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Influence of the reinsertion algorithm on the performance of the Vertex Entanglement method

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The dismantling of complex networks due to targeted attacks has been extensively studied in network science[1]. A method to assess the centrality of nodes to prioritize the targets of a network attack, namely Network Entanglement (NE), has been introduced in 2021[2], and a variation accounting for functional effects has been later introduced[3]. The method is based on the concept of network density matrix, inspired by quantum information theory[4], which has been further developed in the last years[5–8].

Huang et al.[9] introduce another variation of NE and apply it to a range of empirical networks for the same purpose (as them, we name it VE, in the following). In their article,[9] adopt a different nomenclature to refer to the original implementation of NE, erroneously referring to it as Collective Network Entanglement, hence creating confusion when putting the main contribution of Vertex Entanglement in the context of recent literature. In fact, the core implementation of VE (Vertex Entanglement) can be seen as a direct variation of Network Entanglement, as discussed in this Matter Arising and shown in a direct comparison between NE and VE.

Huang et al.[9] report that such a variation is extensively superior to other computational methods for network dismantling, as well as to NE. We discuss here that most of the performance improvement reported by[9] is due to one/two sources that are independent of the merits of the core methodology: Firstly,[9] choose the wrong model parameters— instead of those recommended— for the methods considered as competitors used to compare the performance of VE. Secondly,[9] adopt reinsertion that boosted the performance of VE and the other methods in the comparison. Reinsertion is a supplemental algorithm that can work on top of any centrality measure, a posteriori. While the adoption of reinsertion is presented by[9] as integral part of the VE entanglement method, we discuss here that the performance improvement that VE presents over NE and other competitors is mostly due to reinsertion only, rather than a major differences between NE and VE.

Theoretical assessment

Like the original and second framing of NE,[9] quantify the effect of structural perturbations on the network's Von Neumann entropy[4].

Let the network be indicated by G. The structurally perturbed network would undergo a transformation: $G \rightarrow G' + \delta G$ where G' is the remainder of the network that remains unchanged and δG encodes the perturbed node and its locality.

In the original NE[2], the structural perturbation δG is removing a node with its emanating links, like a star graph. In the second variation[3], the emanating links are also removed. In[9], the structural perturbation is through the formation of a weight-distributed clique around the node.

Indicating the network entropy by $\mathcal{S}$ and the network entropy after perturbing node i as $\mathcal{S}'_i$, the network entanglement for node i is given by $\mathcal{S}'_i - \mathcal{S}$, for all three variations. Accordingly, the background theory is exactly the same.

Huang et al.[9] do not refer to "Network Entanglement (NE)", in accordance with the existing literature, and they relabeled the original method as "Collective Network Entanglement." Note that NE, in general, quantifies the node importance for information flow, while collective entanglement has been used in the original article to refer to the average NE over all nodes[2].

Overall, VE has a similar definition and purpose of NE: it cannot be considered an independent measure from a theoretical perspective, but a variation of the original NE.

Technical assessment

Based on an analysis of four empirical networks,[9] claim the superiority of VE over a range of other methods.

In Fig. 4,[9] claim that the original NE's performance is incredibly poor, leveraging this result to motivate the need for developing another variation of NE.

However, using the methodology described in ref. 9, the same data they have made available after the publication of their work, their and our own

