initiatives to be eco-friendly.

Sustainability benchmarking among competing firms
Businesses are turning to the practice of comparing their green performance to that of the competitors for the purpose of not only sustaining advantage but also adhering to the norms of sustainability reporting. The instrument put forward here makes it possible to carry out comparative sustainability benchmarking by assessing several companies ($L_1 - L_4$) against the twelve GRSCM attributes set and described.

Information derived from entropy-based measures ensures fairness in setting the criteria weights, while the Intuitionistic Fuzzy Yager Aggregation Operators combines the multidimensional evaluations into one sustainability score.

The final rankings provide support to both the public decision-makers and the corporate strategists by giving them a better understanding of which areas are the firm ones (e.g., technological innovation or reverse logistics) and where the weaknesses lie (e.g., collaboration or readiness), thus facilitating their work of coming up with improvement and policy-making strategies grounded on the evidence.

Policy evaluation and strategic sustainability planning
The IF-MADM framework proposed at the macro-policy level is a tool to help the regulatory decision-making process and the sustainability policy design. The decision alternatives can be checked against the attributes like Green Collaboration (K_3), Green HRM (K_{11}), and Sustainable Performance (K_{12}), for example if they are tax incentives, carbon regulations, or green certification schemes.

The entropy device accurately portrays the information variety among the policy alternatives, while the IF combining deals with the ambiguity that is typical of the stakeholders' views.

As such, these policy makers, managers of environmental boards, and corporate sustainability officers can, among other things, put in motion and rank the policy interventions on the basis of the expected green and socio-economic effects, which leads to an increase of transparency and accountability in ecologically conscious governance.

Managerial and strategic implications
The proposed IF-MADM model has practical value which can be briefly outlined as follows:

Holistic Evaluation: The system merges qualitative expert opinions with quantitative sustainability measures, all within a singular intuitionistic fuzzy framework.

Robust decision-support
The entropy Intuitionistic Fuzzy Yager aggregation methods mechanism provides resilience against uncertainty and weight perturbations.

Transparency and Reproducibility Helps to make the decisions taken visible to the public and also keeps a record which can be used for corporate sustainability reporting.

Strategic Adaptability Can be used for green decision-making at the operational, tactical, and policy level. Therefore, the IF-MADM model put forward not only gives the analytics more accuracy but it also brings the advances in fuzzy decision science theory down to the level of practical managerial tools. Thanks to its versatility, it may be used in tasks like supplier evaluation, production planning, technology adoption, benchmarking, and environmental policy formulation which means that it can be regarded as a decision framework ready for real-world application in sustainable industrial ecosystems.

Operational challenges and further developments
Despite the effectiveness of the proposed intuitionistic fuzzy MADM framework, certain limitations should be acknowledged. The current model assumes independence among criteria and considers a static decision environment based on expert-provided evaluations. In future studies, such shortcomings may be overcome by extending the proposed framework to accommodate interdependent criteria with network-based or Choquet-integral-based aggregation structures. Hybrid models incorporating more advanced decision-making models, such as grey systems models, rough set models, and machine learning models for weight calculation, are also expected to improve the flexibility and robustness of the proposed method.

Comparison of the proposed method with existing intuitionistic fuzzy MADM approaches
A comparative evaluation is conducted to assess the effectiveness of the proposed Yager-based intuitionistic fuzzy MADM framework in relation to existing methods reported in the literature. Classical intuitionistic fuzzy MADM approaches typically employ fixed aggregation operators and rely on subjective or predefined weighting schemes, which may limit their ability to capture heterogeneous decision attitudes and complex uncertainty structures[3–6,13,18]. Although such methods have been widely applied, their aggregation behaviour is often rigid and insufficiently adaptive to varying decision contexts.

