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Digital and physical re-creation of ancient Chinese bells: new understandings and discoveries

Kin Sum Li [1]✉

Previous research has focused primarily on replicating bells that were expected to resemble the original closely in terms of material, size, shape, and tone. There has, however, been no effort to use original bells as templates from which to re-create or re-design new bells that deviate significantly in size, yet maintain the shape of the originals in order to address issues in the history of art and in the archeology of ancient China. Experimentation with ancient bells in this field remains largely untapped. This article proposes to re-create bells in enlarged and reduced sizes through casting, based on 3D-printed resin models that have been correspondingly scaled up and down, from a 3D model scanned from a 500 BCE bell that was excavated from Xinzheng in Henan province, China. The study seeks to answer a series of questions, including whether replication of bells was practiced in ancient times, how casters could predict the tones a bell would produce before casting, and how a set of bells used as a musical ensemble could have been developed over history.

[1] Department of History, Hong Kong Baptist University (HKBU), Rm 1155, 15 Baptist University Road, Kowloon Tong, Hong Kong, People's Republic of China. ✉email: kinsumli@hkbu.edu.hk

Introduction and background

This article offers fresh perspectives on our understanding of bronze chime bells produced in China from approximately 1100 to 500 BCE by using digitally and physically re-created bells as research samples. While replicating ancient Chinese bells has been a common practice in conservation science (Hubei, 1981, 1983; Hu et al. 1981; Li, 2012; Ren, 2013; Wang, 2012, 2021; Zhang, 2016; Fang et al. 2022), using re-created bells in art historical and archeological studies, namely, modern scholars designing new bells with reference to old bells, remains relatively underdeveloped. This approach can help us understand how ancient bells were designed and cast, how the unique two-tone feature of a single bell was invented, and how an ensemble of chime bells was formed. As a result, this article focuses on how the re-creation of ancient artifacts benefits, in return, our research into the ancient artifacts themselves.

The re-creation of ancient artifacts can pioneer new methodologies in the field of digital humanities. This process currently involves, both digitally and physically, the re-creation of missing parts from damaged artifacts for restoration purposes, as well as the reproduction of entire artifacts for display and educational purposes (Hu et al. 2009; Zhang, 2016; Weng and Xin, 2016; Debut et al. 2016, 2018; Cekus et al. 2020; Carvalho et al. 2021; Parfenov et al. 2022; Fang et al. 2022). Artifact replication also constitutes a large-scale business sector since these newly created items can be sold as metalcraft items or even as forgeries masquerading as antiques (Zhang, 2016, p. 60). Given this rich background of development in digital humanities, the re-creation of ancient artifacts provides the immense potential to make significant contributions to historical studies, particularly in the non-textual realm. Many current digital humanities projects in historical studies focus primarily on using text-based approaches to extract historical patterns (Burdick et al. 2012, pp. 30–60, 122–123; Svensson et al. 2012). By contrast, the re-creation of ancient Chinese bells does not depend on textual sources. Since casters from the period under discussion in this article left no textual records, in order to explore the mindset of those ancient bell casters, there is a need to establish such new, non-text-based methodologies.

Ancient Chinese bronze chime bells possess many unique characteristics compared to bells that were produced around the rest of the world (Ma, 1981; Shen, 1987; Lehr, 1987, 1988, 2005). One of their most notable features is their almond-shaped cross-section, which is distinct from the round shape of other bells. This unique shape enables each bell to produce two distinct absolute pitches depending on where it is struck (Huang, 1978–1980; Chen and Zheng, 1980; Dai, 1980; Ma, 1981; Shen, 1987; Lehr, 1988; Falkenhausen, 1993, pp. 67–97; 2018, 2023). The almond shape also results in shorter sounds, making these bells suitable for ensemble performances, as opposed to the longer, monotone produced by round bells (Dai, 1980; Falkenhausen, 1993, pp. 67–97, 2023; Bagley, 2005, pp. 54–55). Bronze, being a non-perishable material, ensures the durability of these bells. This enduring nature of this material, if well-preserved, means that the sounds produced by these bells in the present day are likely to be similar to those produced by the same bells in ancient times (Bagley, 2000, p. 36). We can, therefore, use the bell sounds generated in the present day to study how the bells may have sounded in ancient times and how musicians of that era interpreted bell music.

In this article, we aim to answer the following series of questions regarding the design and production of ancient bells by using our newly re-created bells.

Question (Q)1: Was the replication of bells practiced in ancient times?

Q2: If the answer to Q1 is yes, were the replication methods used in ancient times similar to those used today?

Q3: If a bell was re-created, did its two tones remain unchanged, or if not, how were they changed?

Q4: How did casters know before casting what two tones a bell could generate?

Q5: How was the two-tone feature invented and developed?

Q6: How did a set of two-tone bells form a musical ensemble?

While many unsolvable mysteries remain in this historical field, addressing these six questions will provide valuable data for future research.

Modern practices of bell restoration and replication

Conservators employ extensive re-creation methods to reproduce or repair bells. For example, a team of conservators undertook the restoration of a broken bell from the church of S. Pedro de Coruche in Lisbon, Portugal, dating to around the 13th century CE, which was believed to be incapable of generating its original sound (Debut et al. 2016). Using 3D scanning technologies, this team processed the 3D-scanned model of the broken bell in professional 3D software and digitally re-created the missing portion of the bell. They then 3D-printed the missing portion in metal and affixed it to the broken bell, which enabled it to once again emit a clear tone. In China, similar restoration efforts have been made, albeit with greater complexity due to the two-tone feature of their ancient bells (Hu et al. 2009; Falkenhausen 2018, pp. 45–50; Fang et al. 2022). It is crucial that the shape and pitch of a bell after repair need to be as close as possible to those of the original bell. A team of conservators in the Hubei Provincial Museum, who repaired a group of large bells by soldering them, did successfully make them clearly produce their distinctive two tones. Another team of conservators at the same museum used 3D printing in their restoration process. They digitally modeled the shape and size of the missing sections and 3D-printed them in resin. Those resin models were used to create wax models, which in turn were used in a lost-wax casting method (Fang et al. 2022), with molds prepared from the wax models, into which molten bronze was poured. When solidified, the newly-cast sections were soldered onto the broken bell. These cases serve as classic examples of repairing bells as musical instruments.

Another prevalent practice is the reproduction of whole bells. The 3D scanning and printing technologies outlined above can be applied to the reproduction of entire bells, and other techniques and materials such as plaster and silicone have also been used (Hubei, 1981, p. 4; Hu et al. 1981). In historical and modern Europe, church chime bells can be replicated by using wooden strickles of pre-determined sizes and shapes in order to form the shapes and sizes of bell models (Lehr, 2005, pp. 12–18; O'Brien, 2021). Any model could subsequently be produced in exactly the same size and shape as the desired bell. While there may or may not be a pre-designed bell model for direct replication, this process can still be considered a form of replication.

