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A hybrid decision-support framework for selecting sustainable domestic heating systems in cold climates

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A hybrid decision-support framework for selecting sustainable domestic heating systems in cold climates

Abstract

This study proposes a probabilistic hesitant fuzzy multi-criteria decision-making approach for assessing sustainable heating system alternatives under uncertain situations. The selection of heating systems is a major issue in sustainability in cold climate regions due to its high energy requirements. The proposed approach integrates the Probabilistic Hesitant Fuzzy Analytic Hierarchy Process (PHF-AHP) for determining the weights of decision criteria and the Evaluation Based on Distance from Average Solution (EDAS) for ranking alternatives. The probabilistic hesitant fuzzy approach allows decision-makers to reflect uncertainty and hesitation in decision-making under expert judgments by considering various membership values with their respective probabilities. To validate the proposed methodological framework, a case study of five heating system alternatives is presented. The results reveal that the proposed PHF-AHP-EDAS approach is reliable in ranking alternatives under ambiguity and uncertainty in sustainability decision problems.

Keywords: Sustainable heating systems; Domestic heating selection; Energy efficiency; Probabilistic hesitant fuzzy sets; Hybrid MCDM; AHP–EDAS

1 Introduction

Heating is necessary in the winter to provide a comfortable and safe living environment. Effective heating systems are essential in nations like Canada, Russia, Turkey, and others where temperatures can fall below 0 degrees Celsius. From antiquated fireplaces to contemporary technology, heating systems have undergone substantial change. Different heating systems may be classified according to the energy source, heating unit location, heat transport method, and technique of heat transmission. Systems are categorised as local, central, or district heating systems based on where the heat-generating equipment is located. The majority of heating systems use solid fuels, gaseous fuels, diesel, or electricity. As a result, solar heating systems depend mostly on solar radiation and, occasionally, waste energy. Ranjan and Kanitkar [33] assessed the energy thresholds necessary to promote sustainable human growth, providing information on fair access to energy.

Convection, radiation, or the movement of heated air across the space are the three ways that heat is transported. Electric current, water, and air are common heat transporters. Buildings use about 40% of the world's total power output and 32% of its ultimate energy consumption, according to global energy statistics [34]. The overwhelming majority of heating systems continue to depend on fossil fuels, which greatly contribute to pollution and emissions that worsen the environment. By presenting energy as a "community currency," Anto et al. [49] offered an innovative perspective and demonstrated its revolutionary influence on the socioeconomic development of recently electrified communities. China, the US, the EU27, India, Russia, and Japan were among the largest emitters of carbon dioxide in 2021. Collectively, it accounted for 62.4% of the world's GDP, 66.4% of fossil fuel consumption, and 67% of CO_2 emissions from fossil fuels [36]. In comparison to 2020, the CO_2 emissions of all six nations grew in 2021, with Russia and India exhibiting the largest percentage increases. The importance of renewable energy in accomplishing sustainable development goals has been highlighted by recent research. The intricate relationships between the use of renewable energy and other aspects of sustainable development were studied by Boe et al. [25], emphasising the necessity of coordinated policy approaches.

1.1 Literature review

Multi-Criteria Decision-Making (MCDM) is a branch of operations research that focusses on getting optimal results in problems with many indicators, competing objectives, and different criteria. In order to prepare for uncertainty and imprecision in the decision-making process, MCDM techniques are frequently combined with fuzzy set theory. P-HFEs, or probabilistic hesitant fuzzy elements, were initially presented in continuous form by [42]. subsequently, new fundamental operations for P-HFEs were defined using the Frank t-norm and t-conorm operators [46]. For MCDM issues including probabilistic hesitant fuzzy information, a dominance measure and Best-Worst Method (BWM) within a P-HFE framework were suggested [47]. A unique MCDM method utilising sophisticated aggregation operations in a q-rung dual hesitant fuzzy environment [48]. Additionally, [50] proposed a method for making decisions that uses probabilistic hesitant fuzzy preference relations to rank options more reliably. To ascertain the probability distribution of components in a P-HFE and the likelihood of risk situation, two nonlinear programming models were also included [51]. Recent advances in multi-criteria decision-making research have increasingly focused on hybrid fuzzy frameworks that can handle uncertainty, ambiguity, and complicated interrelationships among assessment criteria. Specifically, spherical fuzzy and probabilistic fuzzy environments have been extensively used to improve the modelling performance of traditional MCDM techniques. For example, for the purpose to prioritise low-carbon policy options in climate change mitigation planning, recent research has used combined SWARA–WASPAS techniques under spherical fuzzy environment. These models show determining if robust ranking strategies may be used with subjective weighting techniques to enable sustained policy evaluation in unpredictability [43]. For the assessment of environmental, social, and governance (ESG) performance in high-tech industrial parks, hybrid decision frameworks that combine fuzzy Delphi methodologies with statistical clustering approaches have been created [44]. These methods enable the discovery of crucial sustainability indicators across intricate corporate structures and offer organised methods for the aggregation of expert information. Sustainability-focused decision frameworks have been explored in the agriculture sector in addition to industrial and policy applications. The significance of rigorous assessment frameworks for ecosystem restoration, sustainable land management, and long-term environmental resilience is highlighted by recent research on regenerative organic agriculture [45]. These studies demonstrate how multi-criteria decision-support models are becoming increasingly important for assessing sustainability plans in a variety of industries.

The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions. It is a well-established MCDM method that ranks alternatives based on a hierarchical framework of criteria and sub-criteria. AHP enables decision-makers to identify the most important factors influencing a decision and evaluate alternatives based on those factors. Due to its simplicity and practical applicability, AHP has been widely adopted in various research domains [4]. For instance, Santos et al. [5] identified AHP as the most suitable method in their study. AHP is increasingly used in construction management to tackle complex problems and guide strategic decisions [4]. The fuzzy AHP was employed to identify relevant sustainability issues [3], while another utilized it to select the optimal vendor for supplying materials for pre-insulated pipe manufacturing [39].

Wolnowska et al. applied AHP to evaluate potential transit routes for oversized freight through the city of Szczecin [35]. The VIKOR method, developed by Serafim Opricovic [21], has also been employed extensively. Ali Ebadi Torkayesh et al. [6] evaluated waste-to-energy technologies using the VIKOR approach. In China, distributed renewable energy systems for restaurants were ranked using interval VIKOR, which accommodates data ambiguity through interval values [31]. VIKOR is particularly beneficial for addressing MCDM problems with incommensurable and conflicting criteria and is known for generating compromise-based optimal solutions [32]. Ming Chang Ong et al. applied the VIKOR technique to rank alternatives in a study on reusing cleaning agents in rubber glove manufacturing [22]. Its utility has been widely documented in numerous studies [15, 16].

The integration of AHP and VIKOR is a popular approach in contemporary research [26, 29, 12]. For instance, Gowri et al. [7] proposed a FAHP-VIKOR model to assess and select optimal system configurations under competing criteria. Another study combined AHP and VIKOR in an MCDM framework to elucidate the mathematical underpinnings of selection comparisons [24]. In recent years, various hybrid MCDM models have been developed by incorporating fuzzy theories to enhance decision accuracy as shown in Table 1. Among these, the Evaluation Based on Distance from Average Solution (EDAS) method, introduced by Keshavarz Ghorabaee et al. in 2015 [47], has gained prominence. Alongside TOPSIS and VIKOR, EDAS is now recognized as one of the most widely used MCDM methods [9].

Numerous studies have explored heating system selection as an MCDM problem [30]. For instance, one paper employed AHP to analyze and compare the performance of various space heating systems for rural homes [40]. In another study, AHP was used to select five phase change materials for solar heating systems, considering both technical specifications and material properties [2]. In Turkey, AHP has been used to evaluate and prioritize different heating system alternatives [23]. Meanwhile, in Serbia, fuzzy AHP has supported

Table 1: Review of Fuzzy-MCDM applications in sustainable energy and heating systems

Author(s) and Year	**Fuzzy Set + MCDM Method**	**Application Domain**	**Key Factors**
Saraswat and Digalwar (2021) [54]	Integrated Fuzzy MCDM (AHP–TOPSIS)	Evaluation of sustainable energy sources	Economic viability, environmental impact, social acceptance, and technical efficiency
Hamza et al. (2023) [55]	Integrated Fuzzy MCDM (AHP–VIKOR)	HVAC system selection for sustainable office buildings	Energy efficiency, lifecycle cost, maintenance, thermal comfort, and environmental performance
Wang and Lahdelma (2020) [56]	Uncertainty-based MCDM	Sustainability ranking of district heating systems	Cost, CO_2 emissions, efficiency, fuel flexibility, and reliability
Ćesić et al. (2024) [57]	Fuzzy decision support system with MCDM	Urban heat island management	Temperature reduction, land-use planning, green infrastructure, and sustainability indicators
Aghazadeh et al. (2022) [58]	Hybrid Fuzzy MCDM (SWARA–COPRAS)	Sustainable structural system selection in mass-housing	Cost, material sustainability, energy consumption, and structural performance
Özdemir (2020) [59]	MCDM with fuzzy assessment (AHP, TOPSIS)	Residential heating system selection	Installation cost, energy efficiency, fuel type, maintenance, and environmental impact

the development of strategic management plans for district heating systems [10].

1.2 Expert panel and data collection procedure

To mitigate the issue of uncertainty and hesitation while conducting evaluation processes, a probabilistic hesitant fuzzy approach was adopted to collect expert opinions. A group of experts was consulted to perform the evaluation processes. The experts were chosen based on their qualifications, experience, and expertise in the domain of energy systems, renewable technologies, and sustainable assessment. During the data collection process, the experts were asked to evaluate the importance of evaluation criteria and the performance of different alternatives for heating system technologies. The experts were asked to express their opinions using multiple values to represent their uncertainty while conducting the evaluation processes. The experts also assigned probabilities to each value to represent their confidence levels for the evaluation processes. The evaluation processes were represented using probabilistic hesitant fuzzy numbers to develop a rigorous mathematical framework for dealing with uncertain expert opinions.

1.3 Aim and motivation of the research

In many practical scenarios of decision-making problems, experts often face challenges in providing an accurate assessment when dealing with multiple criteria evaluation problems. The challenges may be due to a lack of sufficient information, vagueness, or even a lack of experience with specific technologies. Hence, experts may be reluctant to provide precise numerical values for their assessment. In the context of this study, reluctant information is defined as hesitant evaluation results where experts are reluctant to provide a single deterministic value but rather multiple possible evaluation values. Generally, most of the conventional MCDM models assume that experts can provide precise evaluation results for the decision-making problem. However, in many practical scenarios of decision-making problems, vagueness, ambiguity, or even a lack of sufficient knowledge can affect the applicability of conventional deterministic MCDM models. To solve the challenges associated with conventional deterministic MCDM models, fuzzy set theory, hesitant fuzzy sets, or probabilistic hesitant fuzzy sets have been introduced to handle vagueness or even hesitation while making decisions.

