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# 1 Model Predictive Control with Adaptive Kalman Filter 2 for Premixed Turbocharged Natural Gas Engine

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## 10 ABSTRACT

11 Robust control of natural-gas engines under unknown load disturbances remains challenging due to strong couplings and de-  
lays in multi-input multi-output (MIMO) dynamics. This paper presents a control framework that integrates rate-based model  
predictive control (MPC) with a gain-scheduling scheme driven by an adaptive Kalman filter to enhance performance under  
unknown load disturbances. A novel adaptation mechanism enables the Kalman filter to rapidly track transient changes in  
load torque while attenuating steady-state estimation noise. The online torque estimate is used to compute local equilibrium  
operating points and generate a gain-scheduling parameter matrix that adaptively adjusts MPC behavior to improve transient  
response. Experimental validation on a laboratory engine demonstrates that the estimator converges quickly during load  
transients and maintains low steady-state noise; when combined with gain-scheduled MPC, the proposed controller signifi-  
cantly reduces speed and air-fuel-ratio deviations and shortens settling time following step load changes. The results indicate  
improved disturbance rejection and practical applicability for power-generation engines.

## 12 Introduction

13 Due to increasingly stringent emission regulations, natural-gas engines—featuring lower carbon intensity and reduced pollutant  
14 emissions—are becoming attractive alternatives to conventional gasoline and diesel engines in industrial and power-generation  
15 applications<sup>1–3</sup>. This paper focuses on natural-gas engines coupled to synchronous generators for power generation, where en-  
16 gine speed directly determines the generated frequency and voltage. When used as distributed or stand-alone generation units,  
17 these engines must reject unknown load disturbances so as to maintain the target speed and to minimize speed deviation and  
18 recovery time; concurrently, the air-fuel ratio (AFR) must be kept close to its reference throughout operation to satisfy emis-  
19 sion limits. Good disturbance-rejection capability therefore shortens the time to restore frequency and voltage to acceptable  
20 levels and ensures reliable power supply. This work seeks a practical control solution that meets these requirements.

21 Heavy-duty natural-gas engines employed in generator sets typically have many cylinders and large displacement. Com-  
22 pared with electronic fuel-injection (EFI) strategies<sup>4</sup>, premixed intake (pre-mixer) configurations<sup>5</sup> are often preferred in  
23 such engines for lower cost and more uniform mixture formation; however, premixing introduces strong actuator-output  
24 couplings, producing a challenging multi-input multi-output (MIMO) control problem<sup>6</sup>. MPC is a promising approach for  
25 such strongly coupled MIMO systems because it coordinates multiple actuators through an explicit cost function. Prior work  
26 has applied MPC to various engine subsystems, including boost-pressure and exhaust gas recirculation (EGR) control for  
27 two-stage turbocharged engines<sup>7</sup>, nonlinear MPC for turbocharged SI engines with dual-loop EGR<sup>8</sup>, heavy-duty diesel con-  
28 trol<sup>9</sup>, and engine-speed regulation<sup>10,11</sup>, demonstrating MPC's advantages in handling multivariable interactions. Engine  
29 dynamics are inherently nonlinear. While nonlinear MPC is conceptually attractive, practical deployment is challenged by  
30 the computational complexity of solving nonlinear optimal control problems on embedded hardware, which limits achiev-  
31 able prediction horizons<sup>6,12</sup>. A common alternative linearizes the nonlinear model at an operating point and employs linear  
32 parameter-varying or gain-scheduling MPC (LPV-MPC)<sup>13,14</sup>. To guarantee zero steady-state error under disturbances, lin-  
33 ear MPC variants have used integral state augmentation<sup>11</sup>, disturbance estimators that compensate model-plant mismatch<sup>15</sup>,  
34 or rate-based MPC formulations<sup>16</sup>. However, integrator augmentation risks windup and complicates constraint handling in  
35 MIMO settings; disturbance-state augmentation increases model order and implementation complexity. Rate-based MPC of-  
36 fers a practical compromise, but for the large and rapidly varying load disturbances considered here, rate-based MPC alone  
37 does not deliver the required transient performance. To address this, we combine rate-based MPC with a simple independent

38 load estimator and a gain-scheduling strategy.

39 The load estimator is intentionally kept simple and aims solely to observe the external load torque; its output, together  
40 with the tracking references, drives the gain-scheduling mechanism. In practice, however, simple observers are sensitive to  
41 measurement noise and can inject irregular perturbations into throttle commands, degrading tracking performance. Thus a  
42 Kalman filter is introduced. While the extended Kalman filter (EKF) is commonly used<sup>17</sup>, its reliance on local linearization  
43 can incur significant linearization error and poor robustness to model uncertainty. The unscented Kalman filter (UKF) im-  
44 proves linearization accuracy via the unscented transform<sup>18</sup> but incurs high computational cost that limits its applicability  
45 on embedded platforms. Hybrid Kalman filters (HKF)<sup>19,20</sup>, which retain key nonlinear terms, strike a balance by reducing  
46 linearization error with modest complexity. Motivated by HKF, this paper develops a compact second-order linear Kalman  
47 filter tailored for load-torque estimation and decoupled from the original fourth-order engine model.

48 A fundamental trade-off for Kalman filters exists between noise suppression and tracking agility: filters tuned for strong  
49 noise rejection tend to track slowly. To overcome this, prior works have used adaptive tuning and strong-tracking modifications  
50 that adapt the estimator covariance according to an adaptive law<sup>21–23</sup>. Building on these ideas, we propose a load-detection-  
51 based adaptive strategy that rapidly increases estimator responsiveness during transients while preserving noise attenuation in  
52 steady state. The resulting adaptive Kalman filter provides fast, robust load-torque estimates that are suitable for online use in  
53 gain scheduling. The main contributions of this paper are:

54 (1) A novel adaptive Kalman filter is presented with a mechanism for rapid tracking of transient load changes and attenua-  
55 tion of steady-state noise.

56 (2) A hybrid control framework is developed that integrates predictive control of the rate-based model with a gain schedul-  
57 ing strategy driven by an adaptive Kalman filter, allowing enhanced rejection of unknown load disturbances in natural-gas  
58 generator sets.

59 (3) Experimental implementation and validation are presented, demonstrating that the proposed adaptive MPC scheme  
60 effectively reduces engine speed and AFR fluctuations and shortens settling time under varying load conditions, confirming  
61 its superior transient performance and suitability for embedded implementation.

## 62 Engine Model

### 63 Engine Physical Model

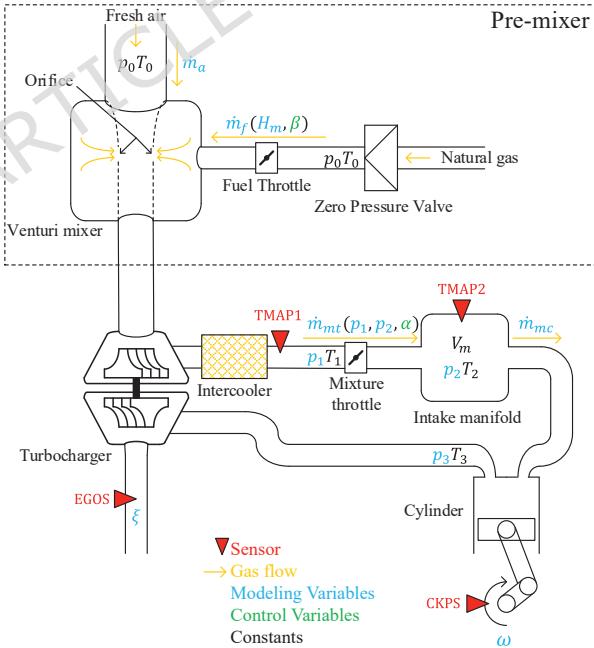


Figure 1. Pre-mixed turbocharged CNG engine.