In contemporary China, bell replication has evolved into a substantial industry that utilizes modern metallurgical knowledge and technology. This development largely stemmed from the discovery and replication of the chime bell set excavated from the famous tomb of Marquis Yi of Zeng (died circa 433 BCE) (Hubei, 1981, 1983; Hu et al. 1981; Zhang, 1985). This bell set comprises sixty-five bronze bells spanning more than three octaves, which are tuned to the chromatic scale. The knowledge at that time of the twelve semitones within an octave is demonstrated by all pitches that are provided by the bells and the then-existing musical theories that are inscribed on them. Modern musicians, recognizing the immense value of replicating this bell set, initiated the practice of their replication in the

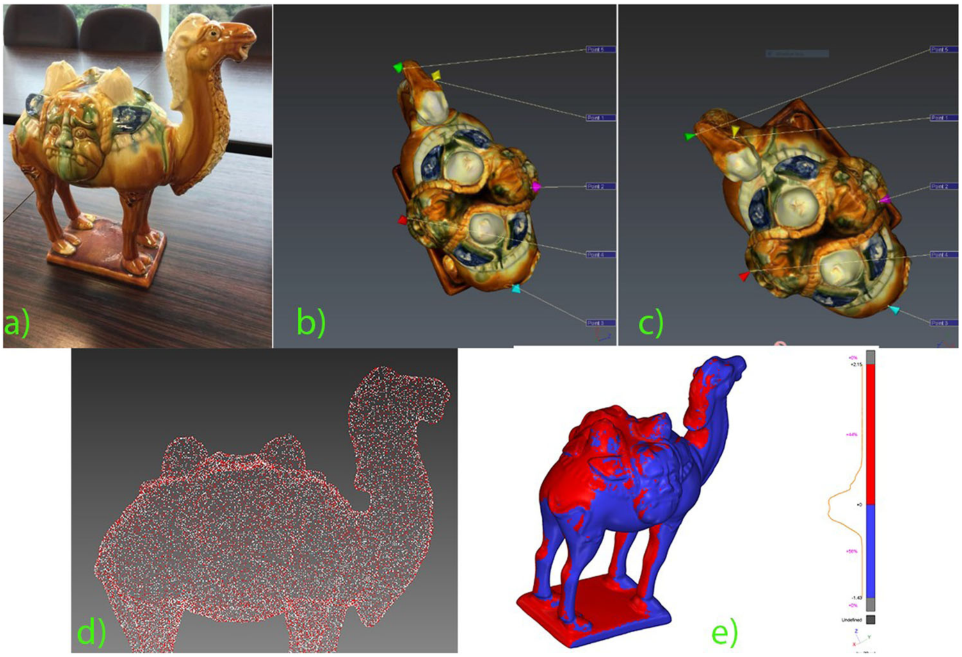

Fig. 1 Experiment to validate the accuracy and precision of our 3D scanning technology. a The modern camel sculpture we 3D-scanned. **b** The first 3D computer model generated. **c** The second 3D computer model was generated. **d** Point cloud of the two 3D models, when superimposed. **e** The two overlapped 3D models are shown in blue and red. Photograph and images by the author. This figure is not covered by the Creative Commons Attribution 4.0 International License. Reproduced with permission of Kin Sum Li; copyright © Kin Sum Li, all rights reserved.

1980s (Hubei, 1981; Wang, 2012, 2021, pp. 1–47; Li, 2012). Collaborative teams consisting of archeologists, musicians, acousticians, metallurgists, and traditional casters were assembled to take part in replication experiments. They employed various types of molding silicone to imitate the exact size and shape of the bells, conducted a comprehensive analysis of the metallurgical composition of the bell alloy, and prepared crucibles of molten bronze according to that alloy ratio. The newly-cast bells did, therefore, closely resemble the original bells in terms of size, shape, wall thickness, and metallurgical composition. These teams also acknowledged, however, that the newly-cast bells required fine-tuning in the post-processing process to ensure that the two tones of each bell would closely match those of the original (Wang, 2012, pp. 48–49). Numerous further bell sets, as well as the Zeng Hou Yi bells, have since been replicated, with the factories undertaking these replication experiments amassing a wealth of experience (Zheng, 1998; Hua, 2004; Li, 2012; Ren, 2013; Wang, 2012, 2021, pp. 1–47, 101–231, 356–385).

Our digital and physical re-creation methods

Our re-creation methods have utilized both digital and physical aspects. The first sample set we selected is from bronze bells excavated from multiple sacrificial pits at the site of the Construction Bank of China in Xinzheng county, Zhengzhou city, Henan, which date to about 500 BCE (hereafter "the Xinzheng bells") (Henan, 2006, vol 1, pp. 311–327; Wang, 2006; Falkenhausen, 2009, 2018, 2023, pp. 230–31). We 3D-scanned some of the Xinzheng bells and generated their corresponding 3D models using professional software. The white-light 3D scanner used was the Artec Eva scanner, which provides an accuracy of up to 0.1 mm. Our 3D models were generated in both formats of mesh and point cloud. In order to validate the accuracy and precision of these 3D models, we scanned a modern camel sculpture twice and superimposed the two 3D models together for comparison (Fig. 1). Using the professional software provided by the scanner company, Artec Studio 11, we demonstrated an ideal overlap between the two 3D camel models. This example implies that each 3D model generated for our subsequent comparison experiments is sufficiently accurate and precise for our study purposes.

In the subsequent simulation experiments, we used the digital 3D computer models of the Xinzheng bells to create 3D-printed resin models and sent these resin models to a foundry for casting. The foundry workers used the resin models as the equivalent of wax models in their lost-wax casting process. As such, they would encase a resin model in soft, wet clay and allow the clay to dry to form the clay mold. The mold was then heated to a temperature at which the resin softened and was drained out, leaving a cavity in the shape of the original bell. The molten metal that was then poured into the mold upon solidification became a new metal bell corresponding to the original bell, referred to as a "*surmoulage*" (Li, 2017, p. 262). They also used the indirect lost-wax casting technique to replicate the resin models. In this way, they could replicate as many as they desired, provided that they retained the original resin model (Peng, 2023).

Due to budget and local environmental issues, brass, an alloy composed of 79–83% copper, 7–9% zinc, and 10–12% tin, was used to cast these new bells, although ancient Chinese casters did not use brass until the tenth to fifteenth century CE. We did not use the same metallurgical composition as the original bronze bells, which contained 73–84.6% copper, 8.5–12.5% tin, and 6.9–11.1% lead (Huang and Li, 2006).[1] These ratios, as tested from the original bells, may have varied slightly, within an acceptable range, due to factors such as corrosion and sampling position. One factor that we initially overlooked was the approximate 1.05% contraction rate of the newly cast bells. In consequence, the new bells are slightly smaller than the originals. For example, for the original bell with a side length of 20 centimeters (cm), its corresponding new counterpart measures approximately 19.8 cm, and the counterpart of a 40 cm original bell measures 39.5 cm. Although metal contraction is inevitable in making *surmoulages*, these were mistakes that we must acknowledge and rectify in future experiments. However, at this stage, we have found that the different alloys and the contraction rate did not result in significant discrepancies in their tones, a point which will be elaborated upon in the following sections.

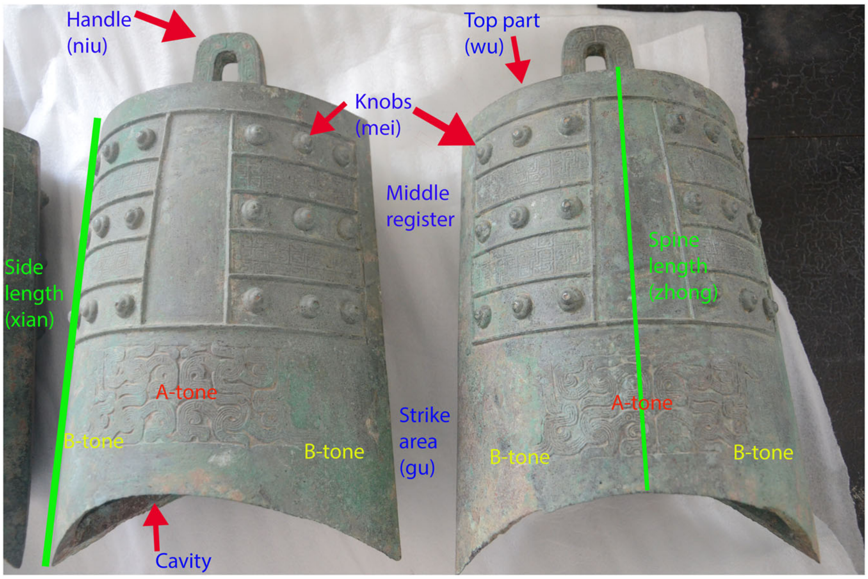

Fig. 2 Nomenclature of ancient Chinese bells. The two shown here are bells Pit16–G1x and –G1y from Xinzheng, Henan. Photograph by the author. After Li et al. (2024a, p. 4 of 14, Fig. 2). This figure is covered by the Creative Commons Attribution 4.0 International License. Reproduced with permission of Kin Sum Li; copyright © Kin Sum Li, all rights reserved.