1.4 Contribution of the research

Recently, developments in fuzzy set theory have led to the concept of PHFSs, which improves the handling of uncertainties in multi-criteria decision problems. Unlike other fuzzy approaches that rely on triangular fuzzy numbers, PHFSs allow decision-makers to present various membership levels with their respective probability measures, which is closer to reality in terms of decision-makers' hesitation. This paper utilizes the PHF-AHP method in determining the relative weights of decision criteria and the EDAS method in ranking heating system alternatives. The proposed hybrid approach is applicable in decision situations where uncertainty and ambiguity exist, offering a better solution in sustainability assessment compared to other conventional approaches.

The organization of this paper is presented as follows: Section 1 presents the basic concepts related to fuzzy set theory, PHFSs, and F-MCDM. Section 2 presents the definition of PHFS along with the basic guidelines for operating the same. Section 3 presents the structure of a unique PHFS-based AHP and EDAS hybrid approach. Section 4 presents the practical applicability of the proposed approach by solving an efficient domestic heating system selection problem. Sections 5 and 6 are dedicated to comparative analysis and sensitivity analysis to ensure the robustness of the proposed model. Finally, Section 7 presents the key findings of the paper and possible future directions for research.

2 Preliminaries

Some PHFS principles that may be used in later sections are introduced in this section.

Definition 2.1. *[41] A PHFS on X produces probabilistic variables with results on a subset of $[0,1]$. We assume that the PHFS' continuous random variable exists. A PHFS can be mathematically represented as follows:*

$$
{}^{\wp}\mathbb{G} = \{\langle x, {}^{\wp}g_x\rangle : x \in X\}
$$

$$
= \left\{ \left\langle x, \bigcup_{\langle g_x, \wp_x\rangle \in {}^{\wp}g_x} \{\langle g_x, \wp_x\rangle\} \right\rangle : x \in X \right\} \tag{1}
$$

where $\langle g_x, \wp_x\rangle$ is a pair such that $g_x \in [0,1]$ and $\sum \wp_x = 1$. Here, g_x indicates the degree to which the element $x \in X$ belongs to the set H, and $\wp_x$ indicates the probability associated with g_x.

Definition 2.2. *[14] Let ${}^{\wp}g_1 = \{\langle g_1^{(i)}, \wp_1^{(i)}\rangle : i = 1, 2, \ldots, m\}$ and ${}^{\wp}g_2 = \{\langle g_2^{(j)}, \wp_2^{(j)}\rangle : j = 1, 2, \ldots, n\}$ be two PHFS elements.*

1. ***Addition:***

$$
{}^{\wp}g_1 \oplus {}^{\wp}g_2 = \left\{ \left\langle g_1^{(i)} + g_2^{(j)},\ \frac{1}{Z} \cdot \wp_1^{(i)} \cdot \wp_2^{(j)} \right\rangle \middle|\ i = 1, \ldots, m;\ j = 1, \ldots, n \right\}
$$

where $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \wp_1^{(i)} \cdot \wp_2^{(j)}$ is a normalization constant to ensure the sum of probabilities is 1.

2. ***Subtraction:***

$$^{\wp}g_1 \ominus {}^{\wp}g_2 = \left\{ \left\langle \max(0, g_1^{(i)} - g_2^{(j)}),\ \frac{1}{Z} \cdot \wp_1^{(i)} \cdot \wp_2^{(j)} \right\rangle \middle|\ i = 1, \ldots, m;\ j = 1, \ldots, n \right\}$$

3. ***Multiplication:***

$$^{\wp}g_1 \otimes {}^{\wp}g_2 = \left\{ \left\langle g_1^{(i)} \cdot g_2^{(j)},\ \frac{1}{Z} \cdot \wp_1^{(i)} \cdot \wp_2^{(j)} \right\rangle \middle|\ i = 1, \ldots, m;\ j = 1, \ldots, n \right\}$$

4. ***Division:***

$$^{\wp}g_1 \oslash {}^{\wp}g_2 = \left\{ \left\langle \min\left(1, \frac{g_1^{(i)}}{g_2^{(j)}}\right),\ \frac{1}{Z} \cdot \wp_1^{(i)} \cdot \wp_2^{(j)} \right\rangle \middle|\ i = 1, \ldots, m;\ j = 1, \ldots, n;\ g_2^{(j)} \neq 0 \right\}$$

Definition 2.3. *[14] The score function of a PHFE $^{\wp}g_x$ is given by:*

$$S(^{\wp}g_x) = \left(\sum_{k=1}^{|g(\wp)|} g^k \wp^k \right) \tag{2}$$

For two PHFEs $g_1(\wp)$ and $g_2(\wp)$, if $S(g_1(\wp)) > S(g_2(\wp))$, then we consider that $g_1(\wp)$ is superior to $g_2(\wp)$, denoted as $g_1(\wp) > g_2(\wp)$ or $g_2(\wp) < g_1(\wp)$. This makes it simple to compare two PHFEs by using their respective scores.

Example 1. *Let $g_k(\wp)$ $(k = 1, 2, 3)$ be three Probabilistic Hesitant Fuzzy Elements (PHFEs), defined as:*

$$g_1(\wp) = \{0.5(0.2),\ 0.8(0.8)\}, \quad g_2(\wp) = \{0.3(0.4),\ 0.7(0.6)\}, \quad g_3(\wp) = \{0.2(0.5),\ 0.9(0.5)\}$$

Following Definition 2, the score values of these PHFEs are calculated as follows:

$$S(g_1(\wp)) = 0.5 \times 0.2 + 0.8 \times 0.8 = 0.74$$

$$S(g_2(\wp)) = 0.3 \times 0.4 + 0.7 \times 0.6 = 0.54$$

$$S(g_3(\wp)) = 0.2 \times 0.5 + 0.9 \times 0.5 = 0.55$$

2.1 Boundedness and Idempotency of aggregation operators in PHFS

Let $^{\wp}\mathbb{G}_i = \left\langle x_i, \left\{ (g_i^{(j)}, \wp_i^{(j)}) \right\}_{j=1}^{m_i} \right\rangle$ for $i = 1, 2, \ldots, n$ be a set of PHFS elements. The expected value for each i-th element is defined as:

$$g_i = \sum_{j=1}^{m_i} \wp_i^{(j)} g_i^{(j)}, \quad \text{with } g_i^{(j)} \in [0, 1],\ \sum_{j=1}^{m_i} \wp_i^{(j)} = 1.$$

Let $\{g_1, g_2, \ldots, g_n\}$ be the representative values, and $\{w_1, w_2, \ldots, w_n\}$ be the weights, where $w_i \geq 0$ and $\sum w_i = 1$.

Theorem 2.4. *The arithmetic mean aggregation operator defined by*

$$AM = \sum_{i=1}^{n} w_i g_i$$

satisfies boundedness and idempotency in PHFS.

Proof. **Boundedness:** Since $g_i \in [0, 1]$ and $w_i \geq 0$ with $\sum w_i = 1$, the convex sum $AM \in [0, 1]$.

Idempotency: If $g_i = g$ for all i, then $AM = \sum w_i g = g$.

$\square$

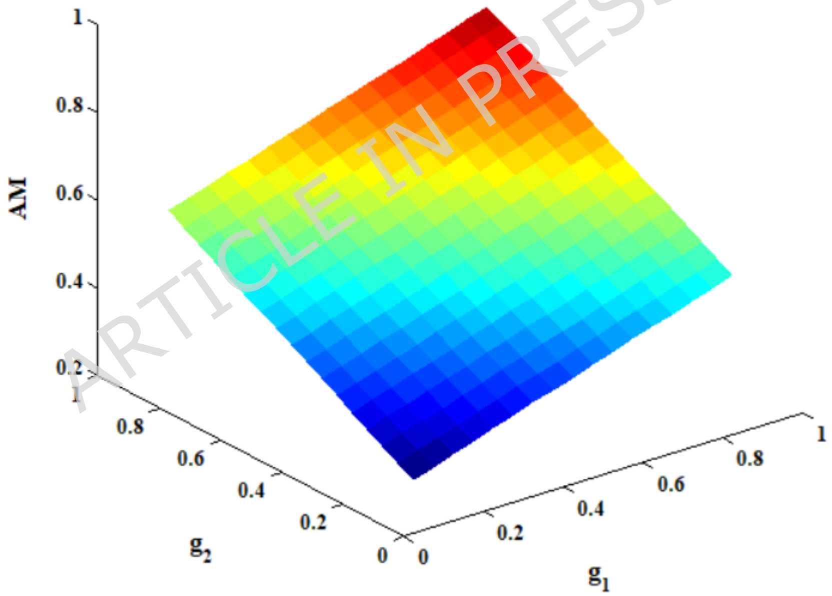


Figure 1: **PHFS Aggregation: Arithmetic Mean**

Theorem 2.5. *The geometric mean aggregation operator defined by*

$$GM = \prod_{i=1}^{n} g_i^{w_i}$$

satisfies boundedness and idempotency in PHFS when $g_i > 0$.

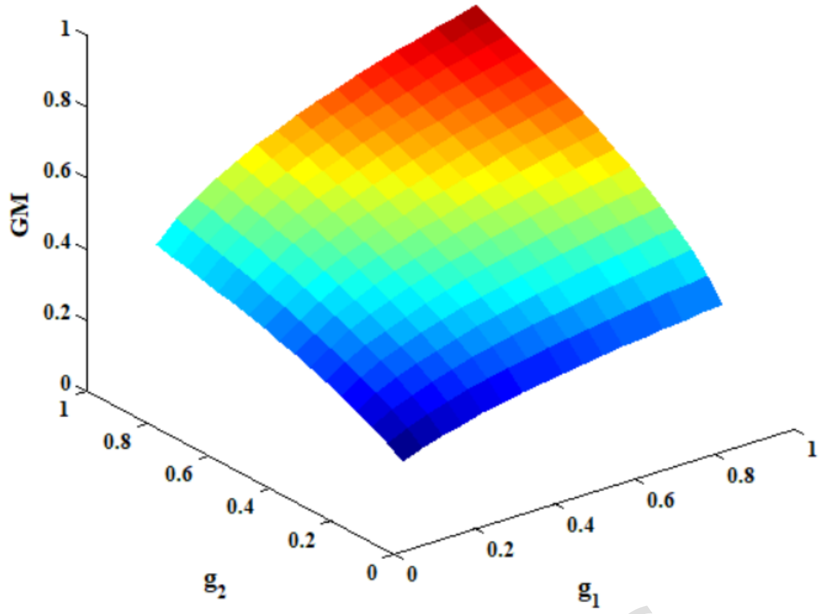


Figure 2: **PHFS Aggregation: Geometric Mean**

Proof. **Boundedness:** Since $g_i \in (0, 1]$ and $w_i \geq 0$, the product of positive numbers in $[0, 1]$ raised to positive exponents summing to 1 lies in $(0, 1]$.