Several studies have proposed advanced intuitionistic fuzzy aggregation operators, including geometric, induced, Einstein, Hamacher, Bonferroni, and Dombi-based operators, to enhance aggregation performance[5,13–15,23,24,26,61]. However, many of these approaches primarily focus on numerical ranking outcomes and provide limited discussion on axiomatic validation or explicit preservation of hesitation information[4,14,18]. In addition, criteria weighting in most existing frameworks is predominantly based on expert judgment, which may introduce subjectivity and affect decision reliability[3,6,26].

In contrast, the proposed framework introduces a parameterized aggregation mechanism based on Yager's t-norm and t-conorm within an intuitionistic fuzzy MADM setting, enabling greater flexibility in modelling diverse decision attitudes. Moreover, the use of entropy-based objective weighting reduces reliance on subjective judgments and enhances robustness in criteria importance determination. From a professional evaluation perspective encompassing uncertainty modelling capability, aggregation flexibility, weighting objectivity,

Professional Indicator	Existing IF-MADM Methods	Proposed Yager-based IF-MADM Framework
Uncertainty modelling capability	Partial representation of uncertainty; hesitation often weakly incorporated ([3,4,23])	Explicit modelling of membership, non-membership, and hesitation
Aggregation flexibility	Fixed or limited aggregation behaviour ([5,13,18])	Parameterized aggregation mechanism
Criteria weighting approach	Subjective or predefined weights ([3,6,26])	Objective entropy-based weighting
Theoretical soundness	Limited or partial axiomatic validation ([4,23,27])	Comprehensive axiomatic verification
Decision stability	Moderate sensitivity to aggregation operators ([23,24])	Stable and consistent ranking results
Practical applicability	Application-specific decision contexts ([21–23,26])	Suitable for sustainability-oriented MADM problems

Table 11. Comparison of the proposed method with existing intuitionistic fuzzy MADM approaches based on professional indicators.

theoretical soundness, decision stability, interpretability, and practical applicability, the proposed approach demonstrates improved overall performance. Its application to Green Supply Chain Management further confirms its ability to produce stable and interpretable rankings while effectively preserving intrinsic uncertainty and hesitation. The comparison analysis is shown in Table 11.

Conclusion

In Conclusion, the Yager-based intuitionistic fuzzy aggregation operators have shown to be very powerful in combining uncertain and vague information, especially in complicated decision scenarios. The key innovation of this study lies in the systematic integration of Yager's parameterized aggregation operators within an intuitionistic fuzzy MADM framework, supported by rigorous axiomatic validation and objective entropy-based weighting, thereby offering a flexible and reliable decision-support mechanism for sustainability driven applications. The use of Yager's t-norms and t-conorms in the proposed system gives a very resourceful and all-round way of aggregating the evaluations of the experts while still keeping the intrinsic hesitation of the intuitionistic fuzzy sets. Six new operators were introduced in this research along with their significant theoretical properties such as idempotency, monotonicity, and boundedness which were verified to ensure mathematical consistency. The operators that were developed were then used in a Multi-Attribute Decision-Making (MADM) model for the evaluation of companies implementing Green Supply Chain Management (GRSCM) based on twelve sustainability-related attributes. The operators IFYWAA and IFYWGA was able to produce consistent and stable rankings of alternatives, thus, confirming the robustness of the proposed approach. The findings emphasize that Yager-based intuitionistic fuzzy aggregation is a flexible framework for sustainability-oriented decision-making, which can handle ambiguity and provide easily understandable results in different practical contexts.

Received: 12 November 2025; Accepted: 27 January 2026
Published online: 13 February 2026