Since we aimed to imitate only the design and production process of the original bells, in order to retain the raw, original two tones of each new bell, we did not fine-tune these newly cast bells.

After casting the new bells, we used a wooden mallet to strike each one in order to generate its two tones. Each bell has two A-tone and four B-tone positions (Fig. 2), and we struck every position three times, recording the tones using the professional software "Audacity".[2] Only the fundamental tone of each vibration was documented, with overtones ignored since they were probably not the sounds sought by ancient musicians. These fundamental tones were then converted from hertz to musical notes and recorded in the international scientific notation format (A4 = 440 Hz). In our analyses, we use the musical notes of the two tones of each bell for comparison with other bells. The same testing method was used to record and analyze all of the ancient and newly cast bells.

We established constants and variables for our experiments. The constants were factors that mostly remained unchanged, including but not limited to the force and gesture used to strike the bells. The shape of the bell was kept proportional. The variables referred to factors leading to changes in the bell tones. For example, we altered the size and mass of the bells to observe how they correspondingly impacted their tones.

Replication phenomenon in 500 BCE

To address the first question (Q1) posed above, we did indeed find evidence of bell replication around 500 BCE in Xinzheng (Fig. 2), as demonstrated by our superimposed 3D models (Fig. 3). This is the first time that the practice of bell replication has been seen in archeological records of ancient China (Li et al. 2021, p. 141). Figure 3 shows the result of superimposing 3D models of two bells discovered in Pit 16 in Xinzheng, specifically bells no. G2x and G2y (originally numbered A2 and B2; see also Table 1). It is evident that the overall size, shape, and wall thickness of the two overlapped bells align perfectly. Table 1 also reveals that the measurement data of their overall size and shape are strikingly similar. Their two tones are nearly the same, as both bells' A-tones are D5, while their B-tones are E5 and F5. Given that E5 and F5 are adjacent semitones and considering the technological level of the era, we can conclude that their two tones are almost the same, indicating that they are a pair of bells that were successfully replicated in around 500 BCE. Figure 3 does not show any protrusions, which suggests that their original model was not substantially different. In other words, they originated from the same rough clay model, and their clay molds are highly similar (Daxi (Shanxi), p. 86).

The reason for stating that they could not have shared the same mold in ancient times is that the clay molds would not have been re-usable having been smashed open to retrieve the bell after the molten bronze solidified (Bagley, 2009). They were, moreover, cast using the section-mold method rather than the lost-wax method. This difference resulted in mold marks on the bells cast by the section-mold method, whereas bells cast by the lost-wax method did not bear any mold marks (Bagley, 2009; Peng, 2023, p. 9). Therefore, while their molds might have appeared to be highly similar, they were not the same actual mold. Casters could further decorate their molds by adding patterns or altering minor parts, such as the patterns on their handles, and as a result, minor parts such as handles may have differed from each other. However, the bells' overall size, shape, and wall thickness remained identical.

From these observations, it is clear that ancient casters did not use the lost-wax method that is commonly used today (Q2). Instead, they used the section-mold method, in which the mold was sliced into sections and re-assembled when the model was removed. They also decorated directly onto the mold. Their final, cast bells would exhibit slight differences from the original model. They merely replicated the general size, shape, cavity, and wall thickness of the original bells. These factors are crucial to the tonal similarities between the original and the *surmoulage* bells. On the other hand, the patterns and minor parts, which did not need to be completely replicated, are not significant factors in the tones that the bells generate.

Our reproduction in 2023 CE

Variable 1: Material. A further question arose above as to whether the two tones of a replicated, *surmoulage* bell would be the same as those of the original bell (Q3). In order to address this question, we selected the Xinzheng Pit16–G2x bell to use as the prototype from which to cast a series of new bells (Fig. 4, Table 2).

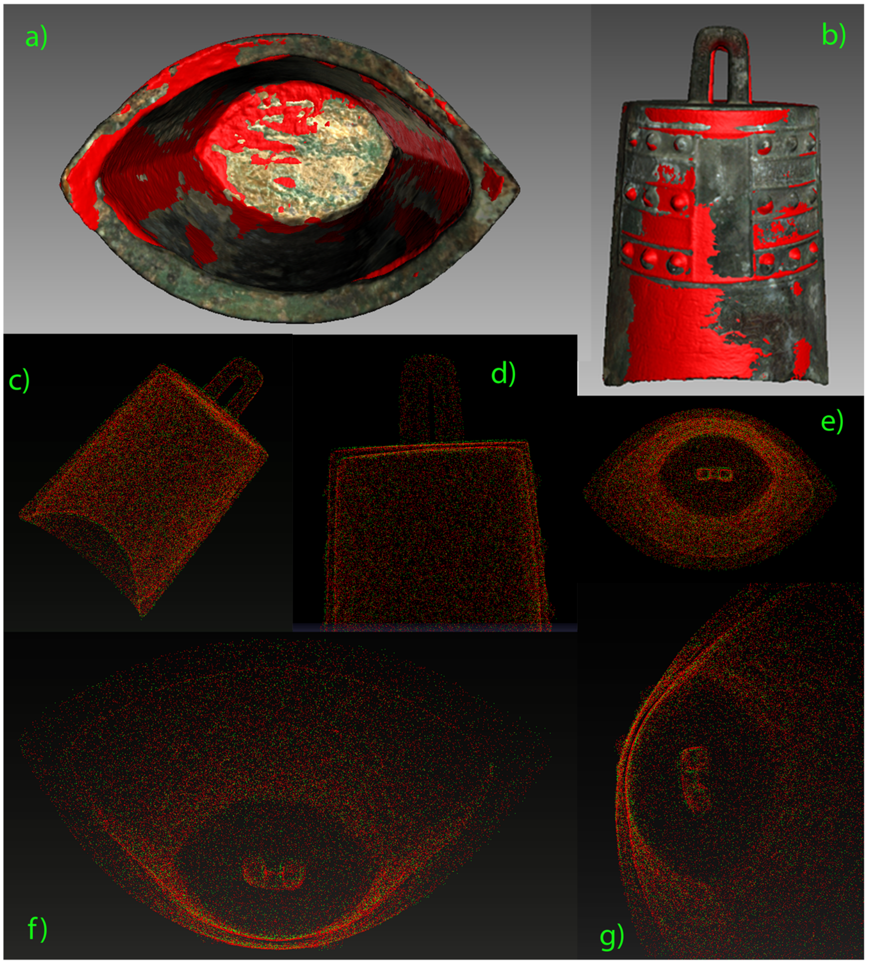

Fig. 3 3D models of bells Pit16–G2x and –G2y, when superimposed. a View of the textured models of the two bells' mouths. **b** Two textured models overlapped. **c**–**g** Different views of the point clouds of the two superimposed 3D models. Images by the author. After Li et al. (2024a, p. 11 of 14, Fig. 9). This figure is covered by the Creative Commons Attribution 4.0 International License. Reproduced with permission of Kin Sum Li; copyright © Kin Sum Li, all rights reserved.

We cast sixteen bells in total, and divided them into four distinct types (A–D), with each type consisting of four bells. We will discuss the Type A bells first.

While other constants were maintained unchanged to ensure that the size, shape, and wall thickness of the new bells would be the same as those of the original bell (Type A in Table 2, intended side length 20 cm), the casting material was set as Variable 1. Conforming to the same proportion, we cast four brass bells for Type A based on the same resin model. The contraction rate may vary slightly in actual casting exercises due to a variety of factors. However, the dimension measurement results shown in Table 2 display that our newly cast brass bells are remarkably similar in size to the prototype bell.

The pitch testing results are documented in Table 3. The tones of these brass bells are notably clear and consistent. Readers are reminded that we struck each bell at least 18 times, with 3 strikes at each of the 6 A- and B-tone positions, and occasionally more. For Type A bells, usually, only one frequency was recorded for each of the two tones, which implies that the Type A bells were very well cast. The A-tones of all four brass bells span from F5 to F#5 (712–749 Hz), while the B-tones range from G#5 to A5 (840–886 Hz). Considering that we did not fine-tune the brass bells and that their tonal variances are confined to only neighboring semitones, it can be inferred that all four bells yield the same two tones. With the two tones of the original bell being D5 and E5 (Table 1), the two tones of each of the newly-cast Type A bells are merely elevated by one minor or major third. Owing to the consistent interval of the two tones of the new bells, we can infer that the tonal disparities are attributable to the different materials used, namely, the substitution of lead with zinc. If ancient casters had used the same alloy as in this study, it is highly plausible that they could have reproduced bells with strikingly similar two tones by replicating a bell's size and shape.