Idempotency: If $g_i = g$, then $GM = g^{\sum w_i} = g$. □

Theorem 2.6. *The harmonic mean aggregation operator defined by*

$$HM = \left(\sum_{i=1}^{n} \frac{w_i}{g_i}\right)^{-1}$$

satisfies boundedness and idempotency in PHFS when $g_i > 0$.

Proof. **Boundedness:** Since $g_i \in (0, 1]$, $\frac{1}{g_i} \geq 1$, and $w_i \geq 0$, we have $HM \in (0, 1]$.

Idempotency: If $g_i = g$, then $HM = \left(\frac{1}{g}\sum w_i\right)^{-1} = g$. □

Theorem 2.7. *The Einstein weighted average aggregation operator defined by*

$$EA = \frac{\sum_{i=1}^{n} w_i g_i}{1 + \sum_{i=1}^{n} w_i g_i - \sum_{i=1}^{n} w_i g_i^2}$$

satisfies boundedness and idempotency in PHFS.

Proof. **Boundedness:** Since $g_i \in [0, 1]$:

$$0 \leq \sum w_i g_i \leq 1, \quad 0 \leq \sum w_i g_i^2 \leq 1$$

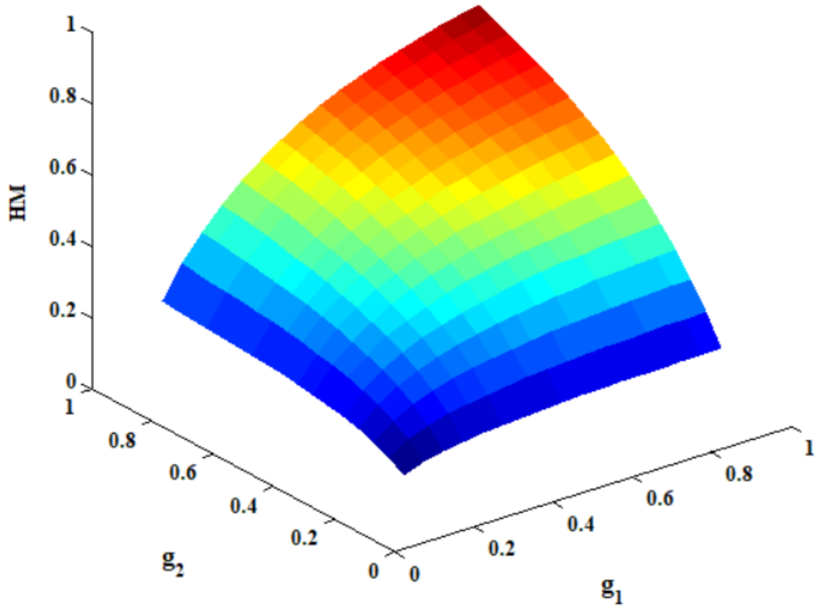


Figure 3: **PHFS Aggregation: Harmonic Mean**

So the denominator $1 + \sum w_i g_i - \sum w_i g_i^2 \geq 1$, ensuring $EA \in [0, 1]$.

Idempotency: If $g_i = g$, then:

$$EA = \frac{g}{1 + g - g^2} = g$$

□

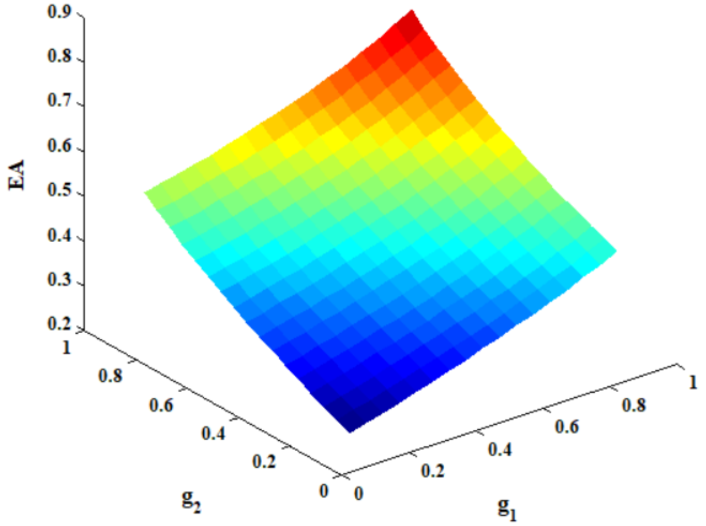


Figure 4: **PHFS Aggregation: Einstein Weighted Average**

Theorem 2.8. *The Dombi weighted average aggregation operator defined by*

$$DA = \left(1 + \left(\sum_{i=1}^{n} w_i \left(\frac{1-g_i}{g_i}\right)^{\lambda}\right)^{1/\lambda}\right)^{-1}, \quad \lambda > 0$$

satisfies boundedness and idempotency in PHFS.

Proof. **Boundedness:** For $g_i \in (0,1]$, we have $0 \leq \left(\frac{1-g_i}{g_i}\right)^{\lambda} < \infty$. The sum is finite and positive, so:

$$DA \in (0,1]$$

Idempotency: If $g_i = g$, then:

$$DA = \left(1 + \left(\left(\frac{1-g}{g}\right)^{\lambda} \sum w_i\right)^{1/\lambda}\right)^{-1} = \left(1 + \left(\frac{1-g}{g}\right)\right)^{-1} = g$$

$\square$

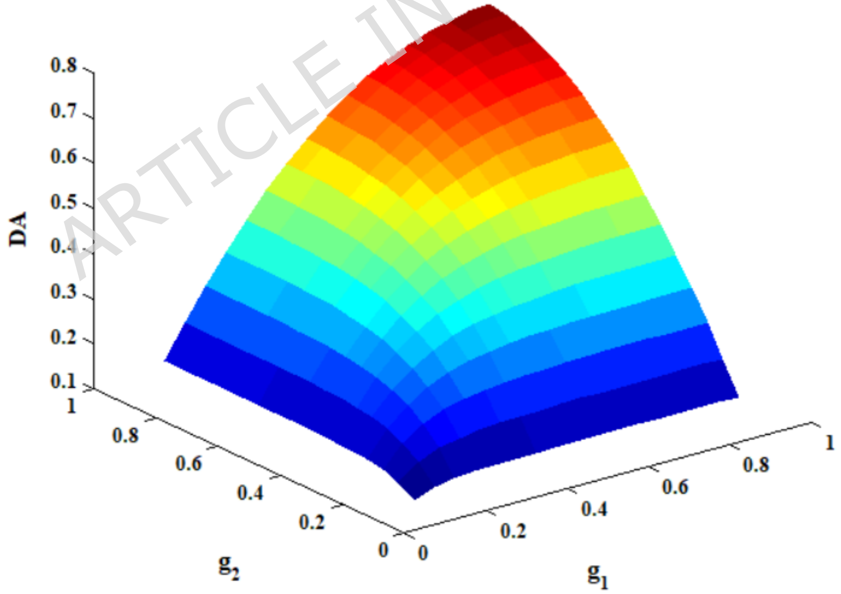


Figure 5: **PHFS Aggregation: Dombi Weighted Average**

Each of the considered aggregation operators maintains the foundational properties of boundedness and idempotency under the probabilistic hesitant fuzzy environment, thereby validating their applicability in PHFS-based decision-making models.

3 Proposed Methodology

In the current research, the applicability of the PHF-AHP method and the EDAS approach is evaluated for decision-making under conditions of uncertainty and expert hesitations. The PHF-AHP method offers the opportunity to consolidate various membership values as well as probabilities. This increases the reliability of the weight structures in the evaluation of the criteria. The EDAS method allows for the evaluation of alternatives based on the calculation of the deviations from the average performance. This approach helps to increase the stability of the ranking results without the use of ideal points. The above approaches deal with the ambiguity as well as the incompatibility of the criteria, which makes the evaluation of sustainable heating systems suitable. The overall structure of the proposed methodology is illustrated in Figure 6.

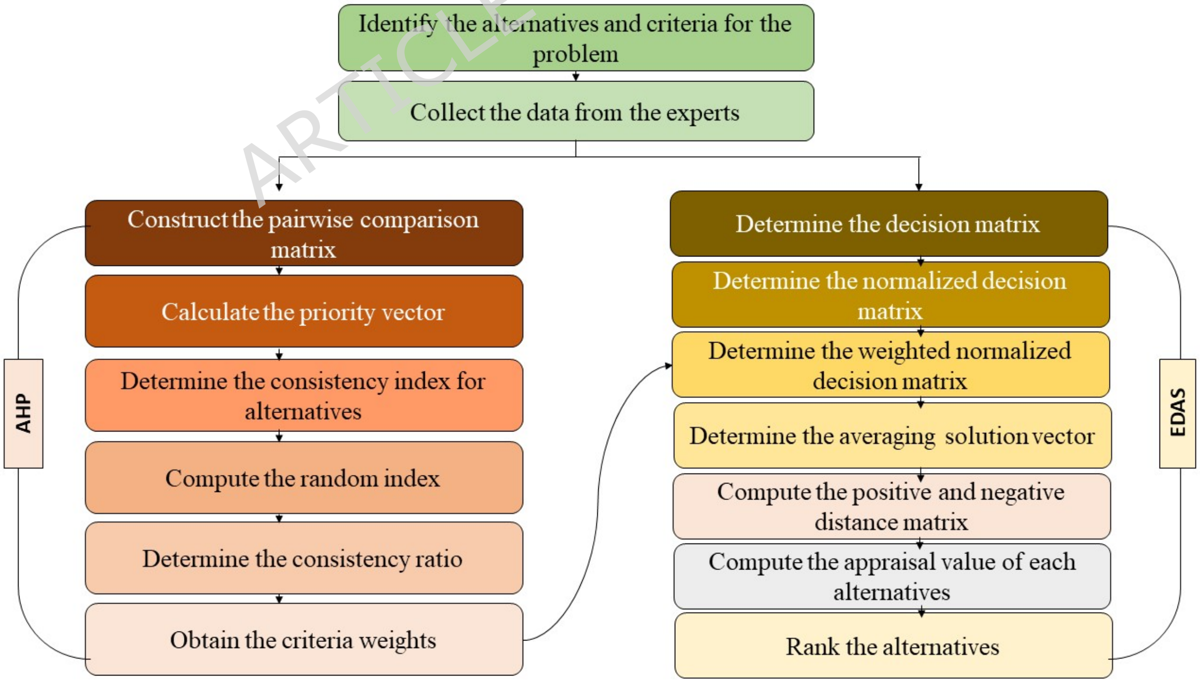


Figure 6: **Framework of the proposed hybrid PHF-AHP–EDAS methodology**

3.1 Algorithm of PHF-based AHP method

The suggested framework, PHFEs are used to represent the judgments of the experts during the pairwise comparison step of the AHP. The experts are asked to provide a range of values with corresponding probabilities, which represent the uncertain nature of the judgments. The PHFEs are used to create the pairwise comparison matrix, which represents the judgments of the experts. The probabilistic hesitant fuzzy score function is used to transform the PHFEs into a numerical form, which can be used for the AHP weighting step. When there are more than one expert, the judgments are aggregated to create a single comparison matrix, which represents the judgments of the experts. The classical AHP is used to obtain the relative weights, with a check for the consistency ratio (CR) to ensure logical consistency, with the evaluations repeated if the ratio is considered too high. This ensures the advantages of probabilistic hesitant fuzzy modeling are retained within the classical AHP framework. When extended under the PHFE, the pairwise comparison matrix is represented as:

$$\mathbb{A} = \begin{bmatrix} 1 & \mathbb{A}_{12} & \cdots & \mathbb{A}_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{A}_{n1} & \mathbb{A}_{n2} & \cdots & 1 \end{bmatrix}$$

The PHF-AHP steps are as follows:

Step 1: Normalize the comparison matrix.

Step 2: Compute the priority vector by averaging the values in each row of the normalized matrix.

Step 3: Compute the consistency vector, and calculate the maximum eigenvalue $\lambda_{\max}$ as the average of the resulting values.

Step 4: Calculate the Consistency Index (CI) using:

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

Step 5: Refer to Table 2 for the Random Index (RI) based on matrix order n.

Step 6: Compute the Consistency Ratio (CR):

$$CR = \frac{CI}{RI}$$

If $CR \leq 0.10$, the pairwise matrix is considered consistent.

Table 2: Random Index (RI) values

Number of alternatives (n)	Random Index (RI)
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41

3.2 EDAS method

The Evaluation based on EDAS method ranks the alternatives using the following steps:

Step 1: Define p alternatives $J_1, J_2, \ldots, J_p$ and q evaluation criteria $C_1, C_2, \ldots, C_q$.

Step 2: Construct the decision matrix $\mathscr{D} = [d_{kl}]$, where d_{kl} denotes the performance of alternative J_k under criterion C_l, derived from PHFEs. Also define the weight vector:

$$\boldsymbol{w} = [\omega_1, \omega_2, \ldots, \omega_q]$$

where ω_l is the importance of criterion C_l.

Step 3: Calculate the average value for each criterion:

$$\mathscr{AV}_l = \frac{1}{p} \sum_{k=1}^{p} d_{kl}$$

Step 4: Determine the Positive Distance from Average (PDA) and Negative Distance from Average (NDA) as:

$$\mathscr{PDA}_{kl} = \frac{\max\left(0,\ d_{kl} - \mathscr{AV}_l\right)}{\mathscr{AV}_l} \tag{3}$$

$$\mathscr{NDA}_{kl} = \frac{\max\left(0,\ \mathscr{AV}_l - d_{kl}\right)}{\mathscr{AV}_l} \tag{4}$$

Step 5: Compute the weighted sum of PDA and NDA for each alternative:

$$\mathscr{SP}_k = \sum_{l=1}^{q} \omega_l \cdot \mathscr{PDA}_{kl} \tag{5}$$

$$\mathscr{SN}_k = \sum_{l=1}^{q} \omega_l \cdot \mathscr{NDA}_{kl} \tag{6}$$

Step 6: Normalize the weighted sums:

$$\mathscr{NSP}_k = \frac{\mathscr{SP}_k}{\max_k(\mathscr{SP}_k)} \tag{7}$$

$$\mathscr{NSN}_k = 1 - \frac{\mathscr{SN}_k}{\max_k(\mathscr{SN}_k)} \tag{8}$$

Step 7: Calculate the appraisal score for each alternative:

$$\mathscr{AS}_k = \frac{1}{2}\left(\mathscr{NSP}_k + \mathscr{NSN}_k\right), \quad 0 \leq \mathscr{AS}_k \leq 1$$

Step 8: Rank the alternatives in descending order based on $\mathscr{AS}_k$ values. The highest score indicates the most preferred alternative.

4 Case Study: Identification and Evaluation of Household Heating System Alternatives

The study focuses on identifying appropriate heating system alternatives and evaluating them using a multi-criteria decision-making framework. A detailed assessment of household heating technologies and sustainable energy systems resulted in the selection of five heating system solutions. The review examined prior assessments of heating system performance, energy efficiency, and renewable heating technologies in an effort to identify feasible alternatives. To confirm that these strategies are feasible, discussions with specialists in energy engineering and sustainable energy systems were conducted. These experts validated the significance of the selected alternatives, which are regularly utilised heating methods in modern residential and commercial systems. The study constructed evaluation criteria using a systematic procedure that included both literature analysis and expert assistance. Relevant criteria reflecting financial success, sustainability, and technical proficiency were selected and refined with advice from experts, ensuring that the final criteria set precisely reflect the critical factors influencing heating system selection while retaining the evaluation process' quality. The structure of the decision problem is depicted in Figures 7 and 8.

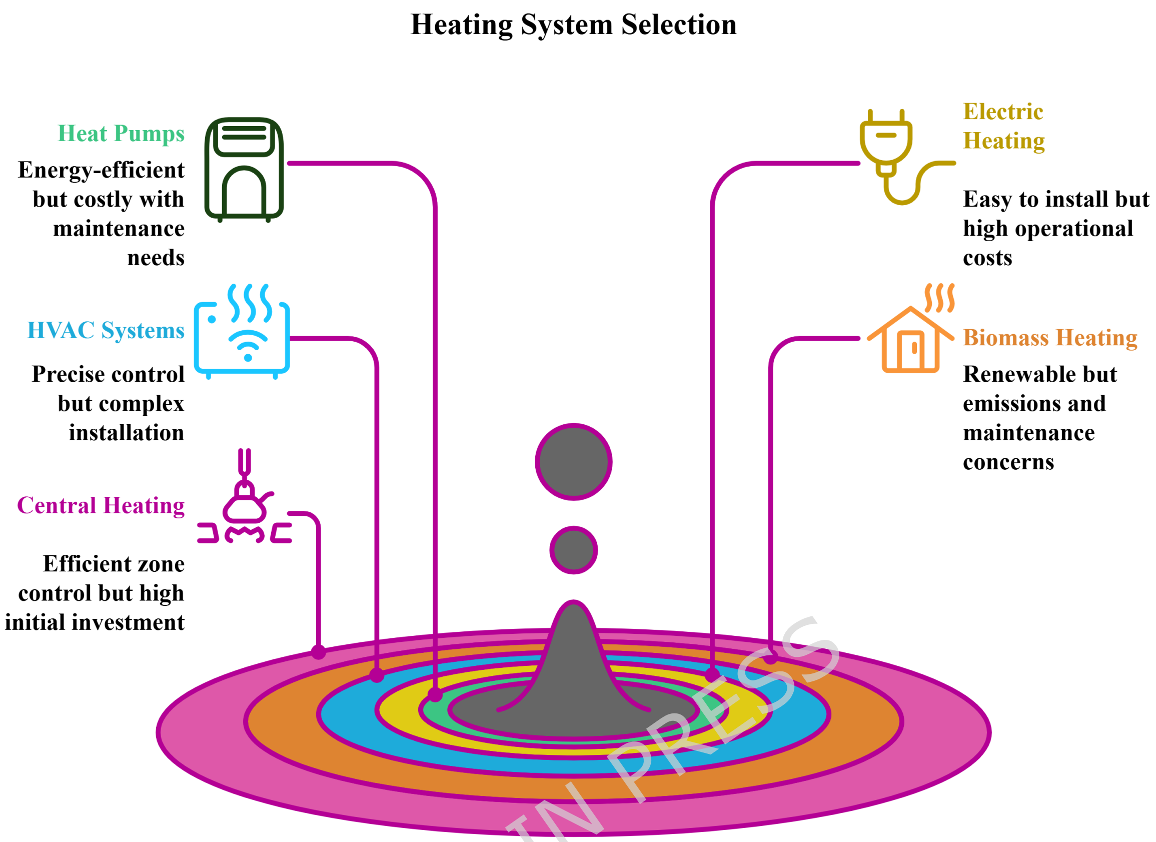


Figure 7: **Factors influencing heating system selection**

Alternatives

- **Heat Pumps ($\mathscr{J}_1$):** Operate by transferring heat from external sources (air, ground, or water) to indoor spaces. Types include Air-Source, Ground-Source, Water-Source, Dual-Fuel, and Mini-Split. They are energy-efficient and environmentally friendly with low operational costs. Limitations include high initial costs, reduced efficiency in extreme cold, noise, and the need for regular maintenance.

- **Electric Heating Systems ($\mathscr{J}_2$):** Include furnaces, baseboard heaters, radiators, and electric floor heating. These systems are easy to install, safe, and quiet. However, they often incur high operational costs, offer limited efficiency and temperature control, and may be unsuitable for large buildings or areas with high electricity tariffs.

- **HVAC Systems ($\mathscr{J}_3$):** Incorporate air conditioning, heating, and ventilation.

These systems are appropriate for contemporary buildings because they provide exact control over temperature and air quality. Even though they are energy-efficient and versatile, they have significant upfront costs, complicated installation, and regular maintenance.

- **Biomass Heating Systems ($\mathcal{J}_4$):** Burn organic resources like wood, agricultural waste, or pellets as fuel. These solutions are locally supplied, renewable, and carbon neutral. Pollutant emissions, reduced efficiency, maintenance requirements, and fuel availability are obstacles. Stoves, pellet systems, and biomass boilers are common varieties.

- **Central Heating Systems ($\mathcal{J}_5$):** Use radiators, ducts, or pipes to disperse heat from a central unit. Forced-air, boiler-based, geothermal, radiant, and heat pump-based systems are common varieties. It offers zone control and long-term efficiency, but they need a large initial investment and work most effectively in newly constructed or significantly modified buildings.