64 The structure of this engine is schematically shown in Figure 1. The air path consists of two main sections: the pre-mixer  
65 and the turbocharged engine section. The pre-mixer primarily includes a Venturi mixer and a fuel throttle. A zero-pressure

66 valve is installed upstream of the fuel throttle to maintain inlet pressure at atmospheric level. Natural gas is drawn into the  
 67 Venturi mixer through orifices via the Venturi effect, with the fuel throttle opening  $\beta$  regulating the natural gas mass flow  
 68 rate. The second section resembles a conventional turbocharged engine, where the mixture throttle opening  $\alpha$  controls the  
 69 mass flow of the mixed gas. Two temperature and manifold absolute pressure sensors (TMAP1 and TMAP2) are employed to  
 70 monitor the temperature and pressure of the boosted mixture downstream of the compressor and the gas in the intake manifold,  
 71 respectively. A crankshaft position sensor (CKPS) measures the crankshaft phase and rotational speed. An exhaust gas oxygen  
 72 sensor (EGOS) is mounted on the exhaust pipe to determine the AFR.

73 The engine model employed in this study is adopted from the authors' prior work<sup>6</sup> and is presented below:

$$\begin{cases} \dot{\omega} = \frac{1}{J} [\tau_e - k_b \omega - \tau_p - \tau_L] \\ \dot{\xi} = H_m \frac{\dot{m}_f}{\dot{m}_a} \frac{1}{\xi + 1} - \frac{\xi}{\xi + 1} \\ \dot{p}_2 = \frac{R_m T_2}{V_m} (\dot{m}_{mt} - \dot{m}_{mc}) \\ \dot{p}_1 = \frac{R_m T_1}{V_t} (\dot{m}_c - \dot{m}_{mt}) \end{cases} \quad (1)$$

74 where

$$\tau_e = \eta_e \frac{\xi(t - t_d)}{\xi(t - t_d) + 1} \dot{m}_{mc} \quad (2)$$

$$t_d = \frac{p_1 V_T}{R_m T_1} \cdot \frac{1}{\dot{m}_{mc}} = \frac{p_1 V_T T_2}{\omega V_d \eta_{ch} p_2 T_1} \quad (3)$$

$$\dot{m}_{mc} = \frac{\omega V_d \eta_{ch} p_2}{4\pi R_m T_2} \quad (4)$$

$$H_m \frac{\dot{m}_f}{\dot{m}_a} = H_m \frac{C_f S_{thvm}}{C_a S_a} \sqrt{\frac{R_a}{R_f}} \left[ 1 - \cos \left( \frac{\beta \pi}{180} \right) \right] \quad (5)$$

$$\dot{m}_{mt} = C_m S_m \left[ 1 - \cos \left( \frac{\alpha \pi}{180} \right) \right] \frac{p_1}{\sqrt{R_m T_1}} \Psi \left( \frac{p_2}{p_1} \right) \quad (6)$$

$$\Psi \left( \frac{p_2}{p_1} \right) = \begin{cases} \kappa^{\frac{1}{2}} \left( \frac{2}{\kappa+1} \right)^{\frac{\kappa+1}{2(\kappa-1)}} & \frac{p_2}{p_1} \leq \kappa_c \\ \sqrt{\frac{2\kappa}{\kappa-1} \left[ \left( \frac{p_2}{p_1} \right)^{\frac{2}{\kappa}} - \left( \frac{p_2}{p_1} \right)^{\frac{\kappa+1}{\kappa}} \right]} & \frac{p_2}{p_1} > \kappa_c \end{cases} \quad (7)$$

$$H_m = \begin{cases} a_1 \dot{m}_{mt}^2 + a_2 \dot{m}_{mt} + a_3 & \dot{m}_{mt} \leq m_{hc} \\ 1 & \dot{m}_{mt} > m_{hc} \end{cases} \quad (8)$$

75 This is a fourth-order nonlinear model with state variables of engine speed  $\omega$ (rad/s), the fuel-air ratio  $\xi$  in the Venturi mixer,  
 76 intake manifold pressure  $p_2$ (Pa), and boost pressure  $p_1$ (Pa). The state variable is chosen as the fuel-air ratio  $\xi$  rather than the  
 77 AFR to avoid the fuel throttle opening  $\beta$  appearing in the denominator, which would introduce additional nonlinearities into  
 78 the model dynamics. The first equation in Eq.1 describes the crankshaft rotation dynamics. Here,  $J$  is the rotational inertia,  $k_b$   
 79 is the friction coefficient,  $\tau_p$  is the mechanical and pumping loss torque, and  $\tau_L$  is the unknown load torque disturbance. The  
 80 indicated torque  $\tau_e$  is defined by Eq.2, where  $\eta_e$  is the engine efficiency coefficient (obtained from look-up tables as a function  
 81 of speed, AFR, and ignition angle). A significant AFR transport delay  $t_d$ , given by Eq.3, is incorporated into  $\tau_e$  because the  
 82 mixture ratio takes time to reach the cylinders. In Eq.3,  $V_T$  is the pipe volume from the mixer to the cylinders,  $T_1$  is the boost  
 83 temperature, and  $R_m$  is the gas constant of the mixed gas. The mass flow rate of the mixture into the cylinders  $\dot{m}_{mc}$ , is given  
 84 by Eq.4, where  $V_d$  is the engine displacement,  $T_2$  is the intake manifold temperature, and  $\eta_{ch}$  is the volumetric efficiency  
 85 (obtained from look-up tables).

86 The second equation in Eq.1 governs the natural gas and air mixing dynamics within the Venturi mixer. The term  $H_m \frac{\dot{m}_f}{\dot{m}_a}$   
 87 is defined by Eq.5. Here,  $H_m$  is an empirical correction factor. Its primary role is to compensate for the deviation between  
 88 the theoretical fuel-air flow ratio  $\frac{\dot{m}_f}{\dot{m}_a}$  and the actual value, which arises from model simplifications, sensor inaccuracies, and  
 89 actual flow characteristics. Introducing this coefficient can significantly improve the model's predictive accuracy regarding  
 90 the actual system behavior. The value of  $H_m$  is a function of the mixture throttle mass flow rate  $\dot{m}_{mt}$ . It is determined by  
 91 identifying parameters from experimental data, and its specific expression is given by the piecewise function shown in Eq.8.

92 The third and fourth equations in Eq.1 describe the gas filling dynamics for the intake manifold pressure and boost pressure,  
 93 respectively. The mass flow rate through the mixture throttle,  $\dot{m}_{mt}$ , is given by Eq.6, where  $C_m$  and  $S_m$  are constants, and the  
 94 flow function  $\Psi(p_2/p_1)$  defined by Eq.7. The compressor mass flow rate is denoted by  $\dot{m}_c$ .

95 As evident from the equations, this fourth-order model represents a dual-input, dual-output system. The control inputs are  
 96 the mixture throttle opening  $\alpha$  and the fuel throttle opening  $\beta$ , and the outputs are engine speed  $\omega$  and fuel-air ratio  $\xi$ . All  
 97 four state variables are measurable. A key characteristic is the strong coupling between inputs and outputs, as both  $\omega$  and  $\xi$   
 98 are simultaneously influenced by  $\alpha$  and  $\beta$ .

### 99 Model Linearization and Discretization

100 The nonlinear engine model is linearized around various equilibrium points to facilitate controller design. Defining the state  
 101 vector  $x = [\omega \ \xi \ p_2 \ p_1]^T$ , control inputs as  $u = [\alpha \ \beta]^T$ , disturbance as  $d = \tau_L$ , and outputs as  $y = [\omega \ \xi]^T$ , the nonlinear  
 102 system described by Eq.1 can be expressed in compact form as:

$$\dot{x} = f(x, u, d) \quad (9)$$

103 Consider an equilibrium point defined by  $x_{ss} = [\omega_{ss} \ \xi_{ss} \ p_{2ss} \ p_{1ss}]^T$  and steady-state inputs  $u_{ss} = [\alpha_{ss} \ \beta_{ss}]^T$ , which  
 104 satisfy the equilibrium condition  $0 = f(x_{ss}, u_{ss}, 0)$ . By introducing deviation variables  $\delta x = x - x_{ss}$ ,  $\delta u = u - u_{ss}$ , and applying  
 105 first-order Taylor series expansion, the system is linearized to obtain the linear time-invariant representation:

$$\begin{aligned} \delta \dot{x} &= A_c \delta x + B_{1c} \delta u + B_{2c} d \\ \delta y &= C_c \delta x \end{aligned} \quad (10)$$

106 where  $A_c$ ,  $B_{1c}$  and  $B_{2c}$  represent the Jacobian matrices of  $f$  with respect to  $x_{ss}$ ,  $u_{ss}$  and  $d$ , respectively, evaluated at the  
 107 equilibrium point  $(x_{ss}, u_{ss}, 0)$ .