If ancient casters started to work from a completely new model, it is highly plausible that they did not know what two tones a bell could generate before it was cast (Q4). Even today, we still do not clearly understand how to cast a bell from scratch with precise control over its two tones. Due to the tremendous effort required in calculation and experimentation, modern acousticians and engineers themselves find this task challenging. As far as the author is aware, to date no one has successfully achieved this feat. Modern European foundry workers, leveraging at least three to five centuries of experience, of which the history and tools of the Whitechapel church bell foundry provide evidence, can cast a round bell with one specific, desired tone (O'Brien, 2021). However, their technical expertise is limited to single-tone bells and they lack the knowledge and technology necessary for casting a two-tone bell. Similarly, modern Chinese foundry workers are unable to work successfully from a blank slate and can only cast a bell with two desired tones because they can replicate an existing

Table 1 Measurement data of bells Pit16-G2x and -G2y.

Name	Pit16-G2 x and y	
A-tone musical note (Hz)	D5-43 (573)	D5-23 (578)
B-tone (Hz)	E5 + 31 (671)	F5 + 1 (699)
Original no.	16A2	16B2
Height of the handle	4.8	4.8
Side length (*xianchang*)	20	Same as bell x
Spine length (*zhongchang*)	17	-
Length and width of the mouth (*xianjian, gujian*)	16, 11.5	-
Length and width of the top section (*wuxiu, wuguang*)	13, 9	-
Girth of lip-mid-hip	35.5-38.7-41	35.2-39-41
Side length (cavity)	18.5	18.5
Spine length (cavity)	16	16
Xianjian, gujian (cavity)	13.3, 9.2	13.5, 9.5

Data measured and tested by the author in July 2022 in the storehouse of the Henan Provincial Institute of Cultural Heritage and Archeology. After Li et al. (2024a, p. 6 of 14, Table 1). Temperature of the tone testing location: 36 °C
The "lip" in the lip-mid-hip section refers to the girth of the bell mouth, "mid" refers to the girth of the section below the top two rows of the knobs, and "hip" the girth of the closed end of the sounding body of the bell. A4 = 440 Hz. Measurements are all in cm unless indicated otherwise. Only the fundamental frequency of vibration is recorded; overtones are not deemed the intended tones of the bells. During the test, the bell was hung on a bar or lifted by the author with its handle on top. The author first used a wooden mallet to strike the A-tone position for one side of the bell 3 times, then 3 times on the other side. The author then turned to the four B-tone positions and struck every position 3 times, or more than 3 if the tone was not clear. The internal recording device of a laptop computer (Samsung Galaxy Book Pro) and the audial software Audacity were used to record and analyze the sounds generated by the bells.

bell with its two already-known tones. When ancient casters intended to cast a bell with two desired tones, they would, very probably, have first referred to a bell with two known tones and reproduced them by replicating that bell.

Variable 2: Size. We know that the almond-shape cross-section is a crucial element in creating a bell's unique vibration, enabling it to generate two different pitches (Chen and Zheng, 1980; Dai, 1980; Ma, 1981; Shen, 1987). The almond-shaped cross-section design has a long history. Some pottery and bronze bells appearing before or around 1500 BCE, predominantly discovered in North China, were designed with this almond shape (Daxi (Henan), pp. 45–48; Bagley, 2000, p. 46). However, their acoustic properties remain largely unknown, due either to a lack of testing or to extensive corrosion that prevents the generation of sound of an acceptable quality. When continuing the search for almond-shaped bells of later eras, there are some dating to approximately the twelfth century BCE that are known, having been found in the Yangzi River areas of South China. These bells are notably larger and heavier than those found in the north (Bagley, 2000, pp. 46–48). More of these bells have been tested. In this era, these bells' tones were clearer and more consistent, and the interval between a bell's two tones could range from a major second to a major third, with occasionally no detectible difference between the two tones (Ma, 1981, p. 134). From 1000 BCE onward, a more consistent minor third or major third interval featured between a bell's two tones. This interval later became a norm in the bell-casting industry for subsequent generations, whose bells also retained their almond shape (Ma, 1981, p. 134). It is impossible to rule out the ability of bells from 1500 to 1000 BCE to have produced two different pitches as, although their consistency is not on a par with those made after 1000 BCE, this characteristic has indeed been observed in them. We can acknowledge that the two-tone feature was invented and widely implemented during this era and that after 1000 BCE it became standardized (Q5).

How musicians and casters used the two-tone feature of each bell to develop music forms another query. While we know that after 1000 BCE, they were able to form a chime bell ensemble, it was not entirely clear whether they had already done so before then, namely in the period from 1500 to 1000 BCE (Q6). In order to understand the technical origins of chime bell ensembles, we need to explore the methods used to manipulate changes in the bell tones. While the exact process of how the first two-tone bells were invented and realized by musicians and casters remains unknown, experiments can be conducted with existing bells to tackle this issue.

In our experiments we focus on exploring how the two tones of a bell change as we adjust its size proportion (Falkenhausen and Rossing, 1995; Asahara, 2000). Since adjusting the shape of a bell would introduce too many variables, its size and mass are set as Variable 2 while keeping other factors, such as bell shape, constant. Using professional computer software, we adjusted the bell size and proportion according to the ratios set in Table 2 (see also Fig. 4a). For instance, we first created the Type B bells. By enlarging the size of the Xinzheng Pit16–G2x bell while maintaining its shape resulted in, for example, the original 20 cm side length of the prototype bell being enlarged to 40 cm. All other parts, including the spine length and wall thickness, were enlarged proportionately, through the use of a simple automated software command. Using the same method, we increased the size slightly to create the Type D bells, changing the side length, for example, from 20 to 30 cm, whilst also proportionately enlarging the other parts. In a similar vein, we experimented with shrinking the size of the prototype bell to create the Type C bells (i.e., side length from 20 to 10 cm). While the bell walls became very thin at that size, the foundry workers did manage to cast from this model. As the size of the bells changed, their mass also changed correspondingly. Since we continued to use the same alloy ratio, the changed mass can also be considered as a form of change in size. We then had the three types of bells, Types B–D, 3D-printed in resin (Fig. 4b), and the foundry workers used their own techniques to cast bells with each type of resin model (Fig. 4c–f). The tones of these twelve bells were then tested, with the results documented in Table 3 (cf. Fig. 4d).

Table 2 demonstrates that the sizes of the actual cast bells closely match the intended dimensions for each type. Similarly, Table 3 shows that the tones produced by these bells are very clear and consistent too. However, it is occasionally hard to control the tones of extremely large or small bells, as tested on the Type B bells. The A-tones of the Type B bells exhibit minimal variation, whereas their B-tones appear to be relatively diverse.

To further analyze the relationship between bell size and tones, we selected one bell from each type and compared their tones in Tables 4 and 5, using the tones of the Type A starter bells for a more lucid comparison. To facilitate readers' understanding of the chromatic scale, the twelve semitones from C to B are listed in the bottom row of Table 5, with the numbers of the octaves listed in the left-hand column. Since these sixteen newly cast bells are not fine-tuned, whereas in ancient China fine-tuning was common in order to adjust final bell pitches to accord with musicians' requirements, variations or ranges in the tones should be allowed for in the pitches of our new bells. In Table 4 the accepted variation rate has been set as a major second. With this in mind, the A-tone of bell A4 (a musical note of F#5) should, therefore, be interpreted as a range of musical notes deviating from F#5. In ancient times, an experienced caster would have been able to fine-tune the A-tone of the bell, and of course, simultaneously its B-tone, to accommodate musicians' demand for a set of harmonious instruments.