Evaluation Criteria

- **Efficiency ($\mathcal{C}_1$):** This is the ratio of heat energy output to the consumption of energy. impacted by operational circumstances, insulation quality, and the structure of the system.

- **Environmental Impact ($\mathcal{C}_2$):** Evaluates the emissions and contaminants in the system. For sustainable development, systems with lower carbon footprints and less air pollutants are preferred.

- **Energy Source ($\mathcal{C}_3$):** Concerns the nature, dependability, availability, as well as security of the energy supply.

- **Life Cycle Cost (LCC) ($\mathcal{C}_4$):** The total cost of ownership for the system during its lifetime, including initial, servicing and replacement expenses.

- **Running Time Cost ($\mathcal{C}_5$):** Continuous expenses include fuel usage, power use, and other operating costs.

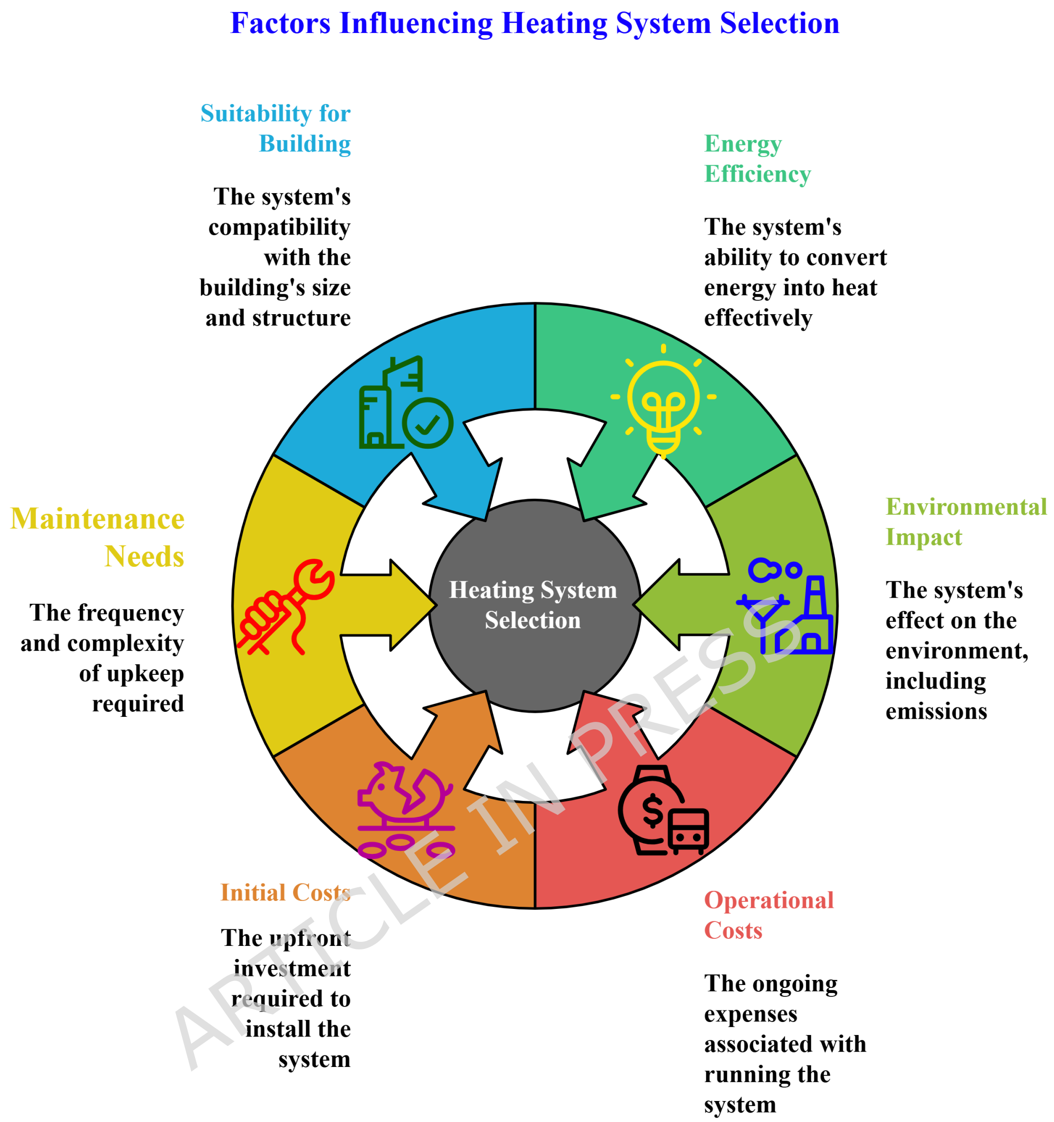


Figure 8: **Comparison of heating system options efficiency, costs, installation, and maintenance**

4.1 Criteria weight evaluation using PHFS-AHP

Using the Saaty scale of relative relevance in Table 3, the pairwise comparison approach is employed to assess the significance of the criteria. The values presented in Table 4 correspond to the score values obtained from the probabilistic hesitant fuzzy evaluations provided by the experts. The weights are calculated as the normalized eigenvector of the pairwise comparison matrix shown in Table 4.

Based on the calculated normalized eigenvector, the criterion with the least importance

Table 3: **Saaty's scale of relative importance**

Saaty Scale	Description
1	Equal significance
3	Moderate significance
5	Strong significance
7	Very strong significance
9	Extreme significance
2,4,6,8	Intermediate values
1/3,1/5,1/7,1/9	Inverse comparisons

Table 4: **Pairwise comparison of the criteria**

	Efficiency	Environmental Impact	Energy Source	Life Cycle Cost	Running Time Cost
Efficiency	1	3	5	3	7
Environmental Impact	1/3	1	3	5	3
Energy Source	1/5	1/3	1	3	3
Life Cycle Cost	1/3	1/5	1/3	1	2
Running Time Cost	1/7	1/3	1/3	1/2	1

is Running Time Cost (weight = 0.0553), and the most significant is Efficiency (weight = 0.4617) as shown in Figure 9.

To validate the consistency of the pairwise judgments, the Consistency Index ($\mathscr{CI}$) and Consistency Ratio ($\mathscr{CR}$) are computed. The values obtained are $\mathscr{CI} = 0.10233$ and $\mathscr{CR} = 0.0913$, which confirms acceptable consistency since $\mathscr{CR} < 0.1$.

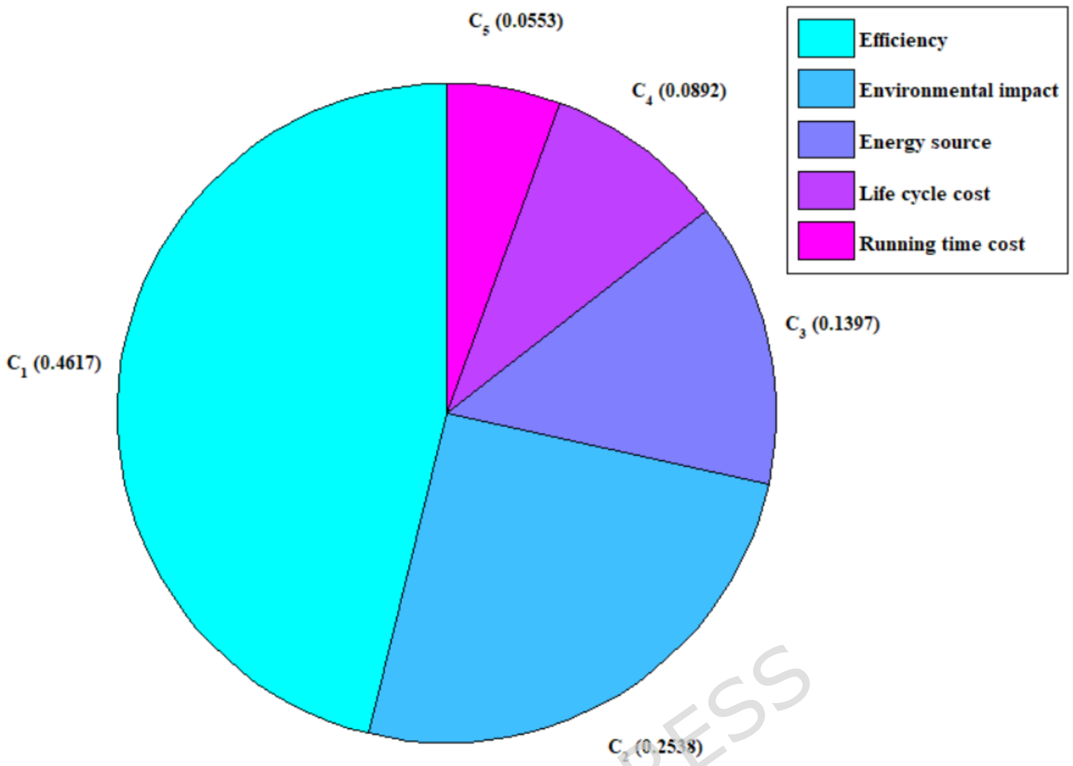


Figure 9: **Relative weights of criteria based on PHFS-AHP**

4.2 Determination of optimal alternative using PHFS-EDAS

Probabilistic hesitant fuzzy decision matrix was developed from the assessments of the expert for the alternatives of the heating system with respect to the predetermined criteria. Initially, the alternatives and the criteria were identified from the literature related to sustainable heating systems. Then, the alternatives and the criteria were refined from the expert consultation. The probabilistic hesitant fuzzy assessments were used for the evaluation of the alternatives, which enabled the consideration of more than one evaluation value with corresponding probabilities. The results were aggregated to obtain the collective decision matrix, which forms the base for the subsequent analysis using the EDAS method. According to Table 5, the decision-maker generates the PHFS decision matrix. Equation (2) is employed to construct the PHFS score decision matrix. Table 6 illustrates the fuzzy average values determined for each criterion. The corresponding $\mathscr{PDA}$ and $\mathscr{NDA}$ values are calculated using Equations (4) and (5), and are presented in Tables 7 and 8, respectively. Subsequently, Equations (6) and (7) are used to compute the crisp $\mathscr{SP}_k$ and $\mathscr{NP}_k$ values, as shown in Tables 9 and 10. Equations (8), (9), and (10) obtain the normalised $\mathscr{NSP}_k$ and appraisal score $\mathscr{AS}_k$ values, which are analysed in Table 11. The alternatives are sorted in accordance with the calculated $\mathscr{AS}_k$ values.