108 The continuous-time linear model in Eq.10 is then discretized using a sampling period of  $t_{s1} = 50\text{ms}$ , yielding the discrete-  
 109 time state-space representation:

$$\begin{aligned} \delta x(k+1) &= A_d \delta x(k) + B_{1d} \delta u(k) + B_{2d} d(k) \\ \delta y(k) &= C_d \delta x(k) \end{aligned} \quad (11)$$

110 where the discrete-time matrices are computed as:

$$A_d = e^{A_c t_{s1}}; B_{id} = \left( \int_0^{t_{s1}} e^{A_c t} dt \right) B_{ic}, (i = 1, 2); C_d = C_c$$

### 111 Time Delay Prediction

112 In the engine model described by Eq.1, the second equation characterizes the mixing dynamics at the Venturi mixer. However,  
 113 the mixed gas must traverse the turbocharger, intercooler, and intake manifold before reaching the cylinders: a process that  
 114 introduces a significant transport delay. This delay, denoted  $t_d$ , must be accounted for in the fuel-air ratio variable  $\xi$ . As given  
 115 by Eq.3,  $t_d$  varies with engine operating conditions, specifically speed  $\omega$  and intake manifold pressure  $p_2$ . Let  $t_d = n_d t_{s1}$ ,  
 116 where  $t_{s1} = 50\text{ms}$  is the sampling period. The fuel-air ratio measured at the Venturi mixer only be observed at the EGOS after  
 117  $n_d$  sampling intervals. To predict the current state affected by this delay, past state variables are utilized. Define a delayed  
 118 state vector  $\delta x'(k - n_d) = [\delta \omega(k - n_d) \ \delta \xi(k) \ \delta p_2(k - n_d) \ \delta p_1(k - n_d)]^T$ , where  $\xi(k)$  is the fuel-air ratio measured at  
 119 the EGOS at time  $k$ , which corresponds to the ratio at the Venturi mixer at time  $k - n_d$ . Using the discrete-time model Eq.11,  
 120 the current state can be iteratively predicted as:

$$\delta x'(k) = A_d^{n_d} \delta x'(k - n_d) + \sum_{i=1}^{n_d} A_d^{n_d-i} B_{1d} \delta u(k - (n_d - i + 1)) + \sum_{i=1}^{n_d} A_d^{n_d-i} B_{2d} d(k - (n_d - i + 1)) \quad (12)$$

121 The second element of  $\delta x'(k)$ , denoted  $\delta \xi'(k)$ , represents the predicted fuel-air ratio at the Venturi mixer at the current time  
 122  $k$ . By replacing all instances of  $\xi(t - t_d)$  in Eq.1 with  $\xi'(k)$ , a new state vector  $x_p(k) = [\delta \omega(k) \ \delta \xi'(k) \ \delta p_2(k) \ \delta p_1(k)]^T$   
 123 is obtained, which is used for subsequent controller design.

124 A common alternative for handling delays in MPC is to incorporate the delay directly into the model, forming an aug-  
 125 mented system<sup>11</sup>. However, this approach increases model dimension proportionally to the number of delay steps. Given the  
 126 large and variable delay in our application, the augmentation method becomes impractical. Instead, the proposed prediction-  
 127 based iteration offers a more efficient solution, wherein variations in  $t_d$  only affect the number of prediction steps  $n_d$ , without  
 128 altering the controller structure.

129 **MPC Controller Design**130 **Linear MPC Design**

131 The reference signal is defined as  $r(k) = [\omega_r(k) \ \xi_r(k)]^T$ , which is treated as constant over the prediction horizon. Conventional MPC for reference tracking typically minimizes a quadratic cost function based on the difference between predicted outputs and target values. While this approach using Eq.11 can theoretically achieve zero steady-state error when there is perfect model-plant match, such ideal conditions are rarely attainable in practice. To address this limitation, this paper employs a rate-based MPC formulation with a difference-scheme augmented model. Define the incremental variables:  $\Delta x_p(k) = \delta x_p(k) - \delta x_p(k-1)$ ,  $\Delta u(k) = \delta u(k) - \delta u(k-1)$ ,  $\Delta d(k) = d(k) - d(k-1)$ , and  $\Delta y(k) = \delta y(k) - \delta y(k-1)$ . The discrete-time model from Eq.11 can be reformulated as:

$$\begin{aligned} \Delta x_p(k+1) &= A_d \Delta x_p(k) + B_{1d} \Delta u(k) + B_{2d} \Delta d(k) \\ \Delta y(k) &= C_d \Delta x(k) \end{aligned} \quad (13)$$

138 A new state vector is constructed as  $x_m(k) = [\Delta x_p(k)^T \ \Delta y(k)^T]^T$ , yielding the augmented state-space model:

$$\begin{aligned} \underbrace{\begin{bmatrix} \Delta x_p(k+1) \\ \delta y(k+1) \end{bmatrix}}_{x_m(k+1)} &= \underbrace{\begin{bmatrix} A_d & 0 \\ C_d A_d & I \end{bmatrix}}_A \underbrace{\begin{bmatrix} \Delta x_p(k) \\ \delta y(k) \end{bmatrix}}_{x_m(k)} + \underbrace{\begin{bmatrix} B_{1d} \\ C_d B_{1d} \end{bmatrix}}_{B_1} \Delta u(k) + \underbrace{\begin{bmatrix} B_{2d} \\ C_d B_{2d} \end{bmatrix}}_{B_2} \Delta d(k) \\ \underbrace{\begin{bmatrix} \delta y(k) \end{bmatrix}}_{y_m(k)} &= \underbrace{\begin{bmatrix} 0 & I \end{bmatrix}}_C \underbrace{\begin{bmatrix} \Delta x_p(k) \\ \delta y(k) \end{bmatrix}}_{x_m(k)} \end{aligned} \quad (14)$$

139 This augmented formulation offers two significant advantages. First, by selecting the equilibrium point to coincide with 140 the reference signal (i.e.,  $y_{ss} = r(k)$ ), the control objective simplifies to driving the output of Eq.14 to zero at steady state. 141 This eliminates the need for explicit reference tracking terms in the cost function. Second, the incremental state formulation 142 inherently ensures zero steady-state tracking error without requiring additional integrator states<sup>24</sup>.

143 The prediction model is constructed as follows. Since the disturbance  $d(k)$  is unknown and assumed constant over the 144 prediction horizon, the term  $B_2 \Delta d(k)$  is omitted from predictions:

$$Y = F x_m(k) + \Phi \Delta U \quad (15)$$

145 where

$$\begin{aligned} Y &= [y_m(k+1|k)^T \ y_m(k+2|k)^T \ \cdots \ y_m(k+H_p|k)^T]^T \\ \Delta U &= [\Delta u(k)^T \ \Delta u(k+1)^T \ \cdots \ \Delta u(k+H_c-1)^T]^T \\ F &= [(CA)^T \ (CA^2)^T \ \cdots \ (CA^{H_p})^T]^T \\ \Phi &= \begin{bmatrix} CB_1 & 0 & \cdots & 0 \\ CAB_1 & CB_1 & \cdots & 0 \\ \vdots & & & \\ CA^{H_p-1} B_1 & CA^{H_p-2} B_1 & \cdots & CA^{H_p-H_c} B_1 \end{bmatrix} \end{aligned}$$

146 Here,  $H_p$  and  $H_c$  denote the prediction and control horizons, respectively. The cost function weights the output and control 147 increments:

$$\min \quad J = Y^T Q Y + \Delta U^T R \Delta U \quad (16)$$

148 where  $Q$  and  $R$  diagonal weighting matrices. The optimization variable  $\Delta U$  contains the sequence of control increments, with 149 the actual control input computed as:

$$u(k) = u_{ss} + \delta u(k) = u_{ss} + \sum_{i=0}^k \Delta u(i) \quad (17)$$

150 where  $\Delta u(k)$  represents the first element of the optimal  $\Delta U$  sequence. Practical constraints are implemented as follows. Rate 151 constraints prevent excessive throttle movements that could destabilize the Venturi mixer flow or cause turbocharger surge:

$$\begin{aligned} \Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max} \\ u_{min} - u(k-1) \leq \Delta u(k) \leq u_{max} - u(k-1) \end{aligned} \quad (18)$$