It can be seen that the relationship between bell size and tones has now become clearer (Table 5). If we proportionally enlarge the size of the bell to a certain degree (Type B), the two tones

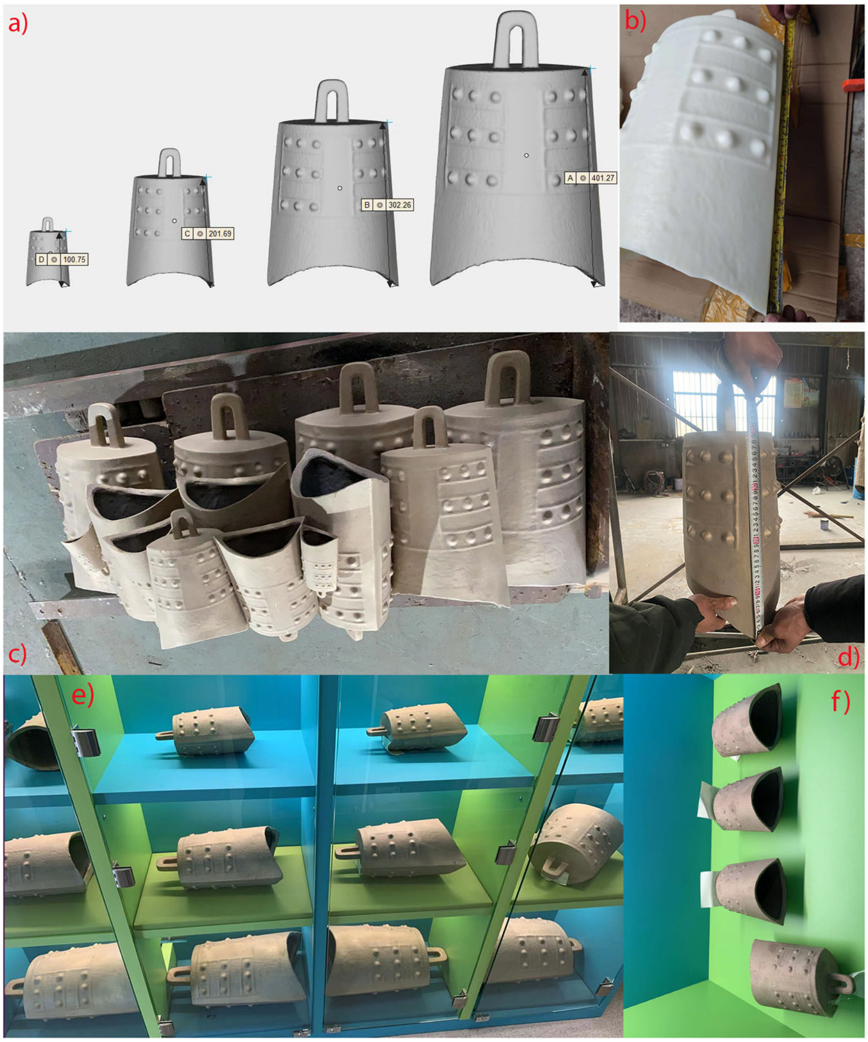

Fig. 4 Brass bells cast by the author's team. a The four different 3D computer models created based on the original Xinzheng Pit16–G2x bell with a side length of 20 cm. The side lengths of the other 3D computer bell models, as specified in the sub-image, are 10 cm (100 mm), 30 cm, and 40 cm respectively. **b** One of the 3D-printed resin models (Type B, side length 40 cm) based on the corresponding 3D computer model. **c** Some of the newly cast brass bells. **d** One of the largest brass bells (Type B, intended side length 40 cm, actual side length 39.6 cm), hung on a frame and ready for testing. **e** Type A, B, D brass bells stored on a wooden shelf (in total 12). **f** Type C bells (in total 4). Images and photographs by Chen Xueqing and the author.

drop by an octave, shifting from F#5 and A5 to F#4 and A4. Conversely, if we proportionally reduce the bell size to a certain degree (Type C), the two tones rise by an octave, reaching G6 and A#6. Fine-tuning would make these tones neater and closer. The two tones of the Type D bells would sit midway between those of the Type B and Type A bells.

This may seem at first glance to align with the principles of Pythagorean arithmetic, which was probably not, however, a viable method for casters in the twelfth century BCE to have been able to discover or realize the chromatic scale (Bagley, 2005, pp. 86–88). According to Pythagorean tuning theory, the vibration frequency, or the pitch, of a string will be an octave higher when plucked in comparison to a string double its length and an octave lower when compared to a string half its length (Adkins, 2001, p. 3). When the string length is shortened according to other certain ratios, such as 2/3 and 3/4 or..., intervals of a perfect fifth and a perfect fourth are obtained, respectively. The Pythagorean method of calculating intervals does, however, fall short when attempting to calculate all the equal semitones of the chromatic scale, since the calculation of other equal semitones becomes significantly more complex (seeking the twelfth root of two in the equal-semitone chromatic scale) (Kuttner, 1975; Adkins, 2001; Cho, 2003). In the context of bell-casting, numerous uncertainties and variables would have surfaced, which would have made the calculation of appropriate bell sizes even more challenging. This remains one of the reasons why modern acousticians and engineers still struggle to precisely manipulate bell size and tones.[3]

The experiments we have undertaken shed light on the initial relationship between bell size and tones. They have shown that enlarging and reducing the size of the starter bell to a certain degree roughly lowers or raises the tones, respectively, by one octave. Simple arithmetic does not, however, hold true for bells of sizes falling between the largest and smallest bells, and casters and musicians would have had to use different methods to create bells in order to attain the other semitones in this range.

Bell casting and the multiple discoveries of the chromatic scale. Achieving desired tones by proportionately adjusting the size

Table 2 Dimension of the 16 brass bells cast by the author's team in 2023 CE.

Bell type	Side length (cm)	Spine length (cm)	Length and width of the mouth (cm)	Length and width of the top section (cm)
Prototype (the Xinzheng Pit16-G2x bell)	20	17	16, 11.5	13, 9
Intended dimension of Type A bells	20	17	16, 11.5	13, 9
A1 (actual bell)	19.8	17	15.9, 11.2	13.1, 9
A2	19.9	16.9	16, 11.3	13.2, 9.1
A3	19.6	16.9	16, 11.3	13.2, 9.2
A4	19.6	17.1	16, 11	13.2, 9.4
Intended dimension of Type B bells	40	34	32, 23	26, 18
B1	39.5	33.9	32, 22.5	26, 18.2
B2	39.4	34	32, 22.3	26.3, 18.2
B3	39.5	33.7	32, 22.5	26.3, 18.5
B4	39.6	34	31.9, 22.3	26, 18.3
Intended dimension of Type C bells	10	8.5	8, 5.75	6.5, 4.5
C1	9.9	8.5	7.9, 5.5	6.4, 4.5
C2	9.9	8.5	8, 5.5	6.4, 4.5
C3	9.9	8.4	7.9, 5.5	6.5, 4.6
C4	10	8.5	7.8, 5.6	6.6, 4.6
Intended dimension of Type D bells	30	25.5	24, 17.25	19.5, 13.5
D1	29.4	25.5	23.9, 16.9	19.5, 14
D2	29.5	25.5	23.8, 16.8	19.6, 14
D3	29.8	25.5	23.8, 16.8	19.8, 14.1
D4	29.6	25.6	23.9, 16.7	19.8, 14

Table 3 Pitch test reports of the 16 brass bells.