As shown in Figure 10, heat pumps are determined to be the best heating alternative after Step 7 of the suggested procedure. Heat pumps are ecologically benign as it transfer heat compared to producing it by using fossil fuels, which results in them extremely efficient. It's an environmentally friendly alternative for their ability to run on electricity, which is often obtained from renewable energy sources like solar power. Heat pumps additionally provide lower running costs because of their minimal reliance on fossil fuels. Its long operating lifespan helps to reduce total life cycle costs, even though their initial installation cost may be quite expensive.

Table 5: **The PHF decision matrix**

	$\mathscr{C}_1$	$\mathbb{C}_2$	$\mathbb{C}_3$	$\mathbb{C}_4$	$\mathbb{C}_5$
$\mathscr{G}_1$	$\{(0.7, 0.2), (0.5, 0.8)\}$	$\{(0.5, 0.4), (0.7, 0.6)\}$	$\{(0.8, 0.1), (0.3, 0.9)\}$	$\{(0.3, 0.5), (0.3, 0.5)\}$	$\{(0.2, 0.6), (0.3, 0.4)\}$
$\mathscr{G}_2$	$\{(0.6, 0.3), (0.8, 0.7)\}$	$\{(0.6, 0.7), (0.2, 0.3)\}$	$\{(0.5, 0.4), (0.4, 0.6)\}$	$\{(0.5, 0.4), (0.6, 0.6)\}$	$\{(0.5, 0.7), (0.6, 0.3)\}$
$\mathscr{G}_5$	$\{(0.4, 0.6), (0.9, 0.4)\}$	$\{(0.5, 0.6), (0.8, 0.4)\}$	$\{(0.7, 0.8), (0.2, 0.2)\}$	$\{(0.3, 0.5), (0.1, 0.5)\}$	$\{(0.8, 0.5), (0.7, 0.5)\}$
$\mathscr{G}_4$	$\{(0.6, 0.4), (0.7, 0.6)\}$	$\{(0.2, 0.8), (0.5, 0.2)\}$	$\{(0.6, 0.4), (0.5, 0.6)\}$	$\{(0.7, 0.2), (0.8, 0.8)\}$	$\{(0.5, 0.4), (0.9, 0.6)\}$
$\mathscr{G}_3$	$\{(0.5, 0.5), (0.5, 0.5)\}$	$\{(0.5, 0.5), (0.3, 0.5)\}$	$\{(0.4, 0.5), (0.5, 0.5)\}$	$\{(0.6, 0.3), (0.7, 0.7)\}$	$\{(0.7, 0.8), (0.5, 0.2)\}$

Table 6: **The fuzzy average values for each criterion**

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{G}_1$	0.5400	0.6200	0.3500	0.3000	0.2400
$\mathscr{G}_2$	0.7400	0.4800	0.4400	0.5600	0.5300
$\mathscr{G}_3$	0.6000	0.6200	0.6000	0.2000	0.7500
$\mathscr{G}_4$	0.6600	0.2600	0.5400	0.7800	0.7400
$\mathscr{G}_5$	0.5000	0.4000	0.4500	0.6700	0.6600
$\mathscr{AV}_l$	0.6080	0.4760	0.4760	0.5020	0.5840

5 Results and Discussion

Five heating systems were evaluated using the suggested PHF-AHP–EDAS methodology: $\mathscr{G}_1$, $\mathscr{G}_2$, $\mathscr{G}_3$, $\mathscr{G}_4$, and $\mathscr{G}_5$. The efficiency ($\mathscr{C}_1$) criterion ported the greatest priority (eigenvalue: 0.4617), demonstrating its dominating role in decision-making, when the

Table 7: $\mathscr{PDA}$ **values**

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathscr{G}_1$	0.0000	0.3025	0.0000	0.4024	0.5890
$\mathscr{G}_2$	0.2171	0.0084	0.0000	0.0000	0.0925
$\mathscr{G}_3$	0.0000	0.3025	0.2605	0.6016	0.0000
$\mathscr{G}_4$	0.0855	0.0000	0.1345	0.0000	0.0000
$\mathscr{G}_5$	0.0000	0.0000	0.0000	0.0000	0.0000

Table 8: $\mathscr{NDA}$ **values**

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathscr{G}_1$	0.1118	0.0000	0.2647	0.0000	0.0000
$\mathscr{G}_2$	0.0000	0.0000	0.0756	0.1155	0.0000
$\mathscr{G}_3$	0.0132	0.0000	0.0000	0.0000	0.2842
$\mathscr{G}_4$	0.0000	0.4538	0.0000	0.5538	0.2671
$\mathscr{G}_5$	0.1776	0.1597	0.0546	0.3347	0.1301

Table 9: $\mathscr{SP}_k$ **values**

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathscr{SP}_k$
$\mathscr{G}_1$	0.0000	0.0768	0.0000	0.0359	0.0326	0.1453
$\mathscr{G}_2$	0.1003	0.0021	0.0000	0.0000	0.0051	0.1075
$\mathscr{G}_3$	0.0000	0.0768	0.0364	0.0537	0.0000	0.1668
$\mathscr{G}_4$	0.0395	0.0000	0.0188	0.0000	0.0000	0.0583
$\mathscr{G}_5$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 10: $\mathscr{NP}_k$ **values**

	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathscr{NP}_k$
$\mathscr{G}_1$	0.0516	0.0000	0.0370	0.0000	0.0000	0.0886
$\mathscr{G}_2$	0.0000	0.0000	0.0106	0.0103	0.0000	0.0209
$\mathscr{G}_3$	0.0061	0.0000	0.0000	0.0000	0.0157	0.0218
$\mathscr{G}_4$	0.0000	0.1152	0.0000	0.0494	0.0148	0.1794
$\mathscr{G}_5$	0.0820	0.0405	0.0076	0.0299	0.0072	0.1672

PHF-AHP technique was initially employed to generate the criteria weights based on expert judgement through pairwise comparison.

PHFEs were then used to calculate the performance values of the alternatives under

Table 11: **Appraisal score** (AS_k) **values**

	$\mathscr{NSP}_k$	$\mathscr{NNP}_k$	$\mathscr{AS}_k$	RANK
$\mathscr{J}_1$	0.8706	0.5059	0.6882	3
$\mathscr{J}_2$	0.6443	0.8836	0.7640	2
$\mathscr{J}_3$	1.0000	0.8785	0.9392	1
$\mathscr{J}_4$	0.3493	0.0000	0.1746	4
$\mathscr{J}_5$	0.0000	0.0676	0.0338	5

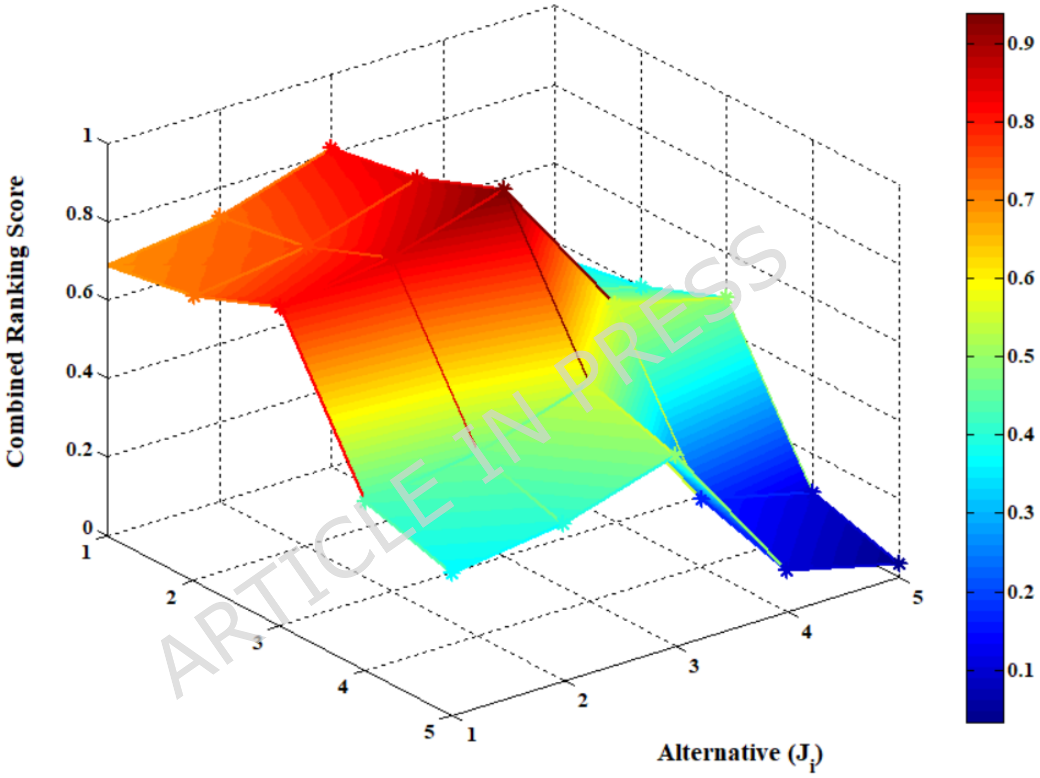


Figure 10: **Ranking results of the alternatives**

each criterion. The PHF-based decision matrix was produced by defuzzifying them using predicted score functions. Positive Distance from Average (PDA) and Negative Distance from Average (NDA) scores were calculated using the EDAS approach on this matrix. These were used to calculate weighted scores ($\mathscr{SP}_k$ and $\mathscr{NP}_k$), normalised values ($\mathscr{NSP}_k$ and $\mathscr{NNP}_k$), and final appraisal scores ($\mathscr{AS}_k$).

The alternatives are ranked as follows based on the final assessment scores: System of HVAC ($\mathscr{J}_3$) > $\mathscr{J}_2$ electric heating system > pumps for heat ($\mathscr{J}_1$) > heating of biomass

($\mathcal{G}_4$) > $\mathcal{G}_5$ central heating system. In comparison to other systems, the HVAC system ($\mathcal{G}_3$) had the highest appraisal score ($\mathscr{A}\mathscr{S}_k = 0.9392$), which was explained by its balance between high efficiency, moderate cost, and comparatively reduced environmental effect. Its supremacy was helped by its success on operating efficiency and energy source criteria, although with greater beginning costs.

The electric heating system ($\mathcal{G}_2$) ranked second, with high rankings for running time cost and installation simplicity. Third place was to the heat pump system ($\mathcal{G}_1$), which performed effectively in terms of the environment however terribly in terms of cost. Central heating systems ($\mathcal{G}_5$) and biomass ($\mathcal{G}_4$) were placed lower because of their high installation complexity and emissions, respectively. These results show that under fuzzy MCDM, contemporary, integrated systems that are compatible with renewable energy (such as HVAC) are more profitable.