152 These constraints apply to all elements in the  $\Delta U$  sequence over the control horizon.

153 **Optimization Problem Solution**

154 The optimization problem defined by Eq. 16 is commonly solved by converting it into a standard quadratic programming (QP)  
 155 formulation, for which efficient and numerically reliable solvers are available. Substituting the prediction model from Eq. 15  
 156 into the cost function, and treating the state vector  $x_m(k)$  as known over the prediction horizon at each control instant, the  
 157 constrained optimization problem can be rewritten as:

$$\min_{\Delta U} J_{QP} = \frac{1}{2} [\Delta U^T (2\Phi^T Q \Phi + 2R) \Delta U] + 2\Delta U^T \Phi^T Q F x_m(k) + x_m^T(k) F^T Q F x_m(k) \text{ s.t. } Z \Delta U \leq W \quad (19)$$

158 This constitutes a typical QP problem. The constraint matrices  $Z$  and  $W$  are readily constructed from the rate and amplitude  
 159 limits given in Eq. 18. Since the weighting matrices  $Q$  and  $R$  are diagonal with positive entries, the Hessian matrix  $2(\Phi^T Q \Phi +$   
 160  $R)$  is symmetric positive definite. Therefore, the optimization problem in Eq. 19 is strictly convex and can be solved efficiently  
 161 online using active-set methods<sup>25</sup>. The solution yields the optimal sequence  $\Delta U$ , from which only the first element  $\Delta u(k)$  is  
 162 applied to the plant via Eq. 17; the remaining elements are discarded in accordance with the receding horizon principle.

163 A key observation is that the QP formulation depends solely on  $x_m(k)$ ,  $F$ , and  $\Phi$ . Since  $x_m(k)$  is measurable or computable  
 164 online, and both  $F$  and  $\Phi$  are functions only of the linearized engine model at a given equilibrium point, these matrices can  
 165 be precomputed offline for a set of operating conditions and stored in look-up tables. As a result, the online computational  
 166 burden of the MPC reduces essentially to solving a QP problem at each sampling instant, making the strategy suitable for  
 167 embedded implementation.

168 **Adaptive Kalman Filter and Gain Scheduling Strategy**

169 While the previous section completed the design of an MPC controller for a single equilibrium point, and prior work<sup>16</sup> has  
 170 demonstrated that rate-based MPC can compensate for certain model-plant mismatches, treating the load torque purely as a  
 171 model mismatch reveals limitations. Although the controller can eventually regulate the system back to the target operating  
 172 point, its inherent compensation capability is insufficient to meet the stringent requirements for speed and AFR fluctuation  
 173 ranges in power generation applications. Therefore, an alternative method to enhance load response performance is necessary.

174 A conventional approach involves treating the load torque as an additional state variable, constructing a fifth-order state-  
 175 space model and observer. However, this method significantly increases model complexity. Re-examining the control design  
 176 context, the natural gas engine studied herein operates primarily at a single nominal working point: 1500 rpm engine speed  
 177 with a stoichiometric AFR. Although this suggests a single equilibrium point, the engine's internal state actually shifts under  
 178 load variations. While the controlled outputs  $\omega$  and  $\zeta$  return to their references in steady state following a load change, the  
 179 pressures  $p_2$  and  $p_1$  settle at new values. This indicates that the equilibrium point itself moves with the load.

180 Consequently, the linearized model corresponding to the new operating point must be updated, and the MPC controller  
 181 should be adjusted accordingly, affecting the incremental control signals  $\Delta u$  and  $\delta u$ . Furthermore, referring to Eq. 17, the shift  
 182 in the equilibrium point also changes the steady-state input  $u_{ss}$ . This implies that during each control interval, in addition to  
 183 the corrective action  $\delta u(k)$  computed by the MPC, an additional feedforward compensation  $\Delta u_{ss}$  can be applied based on the  
 184 updated equilibrium point. This adjustment, which actively accounts for the changing operating condition, is referred to as the  
 185 gain scheduling strategy in this paper. The subsequent challenge, therefore, is to accurately estimate the load torque in order  
 186 to detect and respond to these equilibrium point movements.

187 **Simple Form of Load Estimator Based on Kalman Filter**

188 In practical applications, engine systems are inevitably subject to measurement noise, which can degrade throttle control  
 189 accuracy, induce throttle jitter, and reduce actuator longevity. To address this issue, a Kalman filter is employed as a load  
 190 torque estimator.

191 Reexamining the original engine model in Eq. 1, the load torque  $\tau_L$  appears only in the speed dynamics. Since all four state  
 192 variables are measurable and the remaining three differential equations are independent of  $\tau_L$ , only the speed dynamics equation  
 193 is utilized for estimator design. By treating  $\tau_L$  as an additional state variable and incorporating process and measurement  
 194 noise, a second-order system is formulated as follows:

$$\begin{aligned} \dot{\omega} &= \frac{1}{J} [\tau_e - k_b \omega - \tau_p - \tau_L + v_t] \\ \dot{\tau}_L &= v_{tl} \end{aligned} \quad (20)$$

195 where  $v_t$  and  $v_{tl}$  represent mutually independent, zero-mean Gaussian noise terms associated with the reconstructed engine  
 196 torque and the load torque derivative, respectively<sup>26</sup>. By defining  $u_e = \tau_e - \tau_p$  as an input, the nonlinearities in  $\tau_e$  are avoided,  
 197 resulting in a simple second-order linear model. This formulation is physically justified since the net torque (engine output  
 198 minus load) determines the rotational acceleration.

199 Discretizing the system with a sampling period  $t_{s2}$  yields the discrete-time state-space model:

$$\underbrace{\begin{bmatrix} \omega(k+1) \\ \tau_L(k+1) \end{bmatrix}}_{x_e(k+1)} = \underbrace{\begin{bmatrix} 1 - \frac{t_{s2} k_p}{J} & -\frac{t_{s2}}{J} \\ 0 & 1 \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} \omega(k) \\ \tau_L(k) \end{bmatrix}}_{x_e(k)} + \underbrace{\begin{bmatrix} \frac{t_{s2}}{J} \\ 0 \end{bmatrix}}_{B_e} \underbrace{(\tau_e(k) - \tau_p)}_{u_e} + \underbrace{\begin{bmatrix} \frac{t_{s2}}{J} & 0 \\ 0 & t_{s2} \end{bmatrix}}_L \underbrace{\begin{bmatrix} v_t(k) \\ v_{tl}(k) \end{bmatrix}}_{\Upsilon(k)}$$

$$y_e(k) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_H \underbrace{\begin{bmatrix} \omega(k) \\ \tau_L(k) \end{bmatrix}}_{x_e(k)} + \varepsilon \quad (21)$$

200 where  $\varepsilon$  is the zero-mean speed measurement noise. Since this model is independent of the controller, and this paper has high  
201 requirements for the fast tracking ability of the estimator, the control period of this model can be shorter than the controller,  
202 and it is set to be  $t_{s2} = 10\text{ms}$ . Then the linear Kalman filter can be given by<sup>27</sup>:

$$\begin{aligned} \hat{x}_e^-(k) &= A_e \hat{x}_e^+(k-1) + B_e u_e \\ P^-(k) &= A_e P^+(k-1) A_e^T + M \\ K(k) &= P^-(k) H^T (H P^-(k) H^T + N)^{-1} \\ \hat{\varepsilon}(k) &= \omega(k) - H \hat{x}_e^-(k) \\ \hat{x}_e^+(k) &= \hat{x}_e^-(k) + K(k) \varepsilon(k) \\ P^+(k) &= (I - K(k) H) P^-(k) \end{aligned} \quad (22)$$

203 where

204  $K(k)$ : Kalman filter gain matrix

205  $\hat{x}_e^-(k), \hat{x}_e^+(k)$ : a priori and a posteriori state vector

206  $P^-(k), P^+(k)$ : a priori and a posteriori state estimation error covariance matrices

207  $M$ : Covariance matrix of  $\Upsilon$

208  $N$ : Variance of  $\varepsilon$

209  $\hat{\varepsilon}(k)$ : a priori prediction error

210 Since  $u_e = \tau_e - \tau_p$  is readily available at each control instant, and the output matrix  $H$  is scalar-valued, the matrix inversion  
211 in the gain update reduces to a scalar division. This structure allows for efficient embedded implementation without requiring  
212 the full nonlinear engine model.