Bell	A-tone (Hz)	B-tone (Hz)
A1	712	854
A2	714	840
A3	725; 722, 718	843
A4	749	886
B1	368; 416, 408	428; 433, 425, 423
B2	374; 377, 371	440; 436, 430, 428
B3	373; 377, 370	442; 435, 425, 407, 385
B4	370; 367	439; 436, 432, 429
C1	1602; 1599, 1597	1860
C2	1550	1831
C3	1568	1822
C4	1615	1873
D1	489; 486, 482	574
D2	486; 478	575
D3	490; 483	576
D4	488	573; 569, 564

Conducted on April 18, 2023, from 10:00 to 13:00. *Temperature*: 25 °C; *humidity*: 70–80%. *Assistants*: Yating Liao, Xiaobang Ling, Rongzhen Fu. *Procedure*: one individual grasped a bell in their hands, and another individual struck the designated positions with a wooden mallet. The process involved striking one A-tone position 3 times, then 3 times on the other A-tone position. As a B-tone was harder to generate, each of the four B-tone positions was then struck 3 or more times. If there is only one figure in a grid, this indicates that only one frequency was consistently recorded from all relevant positions. If there is more than one frequency recorded, the first frequency in that grid, before the semicolon, represents the clearest pitch that most frequently occurred. Only frequencies that differ by 3 Hz or more from the clearest pitch will be recorded. For example, a frequency of 369 Hz for the A-tone of brass bell B4 would not be recorded.

of the starter bell presents a straightforward method by which to produce instruments that span two octaves. It was quite possible that ancient casters and musicians were aware of this technique, which would have been possible for them to achieve if they had conducted experiments based on this method. This would, however, only have enabled them to obtain the initial two tones, along with their corresponding two tones an octave higher, and another set of two tones an octave lower. They would still have had to grapple with the pitches within these two octaves. Nevertheless, this method would have enabled them to establish the starter bell, a double-sized bell, and a half-sized bell, thus providing them with a foundation for further exploration.

The discovery of the chromatic scale may have been achieved through a combination of hunting bells with compatible tones, experimentation, and transposition. As Robert Bagley argues, ancient bell musicians may have sought out bells with special tones that were unfamiliar to them but could be integrated into their existing list of tones within an octave (Bagley, 2000, p. 49). Musicians may have added these bells into their ensemble, either as a backup or for future musical experiments. The pursuit of more accurate and precise methods of transposition could also have prompted them to discover more semitones. Bagley posits that musicians in the Ningxiang area in the twelfth century BCE had already realized the chromatic scale since at least seven of the twelve semitones can be found on the ten Ningxiang bells (Bagley, 2005, pp. 79–85). A recent pitch test conducted by the author of this article further confirmed the existence of at least nine to ten consecutive semitones on the Ningxiang bells (Li et al. 2024b, p. 12). This evidence is not likely to be coincidental, but rather bearing testament to the achievement of the chromatic scale by the Hunan casters and musicians of the twelfth century BCE. Considering the maturity of the techniques, the process of attaining the chromatic scale might, in fact, have occurred even earlier than the twelfth century BCE.

Bell casters could well have experimented with the changing relationship between bell size and tones while maintaining the same bell shape. They would have discovered that by slightly altering the size of a bell, its two tones would correspondingly change. For example, if casters had scraped a small quantity of clay from the clay model of the double-sized bell, and used this modified model to cast a new bell, they could have achieved two higher tones. Alternatively, they could have started from the half-sized bell model and gradually, little by little, enlarged subsequent models. They could also have incrementally enlarged or reduced the size of the starter bell. Any of these three ways would have been effective, and the sequence in which they were applied would not have been important. The critical matter would have been to obtain the tones that would meet the needs of an ensemble that had adopted the chromatic scale, and a process of

Table 4 Pitches of selected bells from the four types.

Bell type (actual bell)	Type A, 20 cm (A4)	Type B, 40 cm (B3)	Type C, 10 cm (C2)	Type D, 30 cm (D3)
A-tone (Hz)	F#5 + 21 (749)	F#4 + 14 (373)	G6-20 (1550)	B4-14 (490)
B-tone (Hz)	A5 + 12 (886)	A4 + 8 (442)	A#6-32 (1831)	D5-34 (576)

Table 5 Pitch distribution of the four types of newly cast bells.

6th								G6-20			$A\#6_{-32}$	
5th			$D5_{-34}$				$F\#5_{+21}$			$A5_{+12}$		
4th							$F\#4_{+14}$			$A4_{+8}$		$B4_{-14}$
Octave Pitch	C	C#	D	D#	E	F	F#	G	G#	A	A#	B

The bottom row shows the twelve semitones of the chromatic scale. The pitches of the selected, newly cast bells are shown in their corresponding grids.

experimentation and adjustment would have been key to musical development.

This hypothesis can be corroborated by the evidence presented in Table 6 of bells found in the Middle Yangzi River area that share highly similar patterns and shapes. Despite having been discovered in various locations across the provinces of Hunan and Jiangxi, their strikingly similar decorative patterns indicate that it is highly possible for these bells to have been produced by the same workshop (Fig. 5). Given the geographical locations of the discoveries and the widespread availability of water transportation throughout the region, it is plausible that the bells could have been easily transported between these different locations. As shown in Table 6, even more compelling reasons supporting this notion can be derived from the size and tonal relationships between these bells.

Table 6 links bells based on several criteria, including similar decorative patterns and shape (crucial typological markers), and nearly sequential two tones and size. Starting with the largest bell producing the lowest tones—the Hunan-Ningxiang-Yueshanpu bell, also known as "the *nao* king" and the largest *nao*-bell found until the present day—we see a series of bells gradually decreasing in size and producing progressively higher two tones. We must acknowledge the many missing links, as numerous bells with similar patterns have not had their two tones tested or documented. Many bells may no longer exist, having been melted down for re-casting, destroyed over time, or not having been located. Despite these gaps, Table 6 demonstrates the pattern of increasing two tones correlating with decreasing bell size. A comparison between the Hunan-Ningxiang-Shiguzhai-elephant1 bell and the Hunan-Zhuzhou-collect bell demonstrates that the two tones elevate from G#3 and A#3 to G#3 and B3, while the dimensions (side length, spine length, *xianjian* and *gujian*, *wuxiu* and *wuguang*) decrease. This suggests that the casters of the two bells were attempting to manipulate the size and tones, as by slightly reducing the size of the Zhuzhou bell, they could minutely raise its two tones. Similar hypotheses can be made when comparing the Hunan-Ningxiang-Chenjiawan bell, BPM-Wenwuju-transfer bell, ShanghaiM-collect-19976 bell, and many subsequent bells. Exceptions such as the Hunan-Ningxiang-Shiguzhai-Feng2 bell are few in number. Such anomalies are to be expected, given the then-experimental nature of bell casting and tone control and the levels of technology available during the ancient period, and taking into consideration that bell tones are, in any case, subject to a certain degree of variation. It can, however, be confidently inferred that, from the Hunan-Ningxiang-Chenjiawan bell to the Tianjin-BiKui-collect bell, ancient bell casters were experimenting with size adjustments to achieve their desired tones.

Evidence also suggests that ancient casters experimented with halving a bell size to achieve tones roughly an octave higher, or, conversely, doubling the size to achieve tones approximately an octave lower. In this case the former scenario seems more probable. This hypothesis is based on a comparison between the *nao* king (the Hunan-Ningxiang-Yueshanpu bell) and three other bells listed in Table 7. When the size of the *nao* king was halved, the casters created three bells. The two tones of these half-sized bells, C4 and D4, D4 and E4, and D#4 and F4, are almost an octave higher than the two tones, C3 and D3, of the *nao* king. While the data does not show a perfect relationship between the tones and size, if it is treated as data generated at the experimental stage, it shows that the casters were making a concentrated range of attempts to develop bells with a controlled range of tones. Since the data in Table 6 is from only a small number of bells and it covers tones generated over only a little more than two octaves, this hypothesis must end here and still awaits further research.