The results of the current research confirm the significance of climate awareness and environmental literacy for the implementation of sustainable technologies. Indeed, the efficiency of the alternatives to the heating system is a significant factor. However, it is also important that the stakeholders involved in the implementation of the sustainable technologies should have a strong level of awareness regarding climate change. There is strong evidence that the level of climate literacy is a significant factor for the interpretation of the climate-related information [52, 53]. Additionally, the evaluation of the systematic policy frameworks is a significant factor for sustainable decision-making. Indeed, the recent research highlights the significance of the multi-criteria decision-making models for the selection of the sustainable policies under conditions of uncertainty. The proposed evaluation framework for the selection of the sustainable alternatives to the heating system is based on probabilistic hesitant fuzzy models. The results of the current research can help the policymakers, engineers, and planners to make decisions regarding the selection of the energy-efficient solutions that are consistent with the climate mitigation strategies.

5.1 Comparative analysis

To validate the reliability of the proposed PHFS-EDAS model, a comparative analysis was carried out using some of the existing MCDM methods, namely TOPSIS, VIKOR, WASPAS, COPRAS, MOORA, CODAS, PROMETHEE, and CoCoSo methods. The assessment scores obtained from the nine comparative approaches are presented in Table 12 and these scores are subsequently used to determine the ranking of the alternatives as shown in Figure 11. The results obtained through this analysis reveal that the proposed PHFS-EDAS model is comparable to other MCDM methods. In other words, it is observed that the proposed model is reliable, considering that alternative $\mathscr{G}_3$ ranks first in EDAS, COPRAS, PROMETHEE, and CoCoSo methods, whereas alternative $\mathscr{G}_2$ ranks first in MOORA, WASPAS, and VIKOR methods. On the other hand, it is also observed that alternative $\mathscr{G}_5$ ranks lowest in all methods, ranking fifth in all assessments.

To verify the consistency of the rankings provided by the different MCDM techniques, Spearman's rank correlation coefficient revealed a strong positive relationship between COPRAS, EDAS, MOORA, CODAS, PROMETHEE, and CoCoSo. This revealed a perfect correlation of 1.000 among the techniques, indicating a similar ranking as shown in Figure 12. The proposed approach uses probabilistic hesitant fuzzy evaluation criteria to address the uncertainties involved in the decision-making process. This allows the decision maker to provide a set of values along with their probabilities. This improves the decision outcome when the proposed approach is integrated with the EDAS technique.

Table 12: **Comparative analysis across multiple MCDM methods**

Alt.	TOPSIS	VIKOR	WASPAS	COPRAS	EDAS	MOORA	CODAS	PROMETHEE	CoCoSo
$\mathscr{G}_1$	0.5534	0.6377	0.6197	0.8135	0.6882	0.5211	0.5114	0.5325	0.5406
$\mathscr{G}_2$	0.6715	0.0000	0.7201	0.8778	0.7640	0.6712	0.6931	0.6823	0.7109
$\mathscr{G}_3$	0.6688	1.0000	0.6819	1.0000	0.9392	0.6923	0.7342	0.7493	0.7592
$\mathscr{G}_4$	0.3742	0.4986	0.6343	0.7566	0.1746	0.4021	0.4299	0.4197	0.4308
$\mathscr{G}_5$	0.2548	0.2807	0.5813	0.6956	0.0338	0.3214	0.2932	0.2754	0.3101

5.2 Sensitivity analysis

A complete sensitivity analysis is carried out by varying the criterion weights across 10 distinct cases in order to evaluate the stability and robustness of the suggested decision-making method. For every cases, the ranking values of the alternatives $\mathscr{G}_1$ to $\mathscr{G}_5$ are calculated, and the corresponding optimal option is chosen. The results are summarised in Table 13, while the comparative ranking results have been shown in Figures 13 to 17. The method's preference stability is demonstrated by the sensitivity analysis, which reveals that $\mathscr{G}_1$ is regularly selected as the best alternative. The rankings are significantly affected

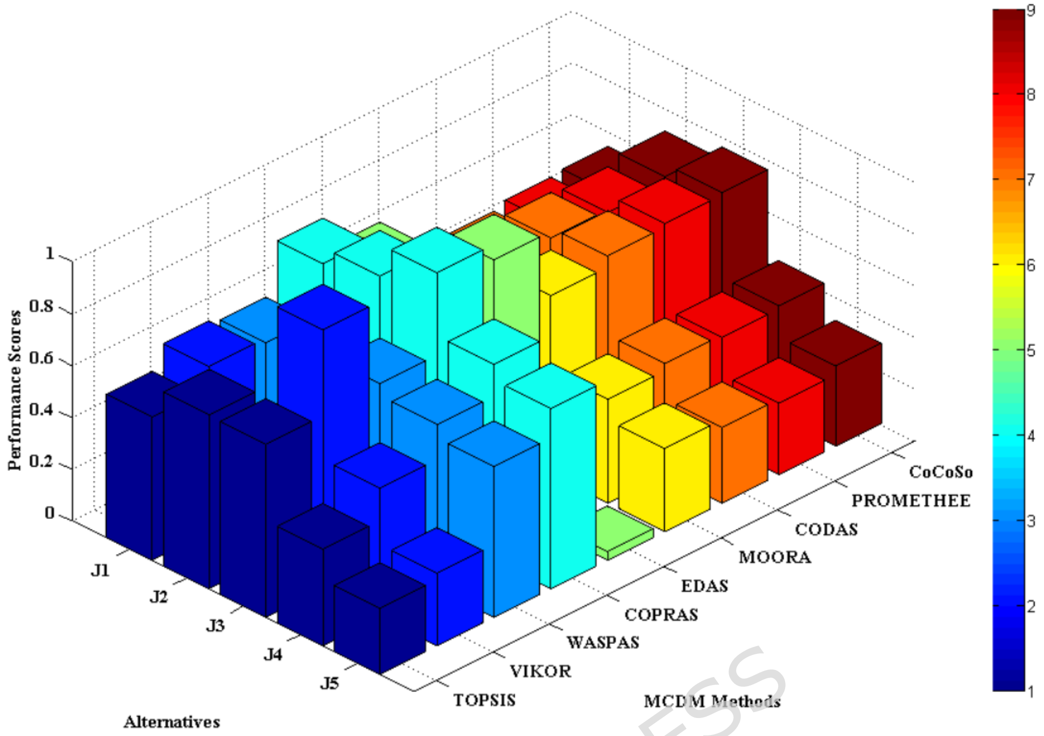
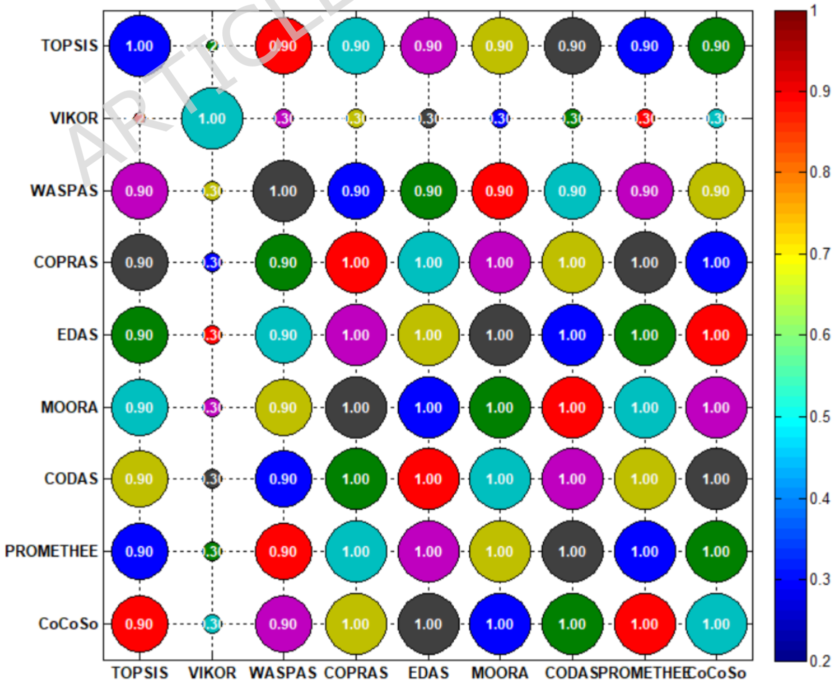


Figure 11: **Comparative ranking values of heating system alternatives across different MCDM methods**



	TOPSIS	VIKOR	WASPAS	COPRAS	EDAS	MOORA	CODAS	PROMETHEE	CoCoSo
TOPSIS	1.00	2	0.90	0.90	0.90	0.90	0.90	0.90	0.90
VIKOR	[illegible]	1.00	.3	.3	.3	.3	.3	.3	.3
WASPAS	0.90	.3	1.00	0.90	0.90	0.90	0.90	0.90	0.90
COPRAS	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00
EDAS	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00
MOORA	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00
CODAS	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00
PROMETHEE	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00
CoCoSo	0.90	.3	0.90	1.00	1.00	1.00	1.00	1.00	1.00

Figure 12: **Spearman rank correlation among multiple MCDM methods**

by changes in weighting, highlighting $\mathscr{G}_2$ and $\mathscr{G}_3$ as competitive alternatives under some circumstances. This validates the responsiveness and discriminating power of the proposed decision-making approach.