### 213 Adaptation Mechanism of Kalman Filter

214 When employing a Kalman filter as an estimator, a well-known trade-off exists between its noise suppression capability and  
215 fast tracking performance, it is difficult to achieve both optimally simultaneously. However, in the context of this study, the  
216 estimator is required to possess a fast convergence speed to promptly detect load variations and trigger equilibrium point  
217 switching, while also maintaining excellent noise attenuation in steady state to ensure accurate and stable throttle opening  
218 tracking. To address these competing requirements, this paper introduces an adaptive tuning strategy.

219 In Eq.22, since  $A_e, B_e$  and  $H$  are constants, the covariance matrix  $M$  and the noise variance  $N$  become the primary tuning  
220 parameters of the Kalman filter. These are defined as:

$$N = [n_\omega] \quad ; \quad M = \begin{bmatrix} m_t & 0 \\ 0 & m_{tl} \end{bmatrix}$$

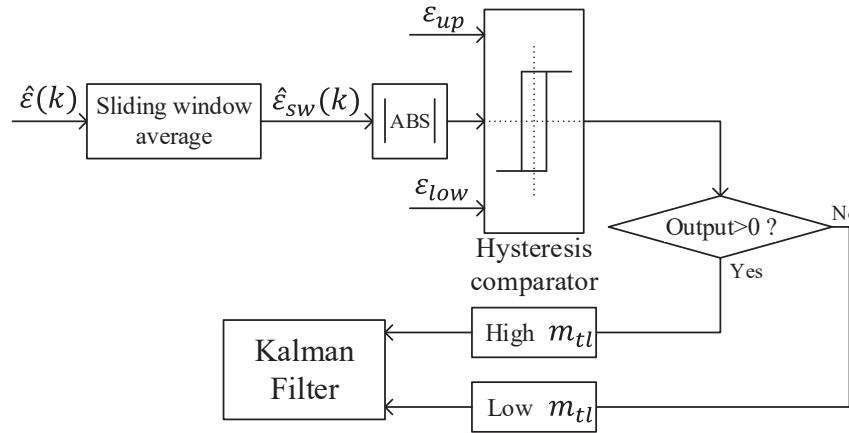
221 According to references<sup>21,27,28</sup>, the parameter  $N$  significantly influences the convergence performance, while  $M$  affects  
222 the convergence rate. In particular, increasing the element  $m_{tl}$  in  $M$  enhances the response speed of the corresponding state  
223 variable  $\tau_L$ . Thus, the core idea of the adaptation strategy is to increase  $m_{tl}$  during load transients to accelerate load torque  
224 tracking, and reduce it after convergence to restore steady-state noise suppression.

225 To detect load changes, the sliding-window average of the a priori prediction error  $\hat{\varepsilon}(k)$  is used<sup>29</sup>:

$$\hat{\varepsilon}_{sw}(k) = \frac{1}{n} (\hat{\varepsilon}(k) + \hat{\varepsilon}(k-1) + \dots + \hat{\varepsilon}(k-n+1)) \quad (23)$$

226 When  $\hat{\varepsilon}(k)$  resembles white noise,  $\hat{\varepsilon}_{sw}(k)$  remains close to zero at steady state. A load change causes  $\hat{\varepsilon}_{sw}(k)$  to deviate from  
227 zero, enabling the detection of loading/unloading events. As illustrated in Figure 2, the absolute value  $|\hat{\varepsilon}_{sw}(k)|$  is fed into a  
228 hysteresis comparator. When  $|\hat{\varepsilon}_{sw}(k)|$  exceeds an upper threshold  $\varepsilon_{up}$ , the comparator outputs 1, triggering an increase in  $m_{tl}$ ;  
229 when it falls below a lower threshold  $\varepsilon_{low}$ , the output becomes 0, and  $m_{tl}$  is reset to its initial value.

230 Furthermore, the following specific and feasible parameter tuning methods are provided:



**Figure 2.** Block diagram of Load change detection adaptation mechanism

- References<sup>30,31</sup> indicate that a sufficient condition for Kalman filter convergence is  $N > HP^+(k)H^T$ . However, an excessively large  $N$  may reduce the Kalman gain  $K(k)$  and slow down convergence. Thus,  $N$  is determined as  $N = HP^+(k)H^T + \zeta$ , where  $\zeta$  is appropriately chosen.
- Both  $m_t$  and  $m_{tl}$  can be set to small values at steady state. Upon detecting a load change, only  $m_{tl}$  is increased to ensure fast convergence of the load torque estimate. Once convergence is achieved ( $|\hat{\epsilon}_{sw}(k)| < \epsilon_{low}$ ),  $m_{tl}$  is reduced back to its initial value.

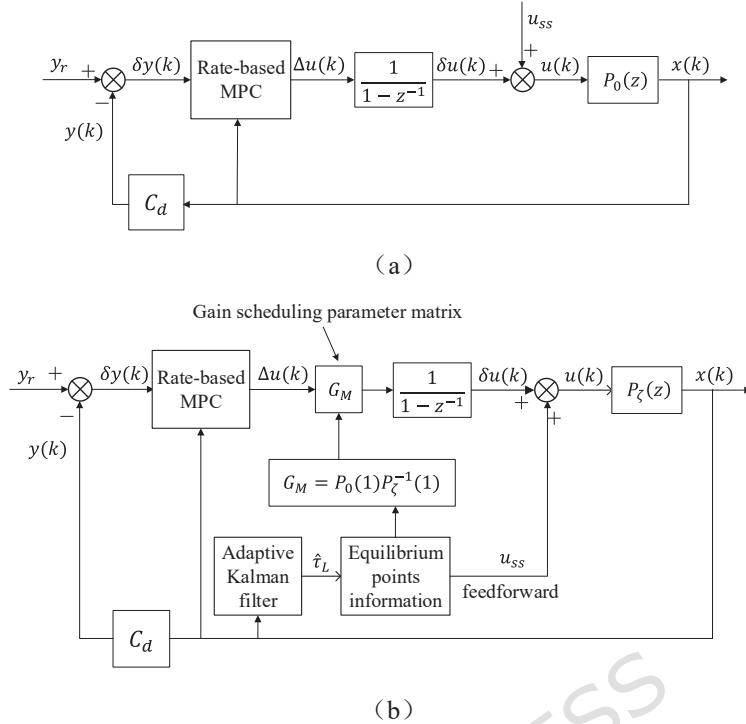
The effectiveness of this parameter tuning strategy is validated experimentally in the subsequent section.

## Gain Scheduling Strategy

The primary objective of gain scheduling is to linearize a nonlinear model at various equilibrium points, thereby generating a family of linear models, each associated with a specific operating condition. A corresponding linear controller is then designed for each equilibrium point. When the operating point shifts, the controller is accordingly switched<sup>32</sup>. In this study, the equilibrium point depends on the load torque, which is considered an unknown disturbance. Thanks to the adaptive Kalman filter proposed in the previous section, an accurate and rapid estimate of the load torque  $\hat{\tau}_L$  can be obtained, enabling fast switching of the equilibrium point and completing the basic gain scheduling framework.

$$G_M = P_0(1)P_\zeta^{-1}(1) \quad (24)$$

Figure 3(a) and (b) illustrate the control block diagrams under the nominal and off-nominal equilibrium points, respectively. The nominal case is straightforward: the steady-state input  $u_{ss}$  is known, and the state  $x(k)$  is measurable, leading to a simple controller implementation. In the off-nominal case (Fig. 3(b)), a Kalman filter quickly provides the load torque estimate, enabling the equilibrium point to be updated. Using the nominal model  $P_0(z)$ , the DC gain is computed to determine  $G_M$ . The MPC remains designed based on the nominal linearized model, and its output is corrected via  $G_M$ . The incremental control  $\delta u$



**Figure 3.** (a) Block diagram under the nominal equilibrium point. (b) Block diagram of gain scheduling under other equilibrium points.