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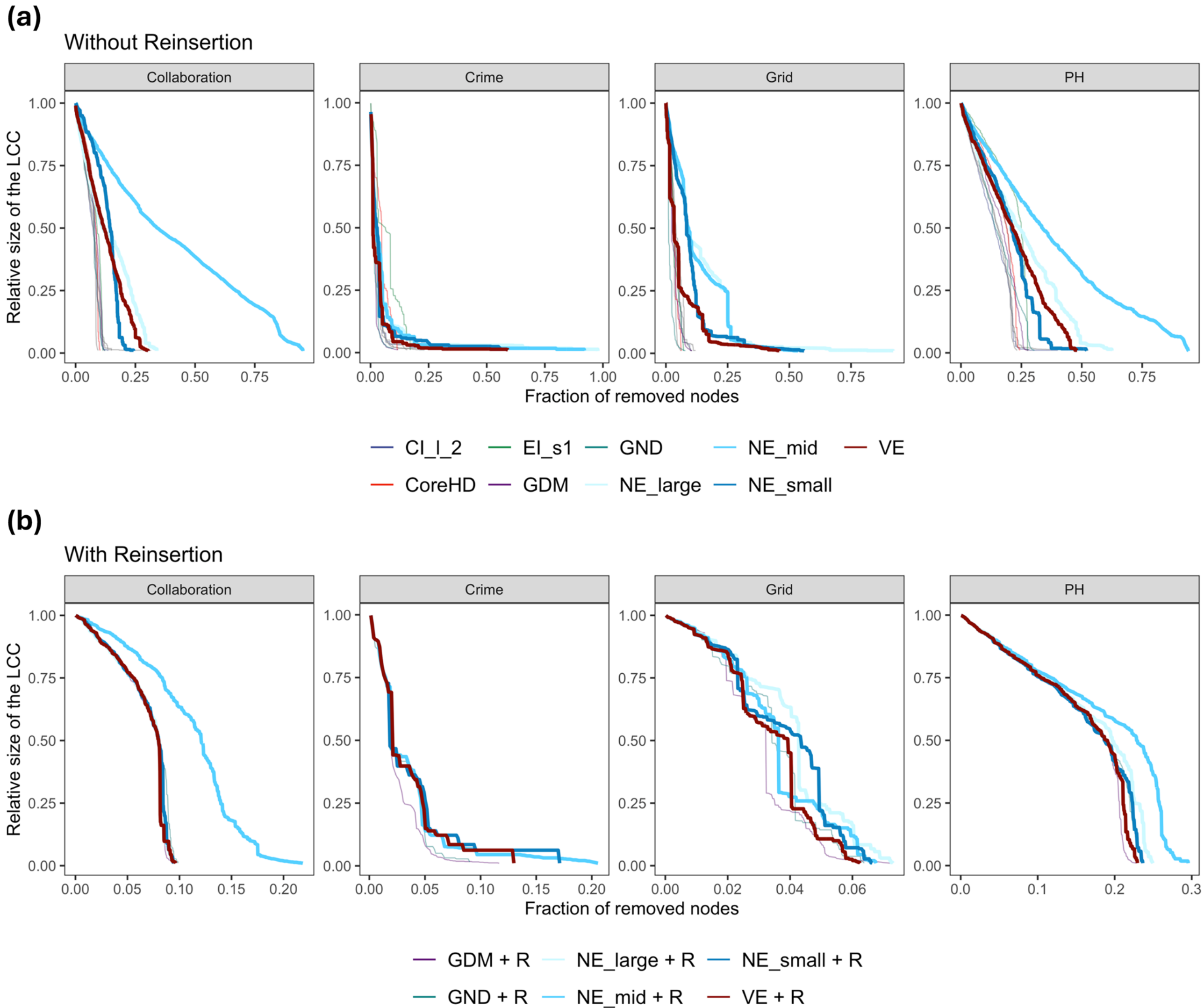

Fig. 1 | Comparison in log scale. The original NE[2] for three choices of the temporal scale parameter (small, middle, and large) is depicted in different shades of blue.[9]'s method (VE) is depicted in red, to facilitate its identification. Other methods (GDM, CoreHD, CI, and EI, see[1] for a thorough review) are also shown with thinner solid lines. The comparison between the methods applied (**a**) without and **b** with reinsertion is shown. Apparently, in their paper,[9] have reported only the worst performing curve for NE, among the three available, thus contradicting the claim that optimal hyper-parameters have been selected for each competitor of VE. Accordingly, VE is far from being the best performing method, while it is perfectly aligned with existing and state-of-the art dismantling techniques.

implementation of VE, we were unable to reproduce their analysis of the four empirical networks.

For our numerical experiments, we use the original implementation of NE[2] and the methods used by ref. 9, while adding a few more methods publicly available from the library accompanying the recent extensive review by Artime et al.[1]— overall including CoreHD, Generalized Network Dismantling (GND), Explosive Immunization (EI), Collective Influence (CI) and Graph Dismantling with Machine Learning (GDM). To show that the performance improvement of VE is due to reinsertion[10], we have considered two sets of experiments: (i) where reinsertion is not applied to any method; (ii) where reinsertion[10] is applied to all methods.

The result of our analysis is shown in Fig. 1a (without reinsertion) and Fig. 1b (with reinsertion). Our analysis shows that the original NE performs as well as[9], as per the aforementioned theoretical expectation, or it is even outperforming[9] in some scenarios. The results confirm that the original NE[2] and[9]'s variation exhibit negligible differences in their assessment of node centrality for network dismantling, in both the experiments (i) and (ii).

To better clarify the points raised above, we add three figures in log-normal scale where the distinction between the methods is more visible (See Fig. 2): (i) all methods without reinsertion, (ii) all methods with reinsertion, and (ii) all methods without reinsertion except for VE that benefits from reinsertion. Here, in addition to the largest connected component, we show the evolution of the second largest connected component. Controlling for the reinsertion algorithm, the minimum of the largest connected component and the maximum of the second largest connected component show no significant difference between VE and NE_small, over the datasets.

Finally, our analysis also show that[9]'s results under-evaluate the performance of other methods, such as GND[1] (Fig. 1b). Overall, the results obtained by the method proposed by[9] are perfectly compatible with state-of-the-art methods, while providing no practical or significant advantage with respect to methods based on network density matrix, such as NE.