Table 13: **Sensitivity analysis results under different weighting scenarios**

Case	**Ranking Values**	**Ranking Order**	**Optimal Rank**
Case 1	0.7388, 0.6259, 0.6095, 0.1234, 0.1379	$\mathscr{G}_1 > \mathscr{G}_2 > \mathscr{G}_3 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 2	0.8523, 0.5479, 0.8334, 0.0850, 0.1640	$\mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 3	0.9122, 0.6311, 0.6885, 0.1012, 0.1335	$\mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 4	0.7564, 0.6933, 0.7121, 0.1356, 0.1842	$\mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_1 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_3$
Case 5	0.6234, 0.7042, 0.6420, 0.1420, 0.1287	$\mathscr{G}_2 > \mathscr{G}_3 > \mathscr{G}_1 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_2$
Case 6	0.8121, 0.6510, 0.7255, 0.1211, 0.1442	$\mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 7	0.8312, 0.5981, 0.8033, 0.0910, 0.1540	$\mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 8	0.6915, 0.7129, 0.6733, 0.1674, 0.1320	$\mathscr{G}_2 > \mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_4 > \mathscr{G}_5$	$\mathscr{G}_2$
Case 9	0.7792, 0.6231, 0.7645, 0.1512, 0.1388	$\mathscr{G}_1 > \mathscr{G}_3 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_1$
Case 10	0.7655, 0.6843, 0.7891, 0.1370, 0.1198	$\mathscr{G}_3 > \mathscr{G}_1 > \mathscr{G}_2 > \mathscr{G}_5 > \mathscr{G}_4$	$\mathscr{G}_3$

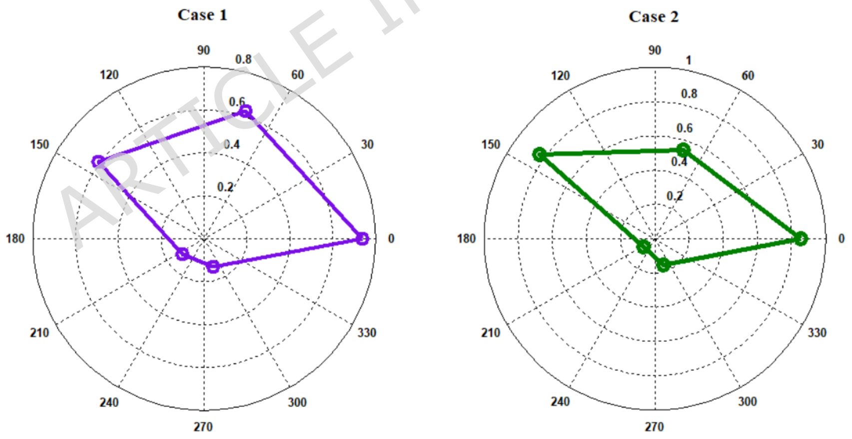


Figure 13: **The variation in ranking scores for sensitivity Cases 1 and 2**

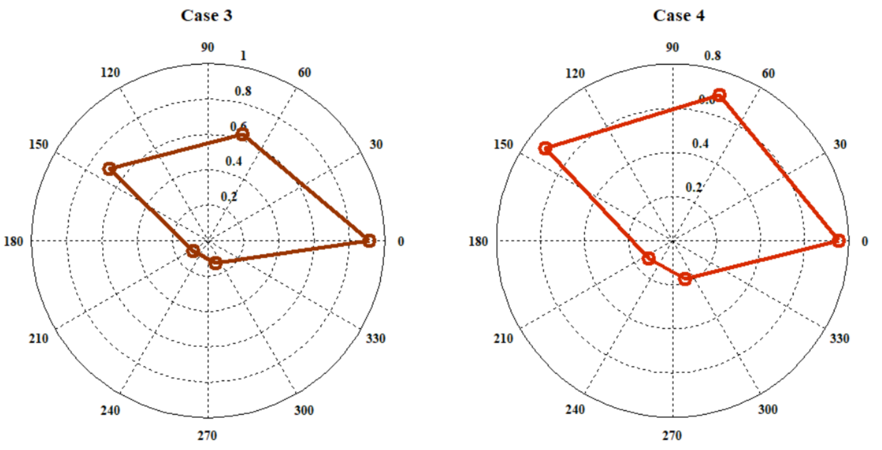


Figure 14: **The variation in ranking scores for sensitivity Cases 3 and 4**

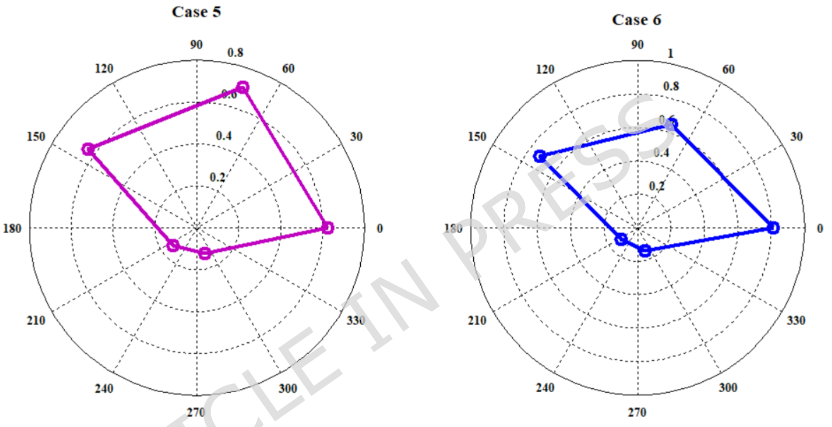


Figure 15: **The ranking scores for sensitivity Cases 5 and 6**

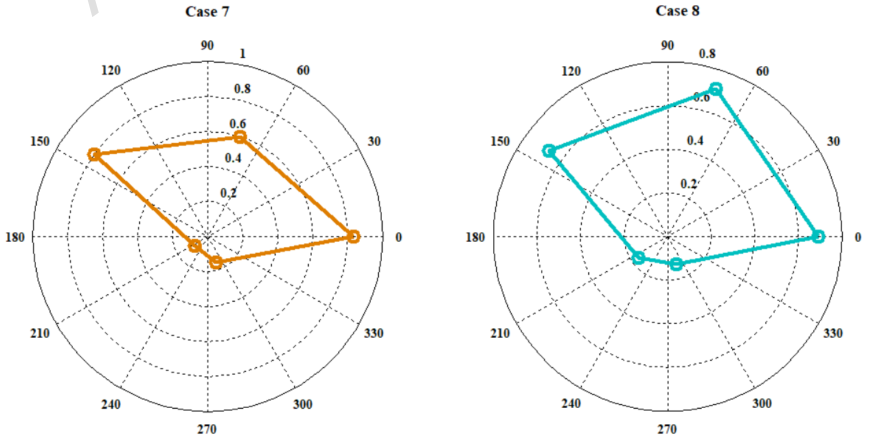


Figure 16: **The ranking scores for sensitivity Cases 7 and 8**

5.3 Practical Implications

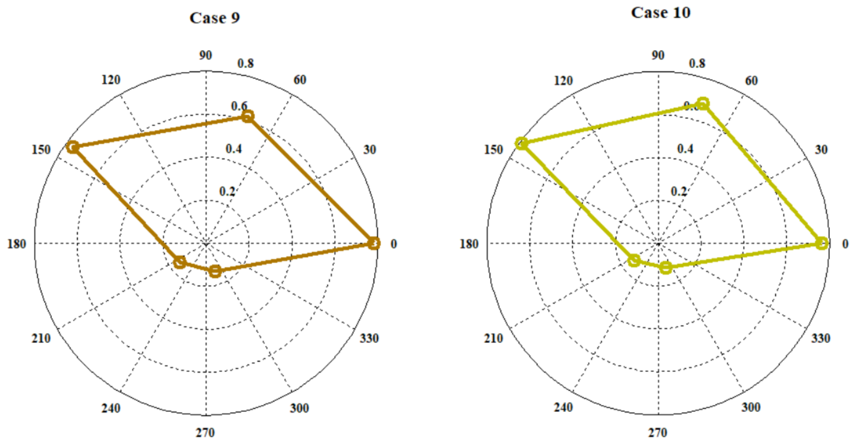


Figure 17: **The ranking scores for sensitivity Cases 9 and 10**

The proposed framework acts as a practical guideline for all the stakeholders involved in sustainable energy planning as well as the selection of the heating system. The framework is particularly important to the policymakers as it acts as a transparent decision support tool that helps in the selection of the heating system based on economic, environmental, as well as technical factors. This is important in the development of energy efficiency policies, the promotion of clean energy, as well as the provision of incentives for sustainable energy solutions. The framework is also important to the engineers as it helps in the evaluation of the heating system based on factors such as energy efficiency, cost, as well as environmental impact. The framework is important under conditions of uncertainty. The framework is important to the manufacturers as it helps in the identification of the strengths as well as weaknesses of the various technologies. This helps in improving the efficiency of the energy system as well as the environment. Therefore, the framework is important as it helps in the provision of a transparent decision support tool for the selection of the heating system for various applications.

6 Conclusion

This study proposes a MCDM approach, which integrates the Evaluation Based on Distance from Average Solution (EDAS) method with the Probabilistic Hesitant Fuzzy Sets (PHFS) concept, while the Analytic Hierarchy Process (AHP) is used to determine the relative importance of the evaluation criteria. The practical application of the proposed approach is demonstrated through the selection of the most suitable residential heating system, according to a variety of technical, financial, and environmental criteria. The results of the evaluation indicated that the heat pumps were the most appropriate option for the specific problem, which can be mainly justified by the high efficiency of the energy transfer, the low environmental effect, and the compatibility with the use of renewable energy sources, such as solar power. Although the use of heat pumps involves a high cost for the installation, the long lifespan of the system, as well as the low maintenance requirements, result in a significantly low cost of ownership.

The advantages of the heat pumps, which are justified for the specific problem, demonstrate the affordability, sustainability, and maturity of the technology, justifying the selection of the heat pumps as the most appropriate option from the available alternatives. The PHFS framework was shown to have a strong capacity for handling the reluctance and ambiguity of the expert evaluation, which results in a more credible and precise decision-making approach. The integration of the AHP with the PHFS framework, which involves a systematic and organized approach for computing the weights of the criteria according to the expert evaluation, enhances the credibility of the decision-making approach.

Future Research Directions

The proposed framework is an appropriate approach for making comparisons regarding heating system options when the conditions are uncertain. However, it is possible to take this approach to the next level by incorporating better computational tools. For example, machine learning and deep neural networks have been found to be promising tools for dealing with complex nonlinear relationships and dealing with complex data sets [19]. Deep learning has been found to be particularly promising for dealing with temporal patterns and complex interactions between features of data sets. For example, deep learning has been found to be particularly promising for dealing with complex data sets in fields like smart healthcare and speech recognition [20]. The future direction of research could be to develop intelligent decision frameworks based on tools like fuzzy multi-criteria decision making coupled with tools like deep learning, convolutional learning, or recurrent learning.

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Author contributions

KS: Conceptualization, Data curation, Visualization, Formal analysis, Methodology. **VM:** Conceptualization, Data curation, Formal analysis, Methodology, Writing—original draft. **RJ:** Conceptualization, Resources, Supervision, Formal analysis, Writing—original draft. **SN:** Investigation, Resources, Methodology, Writing - review and Editing. **NA:** Formal analysis, Methodology, Validation. **HD:** Resources, Validation, Visualization, Writing—review and editing. **SY:** Validation, Writing—review and editing, Funding acquisition. **JJ:** Conceptualization, Investigation, Resources, Formal analysis, Methodology, Writing and Editing.

Ethics declarations

Availability of data and materials

The study utilizes secondary data, which are publicly available and anonymized. The datasets analyzed during the current study are available from the corresponding author on reasonable request.

Ethics approval and consent to participate

This study used publicly available secondary data that did not involve human participants or patient-specific information. Therefore, approval from an institutional or licensing committee and informed consent from participants or their legal guardians were not required. All methods were conducted in accordance with relevant guidelines and regulations.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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