262 is obtained through integration, and the final control  $u(k)$  is formed by adding  $\delta u$  to updated  $u_{ss}$ , which acts as a feedforward  
 263 term. This approach only requires storing the  $G_M$  matrix for each load condition, while the controller itself remains unchanged,  
 264 significantly reducing the need for interpolation and storage. For the dual-input dual-output system considered here,  $G_M$  a  
 265  $2 \times 2$  matrix, further alleviating the computational burden.

## 266 Experimental Validation

267 The experimental investigation was performed on a natural gas engine-generator set, as shown in Figure 4. The system  
 268 comprised an engine directly coupled to a three-phase synchronous generator, which supplied power to a programmable load  
 269 cabinet for precise load control. The engine had a rated power of 155 kW, a rated speed of 1500 rpm, and a maximum torque  
 270 of 1000 Nm. To maintain a stable grid frequency of 50 Hz, the engine speed was controlled at 1500 rpm, while the AFR was  
 271 maintained near the stoichiometric ratio for optimal combustion.

272

## 273 Adaptive Kalman Filter Verification

274 As previously discussed, the Kalman filter operates independently from the fourth-order nonlinear model and MPC controller,  
 275 allowing it to utilize a shorter sampling period ( $t_{s2} = 10$  ms) for enhanced response speed. This study compares three Kalman  
 276 filter configurations: a low-speed response Kalman filter (L-KF), a high-speed response Kalman filter (H-KF), and the  
 277 proposed adaptive Kalman filter (A-KF). Both L-KF and H-KF employ fixed parameters, where  $m_t$  and  $m_{tl}$  remain constant  
 278 throughout operation. The L-KF utilizes small values for both parameters, while the H-KF employs a significantly larger  $m_{tl}$   
 279 value (with  $m_{tl}$  maintained at the same small value as L-KF) to improve load torque tracking performance. In contrast, the  
 280 A-KF dynamically adjusts  $m_{tl}$  based on the load change detection mechanism: it increases  $m_{tl}$  during loading/unloading transients  
 281 and decreases it once convergence is achieved. The parameter  $N$  is identical across all three filters. The sliding window  
 282 length is carefully selected to balance detection sensitivity and noise immunity—excessively long windows delay load change  
 283 detection, while overly short windows introduce more white noise into the averaged signal.

284 The specific parameter configurations for each filter are as follows:



**Figure 4.** Natural gas engine test bench

285

- L-KF:

$$N = HP^+(k)H^T + \zeta$$

$$m_t = 25(N \cdot m)^2$$

$$\zeta = 20(\text{rad/s})^2$$

$$m_{tl} = 100(N \cdot m)^2$$

286  
287

- H-KF:

$$N = HP^+(k)H^T + \zeta$$

$$m_t = 25(N \cdot m)^2$$

$$\zeta = 20(\text{rad/s})^2$$

$$m_{tl} = 5 \times 10^4(N \cdot m)^2$$

288

- A-KF:

$$N = HP^+(k)H^T + \zeta$$

$$m_t = 25(N \cdot m)^2$$

Load change state:

$$\zeta = 20(\text{rad/s})^2$$

$$m_{tl} = 5 \times 10^4(N \cdot m)^2$$

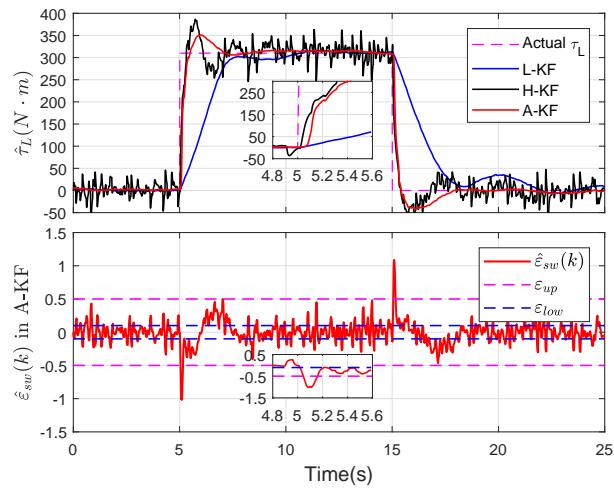
steady state:

$$m_{tl} = 100(N \cdot m)^2$$

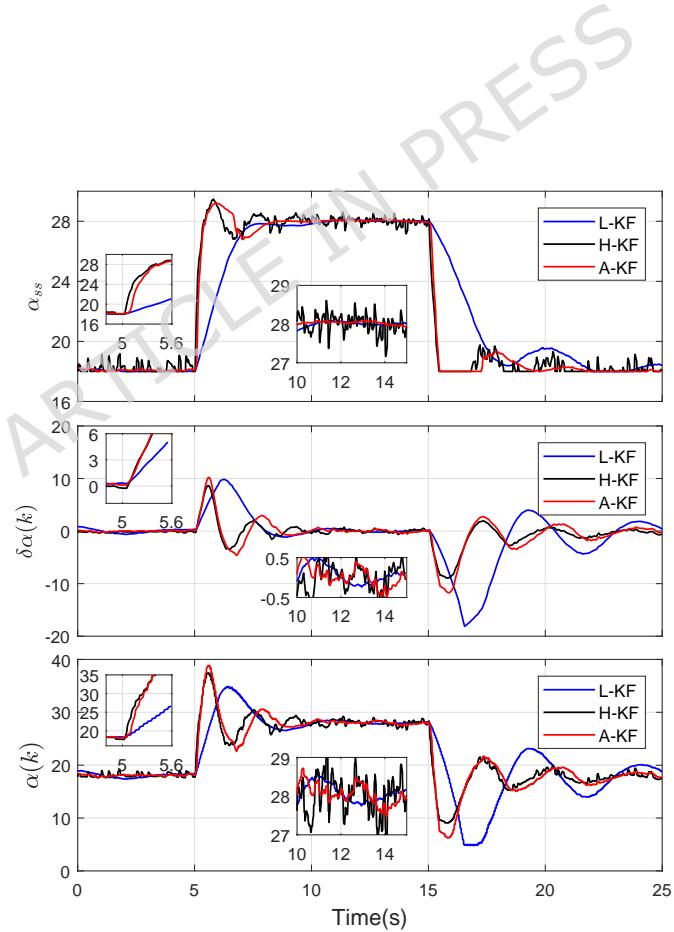
289 The MPC controller operates with a 50ms sampling period in the embedded system. The prediction horizon and control  
290 horizon are set to 6 and 2 steps, respectively. The different sampling periods for the MPC ( $t_{s1} = 50\text{ms}$ ) and the Kalman filter  
291 ( $t_{s2} = 10\text{ms}$ ) were chosen deliberately to achieve an optimal balance between computational feasibility and estimation/control  
292 performance. The MPC algorithm involves computationally intensive online optimization (solving a QP problem). A longer  
293 sampling period is necessary to provide sufficient time for the embedded processor to reliably compute the optimal control  
294 moves within each cycle, ensuring real-time implementation stability. The Kalman filter is a relatively simple second-order  
295 linear observer. Its low computational cost permits a much faster sampling rate. This is crucial for achieving the primary  
296 design goal of the estimator: to rapidly track transient load torque changes. A faster update rate allows for quicker detection  
297 of load steps and more timely adaptation of the gain-scheduling strategy, which directly enhances the transient performance  
298 of the overall control system.