The explanation above reveals that the use of simple arithmetical processing is insufficient to achieve the desired bell size and two tones and that the process of tuning bells to desired tones required numerous trials and adjustments. Casters working during this period made substantial advancements, as evidenced by Tables 6–8, which demonstrate that they had almost mastered the chromatic scale. While Bagley's theory is predicated on the 10 bells found together in one pit, our hypothesis draws upon a typological comparison of bells that may have been made in the same workshop, as well as the progressive changes in size and two tones outlined in Table 6. It must be reiterated that many bells listed in Table 6 have not yet undergone testing, rendering the pitch distribution in Table 8 incomplete. Despite this, Tables 6 and 8 already showcase the ancient casters and musicians' remarkable comprehension and execution of the chromatic scale. While we suspect that the bell tones may extend into the No. 5 octave, to date, we lack sufficient evidence to confirm this. Nevertheless, the numerous tones that fill the grids of the twelve semitones in the Nos. 3 and 4 octaves present a newer perspective, as they indicate that casters were producing bells spanning over three octaves, all the while tuning them to the chromatic scale.[5]

This process of the realization and attainment of the chromatic scale is likely to have occurred around or before the twelfth century BCE. Pythagoras (ca. 570–ca. 490 BCE) and his disciples in the West, as well as proponents of the Pythagorean tuning theory in China, worked for over around two millennia to calculate the twelve semitones within an octave.[6] They forcibly calculated ratio numbers to satisfy their pursuit of harmony in attaining the chromatic scale. It is now known, however, that there was more than one way to achieve the chromatic scale, one of which involved the creation and assembly of bells (Bagley, 2005). Repeated adjustments and experiments with bell size and the corresponding two tones would have allowed casters and musicians to attain the chromatic scale in a manner distinct from the Pythagorean arithmetic approach, as revealed by the impressive bell ensemble that is likely to have been produced by a single workshop in the Hunan area.

Table 6 List of bells decorated with highly similar *taotie*-patterns and shape in the order of their gradually changing two tones and size.

Name of the bell	A-tone	B-tone	Side length (*Xianchang*) (cm)	Spine length (*Zhongchang*) (cm)	Length and width of the mouth (cm)	Length and width of the top section (cm)	A-tone position thickness (cm)	Weight (kg)	Source and note
Hunan-Ningxiang-Yueshanpu	C3 + 26	D3−24	67	61.5	69.4, 48	55, 38.6	2.8	221	Daxi (Hunan), p. 11, Fig. 1.1.4a
Hunan-Ningxiang-Beifengtan-four-tigers (HnM 39201)	?	?	55.1	50	58.7, 40.2	46.4, 29.3	2.8	109	Daxi (Hunan), p. 14, Fig. 1.1.6a
	[Tonal measurement record cannot be found in the *Daxi* but only in Li Chunyi's book (1996, p. 60, A#3 and A#3). But Li does not indicate his source, so they are not listed here.]								
Hunan-Yueyang-Feijiahe	C#3 + 2	D#3−32	46	43.4	53, 37	43.3, 31.4	1.9	82+	Daxi (Hunan), p. 27, Fig. 1.1.12
Hunan-Ningxiang-Shiguzhai-elephant1 (HnM 39202)	G#3-45	A#3−23	44.5	40	46.2, 35.5	37.6, 25.6	2.6	67.3+	Daxi (Hunan), p. 19, Fig. 1.1.8d
Hunan-Zhuzhou-collect (HnM 22225)	G#3 + 36	B3−46	40	38	44, 30	32.5, 24	2	57.5+	Daxi (Hunan), p. 28, Fig. 1.1.13
Hunan-Ningxiang-Shiguzhai-Feng2 (Ningxiang Wenguansuo zong-0667, Huangcai-93)	A#3-31	B3−49	?	?	53, ?	42, 31	?	102	Author's measurement. Daxi (Hunan), p. 22, Fig. 1.1.9b
Hunan-Ningxiang-Chenjiawan	B3-21	C#4−10	42.8	40.9	46.5, 31.7	40, 28	2.5	86	Daxi (Hunan), p. 9, Fig. 1.1.3a
BPM-Wenwuju-transfer	B3 + 15 (*Daxi* record)	?	41.8	38.6	49.5, 30.8	36.5, 25	2.1	82.5	Daxi (Beijing), p. 32, Fig. 1.4.10
	B3 + 15 (Li Chunyi's record)	?	49.5	41.1	?	?	?	80	Li Chunyi (1996, p. 12)
	Perhaps this BPM bell is one of the bells mentioned in Li Chunyi (1996, p. 60). But their size measurements are very different.								
ShanghaiM-collect-19976	A3 + 26	C4 + 14	41	38	44, 30.8	34.2, 23.8	?	50.2	Daxi (Shanghai), p. 17, Fig. 1.1.13
Jiangxi-Yifeng-Niuxingshan (Jiangxi Yifeng Museum 001)	C4 + 36	D4−9	37.7	33.7	41.3, 27.4	31.8, 20.3	1.8	40+	Daxi (Jiangxi), p. 19, Fig. 1.2.2
Hunan-Ningxiang-Shiguzhai-10 (CsM)	D#4 + 10	F4−11	31.5	28	35.5, 27.7	29, 20.7	1.8	28.5	Author's measurement. Daxi (Hunan), p. 22, Fig. 1.1.10a
Hunan-Wangcheng-Gaochong	D4 + 33	E4 + 28	29	26.8	32.4, 20.5	25.2, 15.1	1.3	18.8+	Daxi (Hunan), p. 7, fig. 1.1.1
Hunan-HnM-collect-22224	E4-48	F4 + 18	28.4	26.2	33.5, 23.7	25.2, 17.8	1.5	22.8+	Daxi (Hunan), p. 28, Fig. 1.1.14
Hunan-Liuyang-Baijia	E4 + 15	F#4−37	28.5	25.6	32.8, 22.3	26, 17	1.3	20.7	Daxi (Hunan), p. 8, Fig. 1.1.2a
Hunan-Xiangxiang-collect (CsM)	E4 + 2	F#4 + 32	29	25.8	27.5, 18	21, 13	?	?	Author's measurement
Hunan-HnM-collect-22226	F4 + 8	G#4 + 20	30.3	27.3	39, 20	27.7, 18.9	1.8	14.9	Daxi (Hunan), p. 29
Hunan-HnM-collect-22227	B4-35	D5-34	28	26.5	31.8, 21.2	25.4, 15.9	2.4	29	Daxi (Hunan), p. 31, Fig. 1.1.16
Tianjin-BiKui-collect (Tianjin Art Museum, 59.3.157)	D5-2	G5 + 35	30.5	28.2	35.5, 24	27.2, 18.6	1.3	25+	Daxi (Shaanxi, Tianjin), p. 198, Fig. 1.1.2

Table 6 (continued)

Name of the bell	A-tone	B-tone	Side length (*Xianchang*) (cm)	Spine length (*Zhongchang*) (cm)	Length and width of the mouth (cm)	Length and width of the top section (cm)	A-tone position thickness (cm)	Weight (kg)	Source and note
Miscellaneous bells (tones yet unknown)									
Hunan-Ningxiang-Beifengtan-animal (HnM 39200)	?	?	57.8	53	63.5, 44.5	48.5, 30.7	?	154	Daxi (Hunan), p. 16, Fig. 1.1.7
BPM-collect-1959 (NMC)	?	?	56.3	49.8	56.3, 40.5	45, 31.5	2.5	?	Daxi (Beijing), p. 34, Fig. 1.4.13
Hunan-Ningxiang-Shiguzhai-Feng1 (Ningxiang Wenguansuo, 0796; Tanheli Bronze Museum)	?	?	48? (handle 22)	?	34, ?	39, 25.5	?	75	Daxi (Hunan), p. 22, Fig. 1.1.9a.
Hunan-Heshan-Sanmutu (Yiyang-Qianjiazhou, Yiyang Museum)	?	?	46.2	41	50, 39	39.7, 28.6	2.4	90	Daxi (Hunan), p. 32, Fig. 1.1.17a
Hunan-Ningxiang-Shiguzhai-animal (Beijing Military Museum, 1959)	?	?	44.7	?	49.7, ?	?	?	67	Daxi (Beijing), p. 34, Fig. 1.4.12
Beijing-Ningxiang-Shiguzhai-tiger2 (Beijing NMC, 1959)	?	?	43	37.2	47.2, 38.1	38, 29.1	2.1	?	Daxi (Beijing), p. 33, Fig. 1.4.11
Hunan-Ningxiang-Shiguzhai-tiger1 (HnM 39206)	?	?	41.4	36.8	46.8, 34.6	36.6, 26.6	2.1-2.3	55.8	Daxi (Hunan), p. 16, 17, Fig. 1.1.8a
Hunan-Ningxiang-Shiguzhai-elephant2 (HnM 39204)	?	?	40.3	35.3	43, 31.5	33, 21.7	3.1	?	Daxi (Hunan), pp. 16, 21, Fig. 1.1.8h
Zhejiang-Yuhang-Xujiafan	?	?	17? (overall height 29, shank 12)	?	20.2, ?	?	?	?	Peng and Sun (2011, p. 77, Fig. 13)
Toronto-White-ROM (ROM, 931.13.165)	?	?	?	?	?	?	?	?	Royal Ontario Museum
Anhui-Lujiang-Nihe (AnhuiM)	?	?	? Overall height 49.5)	?	?	?	?	31.8	Sun (2022, p. 197, Fig. 6.3)
Hunan-Miluo-Hailuoshan (Miluo Shi Wenwu Kaogu Yanjiusuo; Miluo Shi Wenguansuo)	?	?	33.4? (overall height 54, shank 21)	?	37?, 23.4?	?	?	33	Hunan (2020)