References

1. Zadeh, L. A. Fuzzy sets. *Inf. Control* **8**(3), 338–353 (1965).
2. Atanassov, K. T. & Stoeva, S. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20**(1), 87–96 (1986).
3. Pei, Z. & Zheng, L. A novel approach to multi-attribute decision making based on intuitionistic fuzzy sets. *Expert Syst. Appl.* **39**(3), 2560–2566 (2012).
4. Xu, Z. Intuitionistic fuzzy aggregation operators. *IEEE Trans. Fuzzy Syst.* **15**(6), 1179–1187 (2007).
5. Xu, Z. & Yager, R. R. Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int. J. Gen Syst* **35**(4), 417–433 (2006).
6. Wei, G. W. Some geometric aggregation functions and their application to dynamic multiple attribute decision making in the intuitionistic fuzzy setting. *Int J Uncertain, Fuzziness Knowl-Based Syst* **17**(02), 179–196 (2009).
7. Feng, L., Chuan-Qiang, F. & Wei-He, X. Type-2 hesitant fuzzy sets. *Fuzzy Inf Eng* **10**(2), 1–11 (2018).
8. Naim, S. & Hagras, H. A type 2-hesitation fuzzy logic based multi-criteria group decision making system for intelligent shared environments. *Soft. Comput.* **18**(7), 1305–1319 (2014).
9. Simić, V., Soušek, R. & Jovčić, S. Picture fuzzy MCDM approach for risk assessment of railway infrastructure. *Mathematics* **8**(12), 2259 (2020).
10. Özlü, Ş & Sezgin, A. Soft covered ideals in semigroups. *Acta Univ. Sapientiae Math.* **12**(2), 317–346 (2020).
11. Özlü, Ş. Multi-criteria decision making based on vector similarity measures of picture type-2 hesitant fuzzy sets. *Granul Comput* **8**(6), 1505–1531 (2023).
12. Alcantud, J. C. R., Khameneh, A. Z. & Kilicman, A. Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information. *Inf. Sci.* **514**, 106–117 (2020).
13. Wei, G. Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making. *Appl. Soft Comput.* **10**(2), 423–431 (2010).
14. Zhao, X. & Wei, G. Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making. *Knowl.-Based Syst.* **37**, 472–479 (2013).
15. Huang, J. Y. Intuitionistic fuzzy Hamacher aggregation operators and their application to multiple attribute decision making. *J Intell Fuzzy Syst* **27**(1), 505–513 (2014).
16. Deschrijver, G., Cornelis, C. & Kerre, E. E. On the representation of intuitionistic fuzzy t-norms and t-conorms. *IEEE Trans. Fuzzy Syst.* **12**(1), 45–61 (2004).
17. Deschrijver, G. & Kerre, E. E. A generalization of operators on intuitionistic fuzzy sets using triangular norms and conorms. *Notes on Intuit Fuzzy Sets* **8**(1), 19–27 (2002).
18. Li, D.-F. Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets. *Expert Syst. Appl.* **37**(12), 8673–8678 (2010).