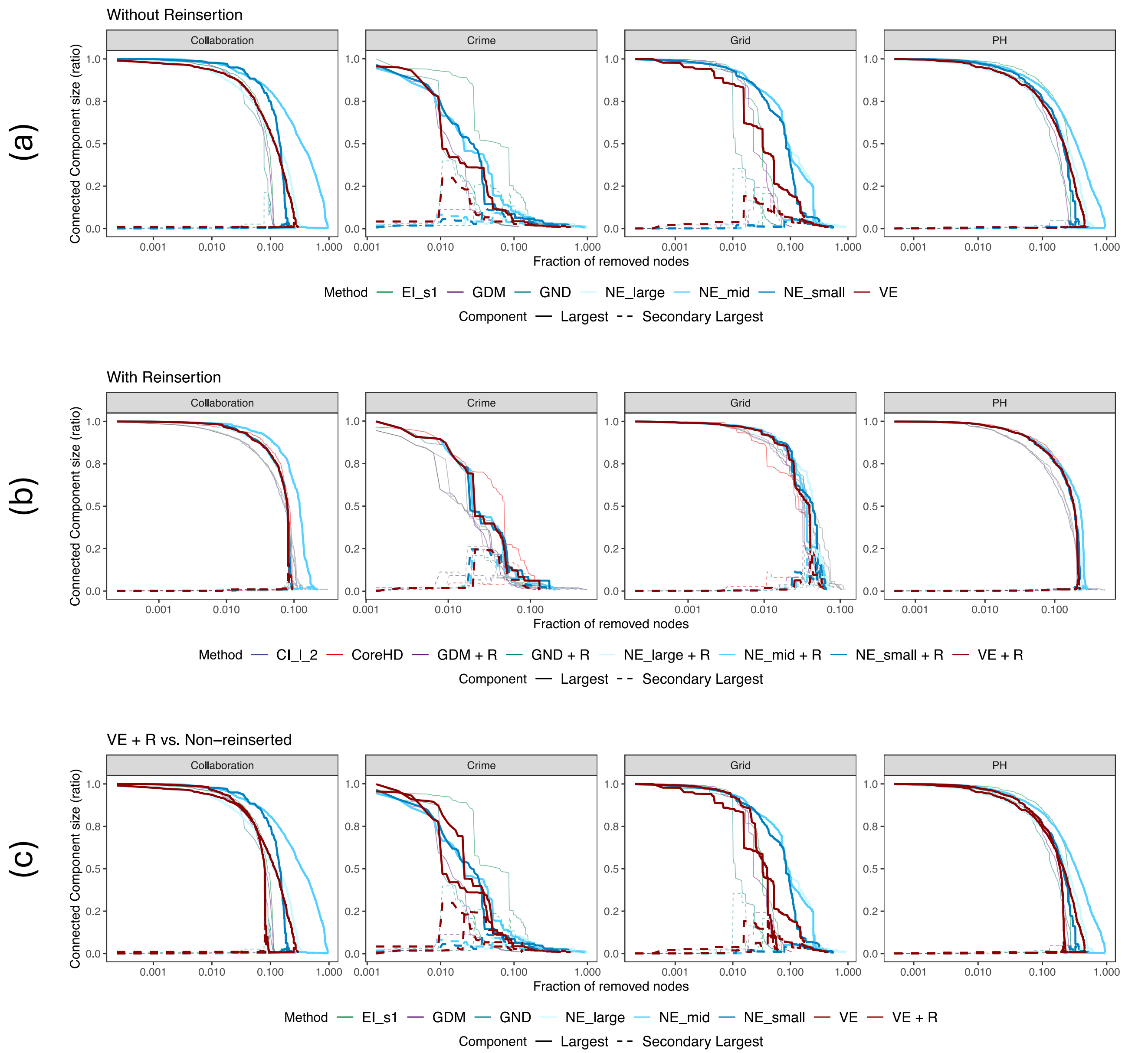

Fig. 2 | Log-normal comparisons of centrality measures. The original NE[2] for three choices of the temporal scale parameter (small, middle, and large),[9]'s method (VE) an other methods (GDM, CoreHD, CI, and EI, see ref. 1 for a thorough review) are shown. The solid lines show the largest connected component while the dashed lines indicate the second largest connected component. **a** When all centrality measures are used without the reinsertion, the minimum of the largest connected component and the maximum of the second largest connected component show no significant difference between VE and NE_small, as previously shown, over the datasets and, interestingly, methods like EI show a better performance than both. **b** Similarly, when all methods benefit from reinsertion, no significant difference between VE and NE_small is observed over the datasets. **c** Considers a special case where only VE benefits from reinsertion and the rest do not, demonstrating that the reinsertion algorithm is the reason why VE slightly outperforms NE_small. Yet, it is interesting to note again that, even in this case, it does not outperform all other methods, especially EI.

Data availability

Datasets analyzed in this matter arising are provided by[9].

Code availability

The code necessary to reproduce our results can be found at: https://github.com/manlius/NE_matter_arising.

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Author contributions

A.G., M.G. and M.d.D. contributed equally to the design, analysis, and writing of this work.

Competing interests

The authors declare no competing interests.

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