299 An equilibrium point lookup table, predetermined from experimental data, enables the proposed gain scheduling strategy  
300 to achieve effective speed and AFR tracking control. Notably, the MPC controller configuration remains identical across all  
301 three Kalman filter implementations, with consistent weighting matrices:  $q = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$ ,  $r = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ ;

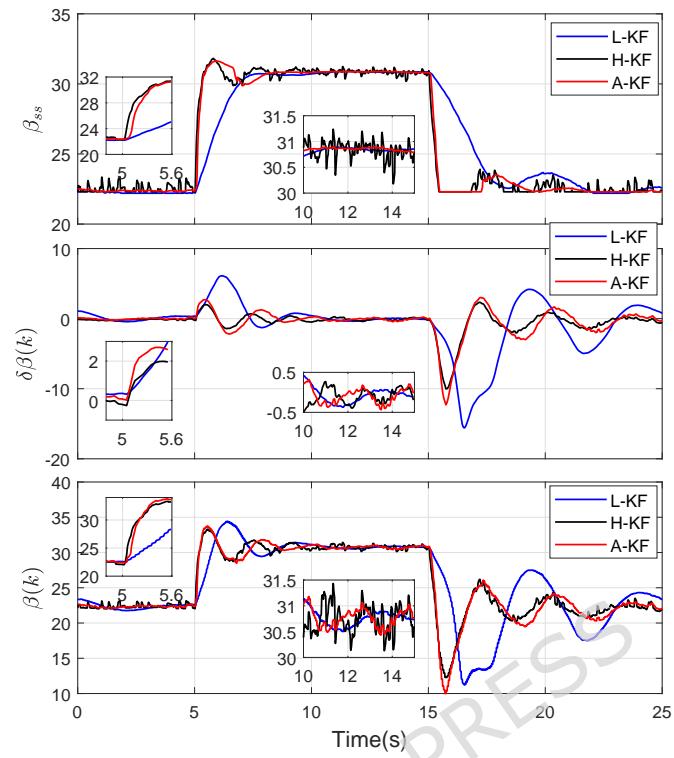
302 The relatively small values of  $q$  and  $r$  reflect the normalization of state variables to comparable orders of magnitude ( $p_2$  and  
303  $p_1$  in kPa,  $\xi$  scaled by 1000). Experimental results are presented in Figures 5-8, focusing on the critical first-loading scenario  
304 where the generator set transitions from no-load to 30% of maximum load before unloading. The excess air coefficient  $\lambda$ , a  
305 normalized value defined as the ratio of the actual AFR to the stoichiometric AFR, is used in the experimental results.



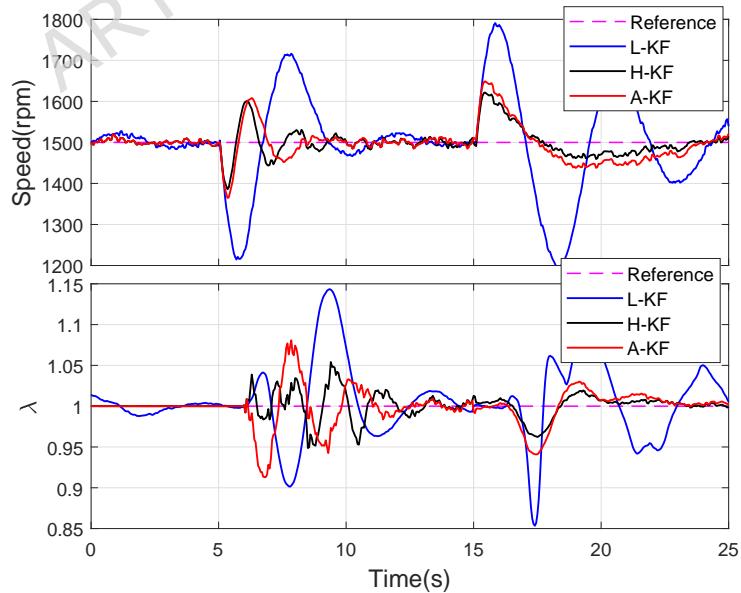
**Figure 5.** Estimated load torque comparison between three Kalman Filter



**Figure 6.** Mixture throttle opening comparison between three Kalman Filter



**Figure 7.** Fuel throttle opening comparison between three Kalman Filter



**Figure 8.** Output comparison of engine speed and AFR between three Kalman Filter

As shown in Figure 5, both H-KF and A-KF achieve rapid load torque tracking, converging within approximately 400ms, significantly outperforming L-KF. The H-KF's immediate convergence stems from its large, fixed  $m_{tl}$  value. The A-KF exhibits a slight delay relative to H-KF due to the detection mechanism's response time – approximately 60ms after the load change,  $|\hat{\epsilon}_{sw}(k)|$  exceeds the upper threshold  $\epsilon_{up}$ , triggering the switch to high  $m_{tl}$  and accelerated convergence. When  $|\hat{\epsilon}_{sw}(k)|$  falls below the lower threshold  $\epsilon_{low}$  around 400ms,  $m_{tl}$  reverts to its low value to ensure steady-state noise suppression. Both H-KF and A-KF provide sufficiently fast load estimates to enable rapid updates of the gain-scheduling matrix and equilibrium input  $u_{ss}$ .

Figures 6 and 7 demonstrate that  $\alpha_{ss}$  and  $\beta_{ss}$  for H-KF and A-KF respond promptly after loading (at 5s), resulting in sharp increases in the final control inputs  $\alpha(k)$  and  $\beta(k)$ . In contrast, L-KF's slow load estimation causes gradual changes in  $\alpha_{ss}$  and  $\beta_{ss}$ , making the final inputs predominantly determined by the MPC outputs  $\delta\alpha(k)$  and  $\delta\beta(k)$ . Consequently, as evident in Figure 8, L-KF exhibits substantial speed and AFR deviations following load changes, with speed excursions exceeding  $\pm 200$ rpm and prolonged AFR settling times. H-KF and A-KF, however, limit speed fluctuations to approximately  $\pm 150$ rpm through prompt input adjustments. (For reference, typical generator sets permit speed fluctuations around  $\pm 10\%$  of nominal speed.)

Regarding noise suppression, H-KF exhibits significant steady-state estimation noise due to its large  $m_{tl}$ , which propagates to  $u_{ss}$  and ultimately to the throttle commands, causing irregular fluctuations detrimental to precise throttle control. Both L-KF and A-KF demonstrate superior noise suppression. Analysis of the final control variables ( $\alpha(k)$  and  $\beta(k)$ ), reveals that noise primarily originates from the equilibrium inputs ( $\alpha_{ss}$  and  $\beta_{ss}$ ), while the MPC outputs ( $\delta\alpha(k)$  and  $\delta\beta(k)$ ) contribute minimal noise across all configurations.

	$\alpha_{ss}$	$\delta\alpha$	$\alpha$	$\beta_{ss}$	$\delta\beta$	$\beta$
H-KF	0.0647	0.0212	0.1238	0.0312	0.0112	0.0757
A-KF	0.0013	0.0189	0.0262	0.0003	0.0098	0.0145

**Table 1.** Comparison of steady-state variance of input variables in H-KF and A-KF (unit:  $degree^2$ )

Table 1 compares the steady-state variances of input variables for H-KF and A-KF during 10-15s (L-KF is excluded as its parameters match A-KF's steady-state values but with inferior dynamic performance). The variances of  $\alpha_{ss}$ ,  $\alpha$ ,  $\beta_{ss}$  and  $\beta$  are substantially larger in H-KF than in A-KF. Specifically, H-KF's  $\alpha$  fluctuates within nearly  $\pm 1$  degree, while A-KF maintains fluctuations within  $\pm 0.5$  degree.

These results confirm the necessity of the adaptive A-KF strategy. While conventional Kalman filters can be tuned to balance tracking speed and noise rejection<sup>27</sup>, static parameter tuning cannot simultaneously satisfy both requirements in this application. H-KF achieves acceptable transient performance but introduces excessive steady-state noise, whereas conservative (low-gain) tuning suppresses noise but fails to track rapid load disturbances. This trade-off is critical in practice since the filter output serves as the reference for throttle positioning – noisy references induce sustained throttle jitter, accelerate actuator wear, and may compromise system stability. The A-KF successfully reconciles these conflicting objectives by enhancing estimator responsiveness during transients while maintaining strong noise attenuation in steady state, thereby ensuring both performance and operational safety.

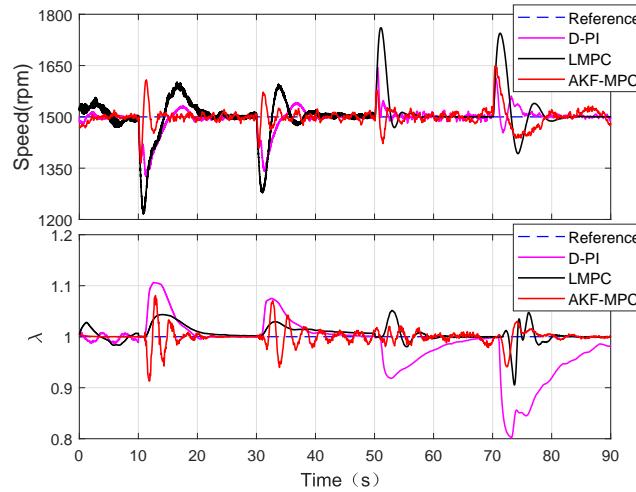
### Comparative Experimental Evaluation

The proposed MPC framework with adaptive Kalman filter (AFK-MPC) was experimentally compared against two baseline methods: a conventional PI control strategy and the standard rate-based MPC without adaptation.