19. Radhakrishnan, S. & Thankachan, B. Single-valued neutrosophic multiple sets and its application in multi-criteria decision-making. *Neutrosophic Sets Syst* **86**(1), 20 (2025).
20. Deli, I. & Şubaş, Y. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *Int. J. Mach. Learn. Cybern.* **8**(4), 1309–1322 (2017).
21. Karaaslan, F. & Hunu, F. Type-2 single-valued neutrosophic sets and their applications in multi-criteria group decision making based on TOPSIS method. *J. Ambient. Intell. Humaniz. Comput.* **11**(10), 4113–4132 (2020).
22. Özlü, Ş. Generalized Dice measures of single valued neutrosophic type-2 hesitant fuzzy sets and their application to multi-criteria decision-making problems. *Int. J. Mach. Learn. Cybern.* **14**(1), 33–62 (2023).
23. Liu, P., Liu, J. & Chen, S.-M. Some intuitionistic fuzzy Dombi Bonferroni mean operators and their application to multi-attribute group decision making. *J Op Res Soc* **69**(1), 1–24. https://doi.org/10.1057/s41274-017-0190-y (2018).
24. Kaur, G. & Garg, H. Multi-attribute decision-making based on Bonferroni mean operators under cubic intuitionistic fuzzy set environment. *Entropy* **20**(1), 65 (2018).
25. Muneeza, G., Abdullah, S., Qiyas, M. & Khan, M. A. Multi-criteria decision making based on intuitionistic cubic fuzzy numbers. *Granul Comput* **7**(1), 217–227 (2022).
26. Rani, D. & Garg, H. Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making. *Expert. Syst.* **35**(6), e12325 (2018).
27. Liu, X., Kim, H. S., Feng, F. & Alcantud, J. C. R. Centroid transformations of intuitionistic fuzzy values based on aggregation operators. *Mathematics* **6**(11), 215 (2018).
28. Shahzadi, G., Akram, M. & Al-Kenani, A. N. Decision-making approach under Pythagorean fuzzy Yager weighted operators. *Mathematics* **8**(1), 70 (2020).
29. Garg, H., Shahzadi, G. & Akram, M. Decision-making analysis based on Fermatean fuzzy Yager aggregation operators with application in COVID-19 testing facility. *Math. Probl. Eng.* **2020**, 1–16 (2020).
30. Khan, S. et al. Decision support technique based on neutrosophic Yager aggregation operators: Application in solar power plant locations—case study of Bahawalpur, Pakistan.. *Math Probl Eng* **202**(1), 6677676 (2020).
31. ÖZLÜ, Ş. Aczel alsina aggregation operators of (M, N, Q)-spherical hesitant fuzzy sets and their applications in multi-attribute decision-making. *Sigma: Journal of Engineering & Natural Sciences/Mühendislik ve Fen Bilimleri Dergisi*, *43*(6) (2025)
32. Liu, P., Shahzadi, G. & Akram, M. Specific types of q-rung picture fuzzy Yager aggregation operators for decision-making. *Int J Comput Intell Syst* **13**(1), 1072–1091 (2020).
33. Özlü, Ş & Aktaş, H. Correlation coefficient of r, s, t-spherical hesitant fuzzy sets and MCDM problems based on clustering algorithm and technique for order preference by similarity to ideal solution method. *Comput. Appl. Math.* **43**(8), 429 (2024).
34. Kamacı, H. & Petchimuthu, S. Navigating decision making with generalized temporal intuitionistic fuzzy sets and soft Sets. *Punjab Univ J Math* **56**(5), 148–174 (2024).
35. Özlü, Ş. New q-rung orthopair fuzzy Aczel-Alsina weighted geometric operators under group-based generalized parameters in multi-criteria decision-making problems. *Comput. Appl. Math.* **43**(3), 122 (2024).
36. Özlü, Ş. Bipolar-valued complex hesitant fuzzy Dombi aggregating operators based on multi-criteria decision-making problems. *Int. J. Fuzzy Syst.* **27**(1), 162–189 (2025).
37. Yager, R. R. Aggregation operators and fuzzy systems modeling. *Fuzzy Sets Syst.* **67**(2), 129–145 (1994).
38. Fortes, J. Green supply chain management: A literature. *Otago Manag Grad Revi* **7**(1), 51–62 (2009).
39. Fahimnia, B., Sarkis, J. & Davarzani, H. Green supply chain management: A review and bibliometric analysis. *Int. J. Prod. Econ.* **162**, 101–114 (2015).
40. Kumari, R. & Mishra, A. R. Multi-criteria COPRAS method based on parametric measures for intuitionistic fuzzy sets: Application of green supplier selection. *Iran J Sci Technol, Trans Electr Eng* **44**(4), 1645–1662 (2020).
41. Govindan, K., Khodaverdi, R. & Vafadarnikjoo, A. Intuitionistic fuzzy based DEMATEL method for developing green practices and performances in a green supply chain. *Expert Syst. Appl.* **42**, 7207–7220 (2015).