Most of the measurement and pitch records are cited from *Zhongguo yinyue wenwu daxi* ("*Daxi*"). The author personally tested the pitches of the 10 Hunan-Ningxiang-Shiguzhai- (CsM), Hunan-Ningxiang-Shiguzhai-Feng2 (Ningxiang Wenguansuo *zong*-0667, Huangcai-93), and Hunan-Xiangxiang-collect bells.
Note: Numerous other bells with similar patterns are not included here because their tonal measurements or their public records are not available.[4]
Note: Entries highlighted in the Simsun font refer to bells of exceptional size, whose two tones can still fit into the list. They merely serve as temporary references and are ready to be abandoned because they do not fit into the progressively diminishing size sequence.

Fig. 5 Six *taotie*-patterned bells. **a** The Hunan-Ningxiang-Beifengtan-four-tigers bell. **b** The Hunan-Ningxiang-Shiguzhai-elephant1 bell. **c** The Hunan-Ningxiang-Shiguzhai-Feng2 bell. **d** The Hunan-Ningxiang-Shiguzhai-tiger1 bell. **e** The Beijing-Ningxiang-Shiguzhai-tiger2 bell. **f** The Hunan-Ningxiang-Shiguzhai-Feng1 bell. Photographs by the author. This figure is not covered by the Creative Commons Attribution 4.0 International License. Reproduced with permission of Kin Sum Li; copyright © Kin Sum Li, all rights reserved.

Table 7 Bells showing the relationship between size and tones.

Name of the bell	A-tone	B-tone	Side length (cm)	Spine length (cm)	Length and width of the mouth (cm)	Length and width of the top section (cm)	Weight (kg)	Source and note	Image
Hunan-Ningxiang-Yueshanpu (the *nao* king)	C3+26	D3−24	67	61.5	69.4, 48	55, 38.6	221	Daxi (Hunan), p. 11, Fig. 1.1.4a.	
Almost half the size of the *nao* king									
Jiangxi-Yifeng-Niuxingshan (Jiangxi Yifeng Museum 001)	C4 + 36	D4−9	37.7	33.7	41.3, 27.4	31.8, 20.3	40+	Daxi (Jiangxi), p. 19, Fig. 1.2.2	NA
Hunan-Wangcheng-Gaochong	D4 + 33	E4 + 28	29	26.8	32.4, 20.5	25.2, 15.1	18.8+	Daxi (Hunan), p. 7, Fig. 1.1.1	
Hunan-Ningxiang-Shiguzhai-10 (CsM)	D#4 + 16	F4−11	32	28.5	35.5, 27.7	29, 20.7	28.5	Author's photograph	

Conclusion

This article has endeavored to address the six questions we initially posed. Through our own experimental work, we have verified that bells were replicated around 500 BCE by using superimposed 3D bell models to develop evidence. While the replication methods used by casters at that time were distinct from ours, they successfully replicated the size and shape of the original bells, which were the crucial factors determining the two tones of the newly cast bells. Our simulation experiments show that the two tones of the replicated bells are very close to those of the original bells, albeit fine-tuning would be required to generate an exact match. This has led to the confirmation of our hypothesis that ancient casters could have replicated bells bearing two tones resembling those of the original ones. These ancient casters discovered the existence of the chromatic scale by seeking bells with tones that could be incorporated into their ensemble, by transposition, and by experimenting with bells of varying sizes. Our simulation experiments show that by changing the size of the starter bells, they could create new bells with tones

Table 8 Pitch distribution of the bells listed in Table 6.

5th			D5-34 D5−2					G5 + 35				
4th	C4 + 14 C4 + 36	C#4−10	D4−9 D4 + 33	D#4 + 10	E4−48 E4 + 28 E4 + 15 E4 + 2	F4−11 F4 + 18 F4 + 8	F#4−37 F#4 + 32		G#4 + 20			B4−35
3rd	C3 + 26	C#3 + 2	D3−24	D#3−32				G#3−45	G#3 + 36	A3 + 26	A#3−23 A#3−31 B3−49	B3−21 B3 + 15 B3−46
Octave Pitch	C	C#	D	D#	E	F	F#	G	G#	A	A#	B

roughly an octave lower or higher. They gradually adjusted the size of the bells, and through this method, rather than by calculation, they managed to produce bells that could generate the 12 semitones within an octave. They could subsequently use this method to cast many more bells in order to form an ensemble tuned to the chromatic scale. Our digital and physical re-creation of ancient Chinese bells has proved to be an innovative and successful approach to generating new knowledge about ancient Chinese bell music and bell casting.

Data availability

All data generated or analyzed during this study are included in this published article.

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Notes

1 This report describes that fragments or damaged areas of the bells were tested (Huang and Li, 2006, pp. 1001–1002). The mouths of eight *niu*-bells from Pits 7 and 16 and two *bo*-bells from Pits 4 and 16 were tested by using a scanning electron microscope (SEM) and energy dispersive spectroscopy (EDS) (Huang and Li, 2006, p. 1026).

2 Audacity Team (2021). Audacity(R): Free Audio Editor and Recorder [Computer application]. Version 3.1.3. Retrieved on August 9, 2021. Audacity® software is copyright © 1999–2021 Audacity Team. Website: https://audacityteam.org/. It is free software distributed under the terms of the GNU General Public License. The name Audacity® is a registered trademark.

3 Coincidentally it is said that Pythagoras discovered the relationship between tones and hammers of different weights when striking a blacksmith's anvil (Hunt, 1992, p. 13; Bagley, 2015, p. 58).

4 For the purpose of distinction, the spirals on the bell handles are referred to as "G pattern," and the spirals on the main register of the bell bodies as "spiral pattern."The upper register of the body of the Hunan-Yueyang-Feijiahe bell and many of its counterparts, along with the G pattern on the handles of numerous bells, show a close relationship with the *taotie* pattern on the main register of the bells. It would have been visually and mentally easy and straightforward for casters to adopt the spirals as another main decorative pattern. This implies that bells adorned with the spiral pattern on their main register have a close association with the *taotie*-patterned bells. A list of the spiral-patterned bells will demonstrate another exciting version of pitch distribution.

5 It will be demonstrated in another article that the ten Ningxiang bells found in the same pit on the Shiguzhai hill were merely a portion of a larger ensemble.

6 Historically, we know that in the 16th century, Prince Zhu Zaiyu (1536–1611 CE) was the first to successfully attain the twelve equally spaced semitones of the chromatic scale (equal-tempered chromatic scale) by seeking the twelfth root of two (Kuttner, 1975; Cho, 2003).

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Author contributions

The author is responsible for the content of this article and bears all responsibilities.

Competing interests

The author declares no competing interests. The author was a member of the Editorial Board of this journal at the time of acceptance for publication. The manuscript was assessed in line with the journal's standard editorial processes.

Ethical statement

Ethical approval was not required as the study did not involve human participants.

Informed consent

This article does not contain any studies with human participants performed by the author.

Additional information

Correspondence and requests for materials should be addressed to Kin Sum Li.