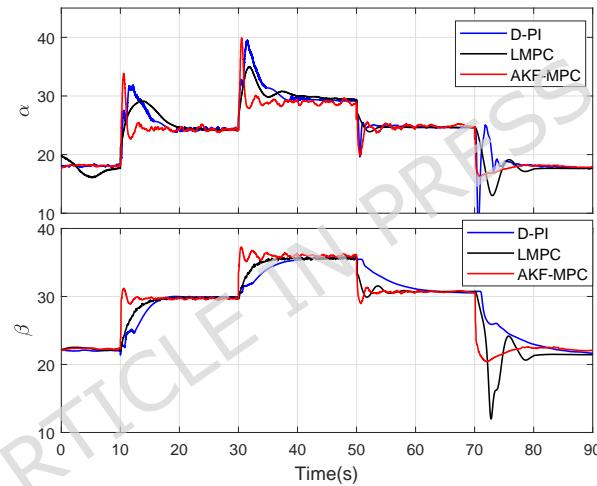
The conventional PI control method employs two decoupled PI controllers to regulate engine speed and AFR, respectively. For speed control, the PI controller directly computes the mixture throttle opening based on the error between the actual and target speed. The AFR control loop is more complex: to address the significant transport delay, a feedforward term based on intake manifold pressure ( $p_2$ ) is incorporated, as suggested in<sup>34</sup>. In each control cycle, a base fuel throttle opening is determined from a  $p_2$  lookup table. A PI controller then acts on the deviation between the measured AFR and its target, and its output is added to the base opening to yield the final fuel throttle command. This approach is referred to as Double-PI (D-PI) control. The final tuned parameters are:  $k_{sp} = 1.3$ ,  $k_{si} = 0.03$  for the speed PI controller, and  $k_{ap} = -5$ ,  $k_{ai} = -4$  for the AFR PI controller.

The rate-based MPC controller follows the design detailed in Section "MPC Controller Design", with its control block diagram identical to that shown in Fig. 3(a). It utilizes the same parameter matrix as the AKF-MPC controller, which is derived from the A-KF configuration. As this approach employs a linear MPC formulation, it is referred to as LMPC throughout this paper.

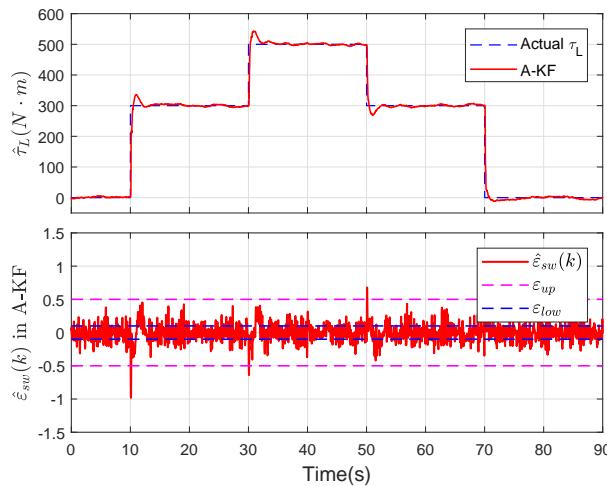
Figures 9 - 11 present a comparative analysis of the load response performance among the AFK-MPC, LMPC, and PI controllers. The test sequence involved applying 30% of the maximum load at 10s, followed by an additional 200Nm at 30s,



**Figure 9.** Comparison of engine speed and AFR responses for D-PI, LMPC, and AFK-MPC controllers.



**Figure 10.** Comparison of control inputs for D-PI, LMPC, and AFK-MPC controllers



**Figure 11.** Comparison of the estimated load torque and its sliding-window average for the AFK-MPC.

**Table 2.** Quantitative comparison of controller performance during loading and unloading transients

Index	D-PI	LMPC	AFK-MPC
Average of speed settling time ( $\pm 2\%$ ) [s]	6.875	7.943	5.146
Max deviation of speed [rpm]	-167 +155	-281 +269	-138 +149
Average of $\lambda$ settling time ( $\pm 2\%$ ) [s]	12.95	11.26	12.136
Max deviation of $\lambda$	-0.2 +0.11	-0.098 +0.052	-0.086 +0.084

354 with sequential unloading thereafter. Table 2 summarizes quantitative metrics for these loading and unloading processes.

355 In terms of speed control performance, the tuned D-PI controller achieves a level close to the power generation requirements. However, the LMPC, which relies solely on the rate-based MPC without load torque information, exhibits significantly 356 degraded speed performance, similar to the L-KF results observed earlier. In contrast, the AFK-MPC, benefiting from the gain- 357 scheduling parameter matrix and feedforward term  $u_{ss}$ , demonstrates markedly improved speed response, outperforming D-PI. 358 The speed fluctuation of AFK-MPC remains within  $\pm 150$ rpm, satisfying practical generation demands, and its settling time 359 is significantly shorter than those of D-PI and LMPC. 360

361 Regarding AFR control, both LMPC and AFK-MPC show substantially smaller AFR fluctuations compared to D-PI. This 362 improvement stems from the ability of MPC to coordinate multiple inputs in MIMO systems, whereas D-PI lacks inherent 363 decoupling capability. Although the settling times for AFR are comparable across controllers, the overall AFR performance 364 of LMPC and AFK-MPC is superior.

365 Based on these results, the AFK-MPC strategy delivers the best comprehensive performance: speed fluctuations remain 366 within the required range, settling time is short, and AFR variations are minimal.

367 In practical applications, the adaptive load detection mechanism is typically triggered during transients. If not activated 368 immediately upon loading, the controller's initial response is slower, leading to rapid speed deviation. This causes  $|\hat{\epsilon}_{sw}(k)|$  to 369 exceed the upper threshold  $\epsilon_{up}$ , activating the adaptation. Only when the load is small enough that the rate-based MPC alone 370 can quickly restore speed will the detection remain inactive, in such cases, speed deviation is negligible.

## 371 Conclusion

372 The main contribution of this paper lies in the proposal of a control method that integrates an adaptive Kalman estimator 373 with a gain scheduling strategy, combined with rate-based model predictive control (MPC), which significantly enhances the 374 system's anti-disturbance performance under unknown load variations. The paper elaborates on the design process of the 375 MPC controller and the adaptive Kalman filter: by introducing an adaptive strategy and a load variation detection mechanism, 376 the Kalman filter achieves rapid tracking of load torque changes under transient conditions while effectively suppressing 377 estimation noise in steady-state operation, thereby balancing dynamic response speed and steady-state accuracy.

378 The proposed method exhibits strong engineering applicability. Since model linearization, discretization, and prediction 379 model computations can be performed offline, the online computational burden of MPC is reduced to solving a QP problem, 380 significantly lowering the computational load. Moreover, the Kalman filter features a simple structure with minimal online 381 computational requirements, making it suitable for implementation in embedded systems. Experimental results demonstrate 382 that the control framework effectively suppresses speed and AFR fluctuations caused by step load changes, shortens the settling 383 time, and provides a feasible solution for improving the load adaptability of natural gas engines used in power generation.

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## 469 Author contributions statement

470 Wenyu Xiong conceived the study, designed the experiments, and wrote part of the manuscript. Qichangyi Gong conducted  
471 the experiments, analyzed the data and wrote part of the manuscript. Songtao Huang, Jie Ye, and Jinbang Xu performed data  
472 curation, validation, and provided critical feedback on the manuscript. All authors reviewed and approved the final manuscript.

## 473 Data availability statement

474 The datasets generated during and/or analysed during the current study are available from the corresponding author on reason-  
475 able request.

## 476 Additional information

477 The authors declare no competing interests.