42. Rouyendegh, B. D., Yildizbasi, A. & Üstünyer, P. Intuitionistic fuzzy TOPSIS method for green supplier selection problem. *Soft. Comput.* **24**(3), 2215–2228 (2020).
43. Sarkis, J. A strategic decision framework for green supply chain management. *J. Clean. Prod.* **11**(4), 397–409 (2003).
44. Srivastava, S. K. Green supply-chain management: A state-of-the-art literature review. *Int. J. Manag. Rev.* **9**(1), 53–80 (2007).
45. Uygun, Ö. & Dede, A. Performance evaluation of green supply chain management using integrated fuzzy multi-criteria decision-making techniques. *Comput. Ind. Eng.* **102**, 502–511 (2016).
46. Jalil, F., Yang, J., Al-Okaily, M. & Rehman, S. U. E-commerce for a sustainable future: integrating trust, green supply chain management and online shopping satisfaction. *Asia Pacific J Market Logist.* **36**(10), 2354–2370 (2024).
47. Letunovska, N. et al. Green supply chain management: The effect of procurement sustainability on reverse logistics. *Logistics* **7**(3), 47 (2023).
48. Ahmad, A., Ikram, A., Rehan, M. F. & Ahmad, A. Going green: Impact of green supply chain management practices on sustainability performance. *Front. Psychol.* **13**, 973676 (2022).
49. Bhatia, M. S. & Gangwani, K. K. Green supply chain management: Scientometric review and analysis of empirical research. *J. Clean. Prod.* **284**, 124722 (2021).
50. Fahimnia, B., Sarkis, J. & Eshragh, A. A trade-off model for green supply chain planning: A leanness-versus-greenness analysis. *Omega* **54**, 173–190 (2015).
51. Lee, V. H., Ooi, K. B., Chong, A. Y. L. & Seow, C. Creating technological innovation via green supply chain management: An empirical analysis. *Expert Syst. Appl.* **41**(16), 6983–6994 (2014).
52. Mathiyazhagan, K., Govindan, K., NoorulHaq, A. & Geng, Y. An ISM approach for the barrier analysis in implementing green supply chain management. *J. Clean. Prod.* **47**, 283–297 (2013).
53. Iddik, S. The role of cultural factors in green supply chain management practices: A conceptual framework and an empirical investigation. *RAUSP Manag J.* **59**(2), 96–122 (2024).
54. Abdallah, A. B., Al-Ghwayeen, W. S., Al-Amayreh, E. A. M. & Sweis, R. J. The impact of green supply chain management on circular economy performance: The mediating roles of green innovations. *Logistics* **8**(1), 20 (2024).
55. Xing, Y. & Liu, Y. Integrating product-service innovation into green supply chain management from a life cycle perspective: A systematic review and future research directions. *Technovation* **126**, 102825 (2023).
56. Saini, N., Malik, K. & Sharma, S. Transformation of supply chain management to green supply chain management: Certain investigations for research and applications. *Clean Mater* **7**, 100172 (2023).
57. Le, T. T. The association of corporate social responsibility and sustainable consumption and production patterns: The mediating role of green supply chain management. *J. Clean. Prod.* **414**, 137435 (2023).
58. Roh, T., Noh, J., Oh, Y. & Park, K. S. Structural relationships of a firm's green strategies for environmental performance: The roles of green supply chain management and green marketing innovation. *J. Clean. Prod.* **356**, 131877 (2022).
59. Selepe, R. L. & Makinde, O. A. Analysis of factors and solutions to poor supply chain quality in a manufacturing organisation. *J Trans Supply Chain Manag* **18**, 989 (2024).
60. Yu, B., Zhao, X., Zheng, M., Yuan, X. & Hou, B. Entropy on intuitionistic fuzzy sets and hesitant fuzzy sets. *J Math* **2022**(1), 1585079 (2022).
61. Seikh, M. R. & Mandal, U. Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making. *Granul Comput* **6**, 473–488 (2021).

Author contributions
All authors contributed to the study conception and design. Conceptualization, Formal Analysis, Investigation, Data Curation, Writing-Original Draft were performed by YK and SR; Resources and Supervision, SR. Methodology, YK, SR and GBZ. Writing-Review &Editing, Visualization, SR and GBZ. All authors have read and agreed to the published version of the manuscript.

Declarations

Competing interests
The authors declare no conflicts of interest.

Ethical approval
This study did not involve human or animal subjects.

Consent for publication
All the authors agree with this manuscript's publication.

Additional information
Correspondence and requests for materials should be addressed to S.R. or G.B.Z.